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Dispersion relations in κ-noncommutative cosmology

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Abstract. We study noncommutative deformations of the wave equation in curved backgrounds and discuss the modification of the dispersion relations due to noncommutativity combined with curvature of spacetime. Our noncommutative differential geometry approach is based on Drinfeld twist deformation, and can be implemented for any twist and any curved background. We discuss in detail the Jordanian twist — giving κ-Minkowski spacetime in flat space — in the presence of a Friedman-Lemaître-Robertson-Walker (FLRW) cosmological background.

We obtain a new expression for the variation of the speed of light, depending linearly on the ratio E_{ph}/E_{LV} (photon energy/Lorentz violation scale), but also linearly on the cosmological time, the Hubble parameter and inversely proportional to the scale factor.

Keywords: quantum gravity phenomenology, gamma ray bursts theory, quantum cosmology, quantum field theory on curved space

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Contents

1 Introduction

Recent years provided us with experimental confirmations of long existing theoretical models, from the Higgs boson discovery at the LHC to the gravitational waves detection by LIGO. The experiments providing evidence for quantum gravity are yet to be found. The difficulty with finding measurable implications of quantum gravitational models lies within the energy scale of the theory. Nevertheless, the physics at the Planck scale, describing gravitational interactions at the quantum level, might be indirectly investigated in the cosmological and astrophysical context. Approaches to quantum gravity phenomenology have considered the possibility that the Planck scale quantum structure of spacetime induces a modification of the wave dispersion relations including dependence of the velocity of photons on their energy [1–3]. Gamma ray bursts (GRBs) are the brightest electromagnetic events in the universe, they are emitted also at relatively high redshifts ($z \sim 9$) and offer an opportunity for testing dispersion relations associated with a Planck scale breaking of Lorentz symmetry, which even though very small may be amplified by the cosmological distances. Indeed, data analysis related with the time delays of photons arriving from distant GRBs are consistent with the velocity of light having a tiny dependence on its energy, cf. [4, 5] and references therein, and the recent studies [6–8] (see also [9] for slightly more stringent lower limits).

Lorentz invariance violating (LIV) theories generically provide modified dispersion relations; among them there is an interesting class where the Lorentz group (or its realization) is modified, so that a new relativistic symmetry replaces the classical one, these theories go under the name of Deformed (or Doubly) special relativity theories (DSR theories) [10–15] and are more appealing since they preserve a relativity principle; moreover, the deformed

Lorentz symmetry allows for milder deviations from special relativity kinematics. Many of these phenomenological models describe spacetime features and wave equations that are typical of noncommutative spacetimes, the prototypical example being κ-Minkowski spacetime, where coordinates satisfy the relations $x_0 \star x_j - x_j \star x_0 = i\kappa x_j, x_i \star x_j - x_j \star x_i = 0$; here we consider $1/\kappa$ to be related to the Planck length. See [12] for an early relation between DSR and κ-deformed symmetries. Noncommutative geometry, as the generalised notion of spacetime geometry were a minimum length emerges due to spacetime noncommutativity, and were groups of symmetry transformations are deformed in quantum symmetry groups can indeed be helpful in providing models quantifying the effects of quantum gravity without full knowledge of quantum gravity itself, but incorporating at a kinematical level the key dynamical aspect of existence of spacetime uncertainty relations. Notice that these latter are generically inferred from gedanken experiments probing spacetime structure at Planck scale and independently arise in String Theory and as minimal area or volume elements in Loop Quantum Gravity, see e.g. [16].

The interplay of the Planck scale effects on the dispersion relations in quantum (noncommutative) spacetime were investigated in the curved backgrounds of expanding universe in [4, 17–20].

In this paper we use a top-down approach that complements the bottom-up one of phenomenological models. We apply noncommutative differential geometry to derive the propagation of waves in noncommutative cosmology. We study a noncommutative deformation of the wave equation in curved background and we discuss the modification of dispersion relations due to the presence of both noncommutativity and curvature of spacetime. As a first approximation we turn on noncommutativity in the usual (classical) homogeneous and isotropic gravity solution given by FLRW spacetime, and derive the wave equation for massless particles in this context. This is a first step toward a more comprehensive approach that encompasses both the dynamics of light and of gravity in a noncommutative spacetime. We here consider a classical gravity background.

In [21] we have obtained the wave equation on a wide class of noncommutative spaces deriving it from first principles associated with noncommutative differential geometry and the corresponding geometric and physical notion of noncommutative infinitesimal translations, i.e. quantum momenta. We have found that contrary to the generic LIV theories expectations, in flat spacetime no dispersion relations arise (but modified Einstein-Planck relations do arise). The study of dispersion relations in flat spacetime is a first propaedeutical step in the study of dispersion relations in cosmological spacetime. The present paper can be considered as a sequel to [21], while there wave equations for massless fields in flat noncommutative spacetime were considered in the context of a correspondingly deformed (quantum) Poincaré-Weyl symmetry, here we turn on a nontrivial curvature and focus our attention to the metric of FLRW cosmology. We find that in this curved case modified dispersion relations for massless fields do indeed arise, the modification is proportional to E_{ph}/E_{LV} (the travelling photon energy over the Lorentz violation scale related to Planck energy) to the cosmological time, to the Hubble parameter H and to the inverse scale factor $a(t)$. In particular we immediately recover the result of [21]: for flat spacetime, $H = 0$, there is no modified dispersion.

We follow the Drinfeld twist formalism that leads to noncommutative spaces via a star product deformation of their algebra of functions. It also canonically gives a noncommutative differential calculus and wave equations [21]. In section 2 we recall the geometric construction of the wave equation in curved noncommutative spacetime this is given by the twist deformed Laplace-Beltrami operator for arbitrary curved metric in the presence of the noncommutativity. The deformed wave equation proposed here can be constructed for any twist and for any curved background.

In section 3 we consider the specific Drinfed twist called Jordanian twist which has built in a minimal length, it is spatially isotropic and induces a noncommutativity that reduces, in the limit of flat spacetime to κ-Minkowski spacetime with its quantum Poincaré-Weyl symmetry for massless particles; moreover, as shown in [21], in this limit it gives the nonlinear realization of the Lorentz generators of the Doubly special relativity theories [13, 14]. The Jordanian twist is defined by, cf. [22]:

$$\mathcal{F} = \exp\left(-iD \otimes \sigma\right) \quad ; \quad \sigma = \ln\left(1 + \frac{1}{\kappa}P_0\right), \tag{1.1}$$

where $D = -ix^\mu \partial_\mu$ is the dilatation generator and $P_0 = -i\partial_0$ is the time translation generator. For other realizations of Jordanian twist see e.g. [23–25].

In section 4 we specialise the metric to the FLRW one, we obtain the first order correction in the noncommutative deformation parameter κ to the velocity dispersion relations; the result does not depend on the spacetime dimensions, we first treat the simpler 2 dimensional case and then the 4 dimensional one. The time arrival lag of energetic photons with respect to low energetic ones is computed. As a first approximation, comparing with estimates coming from recent analysis of GRB data [7, 8], the net result is the constraint $\hbar\kappa \sim$ few 10^{18} GeV, that is one order of magnitude higher than that obtained via usual modified dispersion relations that depend only on E_{ph}/E_{LV}, and very close to Planck energy (1.22×10^{19} GeV). We then further comment on the general mechanisms leading to modified dispersion relations when both noncommutativity and curvature are turned on. After the conclusions, presented in section 5, we provide two appendices offering more informations on the noncommmutative framework considered and the curved spacetime group velocity computation.

2 Differential geometry on κ-spacetime from twist deformation

2.1 κ-spacetime and Jordanian twist

Noncommutative $\kappa-$Minkowski spacetime is the algebra generated by the coordinates x^μ ($\mu = 0, \dots n-1$) that satisfy the commutation relations

$$x^0 \star x^j - x^j \star x^0 = \frac{i}{\kappa} x^j, \qquad x^i \star x^j - x^j \star x^i = 0, \tag{2.1}$$

where i and j run over the space indices $1, \dots n-1$, while κ is the noncommutativity parameter and we have denoted by $\star$ the corresponding noncommutative product. In this paper we do not fix the metric to be $\eta = diag(-1, 1, \dots 1)$ and hence prefer to simply refer to (2.1) as to the defining relations of κ-spacetime.

A far reaching way to obtain the commutation relations (2.1) is via a Drinfeld twist. In this paper we focus on a specific case which is called Jordanian twist $\mathcal{F}$, given in (1.1). The inverse of this twist is $\mathcal{F}^{-1} = \exp\left(iD \otimes \sigma\right) = \exp\left(x^\mu\partial_\mu \otimes \ln\left(1 - \frac{i}{\kappa}\partial_0\right)\right)$. Due to the algebraic properties of $\mathcal{F}$, that follow from the Lie algebra $[D, P_0] = iP_0$ of the time translation P_0 and of the dilatation D, given smooth functions f, $h \in A = C^\infty(\mathbb{R}^n)$, the $\star$-product defined by

$$f \star h = \mu\left\{\mathcal{F}^{-1}(f \otimes h)\right\} \tag{2.2}$$

i.e., $(f\star h)(x) = \exp\left(x^\mu \frac{\partial}{\partial x^\mu} \otimes \ln(1-\frac{i}{\kappa}\frac{\partial}{\partial y^0})\right) f(x)h(y)\big|_{x=y}$, is associative. In (2.2) μ is the usual pointwise product $\mu(f\otimes h)(x) = f(x)h(x)$. We denote by $A_\star$ the algebra of smooth functions on $\mathbb{R}^n$ where the product is given by the $\star$-product in (2.2). One can expand $\mathcal{F}^{-1}$ in power series of $\frac{1}{\kappa}$, see [21, 22],

$$\mathcal{F}^{-1} = 1\otimes 1 + iD\otimes \frac{1}{\kappa}P_0 + \frac{1}{2}iD(iD-1)\otimes \frac{1}{k^2}P_0^2 + \ldots = \sum_{n=0}^{\infty} \frac{(iD)^{\underline{n}}}{n!} \otimes \left(\frac{1}{\kappa}P_0\right)^n \tag{2.3}$$

where $X^{\underline{n}} = X(X-1)(X-2)\ldots(X-(n-1))$ is the so-called lower factorial. In particular, we easily see that when f and h are coordinate functions, the commutation relations (2.1) hold.

The realization of κ-spacetime via a $\star$-product obtained from a twist $\mathcal{F}$ allows to readily construct a corresponding noncommutative differential geometry.

2.2 Differential calculus

It is useful to introduce the following shortcut notation for the twist:

$$\mathcal{F}^{-1} = \bar{\mathrm{f}}^\alpha \otimes \bar{\mathrm{f}}_\alpha\,,$$

where sum over the index α is understood (it corresponds to the sum in (2.3)). In this notation the deformation of the algebra A into the algebra $A_\star$ of κ-spacetime is given by, cf. (2.2),

$$f\star h = \bar{\mathrm{f}}^\alpha(f)\bar{\mathrm{f}}_\alpha(h)\,. \tag{2.4}$$

Following [26] (see also [27, section 7.7] and [21]) there is a canonical construction in order to obtain the algebra of forms and the exterior differential. Similarly to (2.4), the algebra of exterior forms $\Omega^\bullet = A\oplus\Omega^1\oplus\Omega^2\oplus\ldots$ can be deformed to the algebra $\Omega^\bullet_\star$, which as a vector space is the same as the undeformed $\Omega^\bullet$ but has the new wedge $\star$-product

$$\omega\wedge_\star\omega' = \bar{\mathrm{f}}^\alpha(\omega)\wedge\bar{\mathrm{f}}_\alpha(\omega')\;; \tag{2.5}$$

here the action of $\mathcal{F}^{-1} = \bar{\mathrm{f}}^\alpha\otimes\bar{\mathrm{f}}_\alpha$ on forms is via the Lie derivative $\mathcal{L}$ along the vector fields D and P_0 defining $\mathcal{F}^{-1}$. Explicitly, $D(\omega) = \mathcal{L}_D\omega$, $D^2(\omega) = \mathcal{L}_D\mathcal{L}_D\omega$, and iteratively $D^p(\omega) = \mathcal{L}_D D^{p-1}\omega$, and similarly for P_0 instead of D. In particular, when ω' is a zero form f, then the wedge product is usually omitted and correspondingly the wedge $\star$-product reads $\omega\star f = \bar{\mathrm{f}}^\alpha(\omega)\bar{\mathrm{f}}_\alpha(f)$.

Since the Lie derivative commutes with the exterior derivative the usual (undeformed) exterior derivative satisfies the Leibniz rule $\mathrm{d}(f\star h) = \mathrm{d}f\star h + f\star\mathrm{d}h$, and more in general, for forms of homogeneous degree $\omega\in\Omega^r$,

$$\mathrm{d}(\omega\wedge_\star\omega') = \mathrm{d}\omega\wedge_\star\omega' + (-1)^r\omega\wedge_\star\mathrm{d}\omega'\,. \tag{2.6}$$

We have constructed a differential calculus on the deformed algebra of exterior forms $\Omega^\bullet_\star$.

For later purposes we compute the differential of a function f as

$$\mathrm{d}f = \mathrm{d}x^\mu\partial_\mu f = \mathrm{d}x^\mu\star\partial^{\mathcal{F}}_\mu f\,, \tag{2.7}$$

where in the last expression we have introduced the $\star$-product between one-forms and functions, and deformed the partial derivative ∂_μ into the quantum one defined by

$$\partial^{\mathcal{F}}_\mu f = \frac{1}{1-\frac{i}{\kappa}\partial_0}\partial_\mu\,. \tag{2.8}$$

The proof of (2.7) easily follows recalling the explicit expression of the inverse twist.

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3 Wave equations

In order to formulate dynamical theories we need a metric on spacetime, equivalently, using the language of exterior forms, we need a $*$-Hodge operator. We first see how to canonically define the latter in the noncommmutative setting. Then we present the wave equation on κ-noncommutative spacetime with an arbitrary metric.

3.1 Metric and Hodge star operator

For an n-dimensional manifold with metric g the Hodge $*$-operation is a linear map on the space of exterior forms $* : \Omega^r \to \Omega^{n-r}$. In local coordinates an r-form is given by $\omega = \frac{1}{r!}\omega_{\mu_1\ldots\mu_r}\mathrm{d}x^{\mu_1}\wedge\ldots\mathrm{d}x^{\mu_r}$ and the Hodge $*$-operator reads

$$* \,\omega = \frac{\sqrt{g}}{r!\,(n-r)!}\omega_{\mu_1\ldots\mu_r}\epsilon^{\mu_1\ldots\mu_r}{}_{\nu_{r+1}\ldots\nu_n}\mathrm{d}x^{\nu_{r+1}}\wedge\ldots\mathrm{d}x^{\nu_n} \tag{3.1}$$

where $\sqrt{g}$ is the square root of the absolute value of the determinant of the metric, the completely antisymmetric tensor $\epsilon_{\nu_1\ldots\nu_n}$ is normalized to $\epsilon_{1\ldots n} = 1$ and indices are lowered and raised with the metric g and its inverse. There is a one to one correspondence between metrics and Hodge star operators (indeed $\mathrm{d}x^\mu \wedge *\mathrm{d}x^\nu = g^{\mu\nu} \, *1$).

We define metrics on noncommutative spaces by defining the corresponding Hodge star operators on the $\star$-algebra of exterior forms $\Omega^\bullet_\star$. We first observe that the undeformed Hodge $*$-operator is A-linear: $*(\omega f) = *(\omega)f$, for any form ω and function f (of course, since A is commutative we equivalently have $*(f\omega) = f(*\omega)$). We then require the Hodge $*$-operator $*^{\mathcal{F}}$ on $\Omega^\bullet_\star$ to map r-forms into $(n-r)$-forms, and to be right $A_\star$-linear

$$*^{\mathcal{F}}\,(\omega \star f) = *^{\mathcal{F}}(\omega)\star f \tag{3.2}$$

for any form ω and function f. The quantum Hodge operator $*^{\mathcal{F}}$ is then the deformation of the usual Hodge $*$-operator given by:

$$\begin{aligned} *^{\mathcal{F}} : \Omega^\bullet_\star &\longrightarrow \Omega^\bullet_\star \\ \omega &\longmapsto *^{\mathcal{F}}(\omega) = \bar{\mathrm{f}}^\alpha(*)\,\bar{\mathrm{f}}_\alpha(\omega)\,. \end{aligned} \tag{3.3}$$

In (3.3) the action of $\bar{\mathrm{f}}^\alpha$ on the usual Hodge $*$-operator is the adjoint action. Recall that $\bar{\mathrm{f}}^\alpha$ for each index α is a polynomial in the dilatation $D = -ix^\mu\partial_\mu$; then the action of D on the Hodge $*$-operator is defined by $D(*)(\omega) = D(*\omega) - *(D(\omega))$, i.e., $D(*) = D\circ * - *\circ D = [D, *]$. The action of D^2 is hence $[D, [D, *]]$, and iteratively $D^p(*) = [D, D^{p-1}(*)]$. This defines $\bar{\mathrm{f}}^\alpha(*)$ for any index α. From definition (3.3) it immediately follows that in the commutative limit $\kappa \to \infty$ we have $*^{\mathcal{F}} \to *$. From the general theory of twist deformation of maps, cf. [21], [28, Theorem 4.7], it also follows that for any exterior form ω and function f we have the right $A_\star$-linearity property $*^{\mathcal{F}}(\omega \star f) = *^{\mathcal{F}}(\omega)\star f$. We also notice that definition (3.3) of quantum Hodge operator parallels that used to define quantum vector fields and the physical quantum momenta $P^{\mathcal{F}}_\mu$, as we review in appendix A.

Finally, we remark that there is no a priori relation between the metric structure g we have introduced via the Hodge star operator $*$ and the twist $\mathcal{F}$ determining the noncommutativity of spacetime. We comment more on this at the end of section 4.

3.2 Wave equations in curved κ-spacetime

The wave equation in curved spacetime is governed by the Laplace-Beltrami operator

$$\Box = \delta d + d\delta \ .$$

In the case of even dimensional Lorentzian manifolds the adjoint of the exterior derivative is defined by $\delta = *\mathrm{d}*$. In particular, for a scalar field we have

$$\Box\varphi = *\,\mathrm{d}*\mathrm{d}\,\varphi = \frac{1}{\sqrt{g}}\partial_\nu\left(\sqrt{g}g^{\nu\mu}\partial_\mu\varphi\right) \ . \tag{3.4}$$

This is the wave equation for a scalar field minimally coupled to a background gravitational field.

Wave equations in noncommutative spacetime are defined by just replacing the Hodge $*$-operator with the $*^{\mathcal{F}}$-operator introduced in (3.3). For even dimensional noncommutative spaces with Lorentzian metric:

$$\Box^{\mathcal{F}} = *^{\mathcal{F}}\mathrm{d}\,*^{\mathcal{F}}\,\mathrm{d} + \mathrm{d}\,*^{\mathcal{F}}\,\mathrm{d}\,*^{\mathcal{F}},$$

hence for a scalar field we have

$$\Box^{\mathcal{F}}\varphi = *^{\mathcal{F}}\mathrm{d}\,*^{\mathcal{F}}\,\mathrm{d}\varphi = 0\,. \tag{3.5}$$

Notice that this wave equation can be constructed for any twist and any curved background. Using the specific Jordanian twist we can explicitly compute (3.5). From the definition of Hodge $*$-operator we have

$$*^{\mathcal{F}}\,(\mathrm{d}x^1\wedge_\star\ldots\mathrm{d}x^r) = *(\mathrm{d}x^1\wedge_\star\ldots\mathrm{d}x^r)\,, \tag{3.6}$$

i.e., on these forms it equals the commutative Hodge $*$-operator associated with an arbitrary curved metric. Indeed, since for the Jordanian twist each term $\bar{\mathrm{f}}_\alpha$ in the second leg of the tensor product $\mathcal{F}^{-1} = \bar{\mathrm{f}}^\alpha\otimes\bar{\mathrm{f}}_\alpha$ is a power of P_0 and $P_0(\mathrm{d}x^\mu) = 0$, it is immediate to see that

$$\mathrm{d}x^1\wedge_\star\ldots\mathrm{d}x^r = \mathrm{d}x^1\wedge\ldots\mathrm{d}x^r$$

and $P_0(\mathrm{d}x^1\wedge_\star\ldots\mathrm{d}x^r) = 0$, hence $\bar{\mathrm{f}}^\alpha\otimes\bar{\mathrm{f}}_\alpha(\mathrm{d}x^1\wedge_\star\ldots\mathrm{d}x^r) = 1\otimes\mathrm{d}x^1\wedge_\star\ldots\mathrm{d}x^r$, and therefore

$$*^{\mathcal{F}}(\mathrm{d}x^1\wedge_\star\ldots\mathrm{d}x^r) = \bar{\mathrm{f}}^\alpha(*)\bar{\mathrm{f}}_\alpha(\mathrm{d}x^1\wedge_\star\ldots\mathrm{d}x^r) = *(\mathrm{d}x^1\wedge_\star\ldots\mathrm{d}x^r)\,.$$

Because of right $\star$-linearity, $*^{\mathcal{F}}(\mathrm{d}x^1\wedge_\star\ldots\mathrm{d}x^r\star f) = *(\mathrm{d}x^1\wedge_\star\ldots\mathrm{d}x^r)\star f$, recalling (2.7) it follows that

$$*^{\mathcal{F}}\,(\mathrm{d}\varphi) = *^{\mathcal{F}}(\mathrm{d}x^\mu\star\partial^{\mathcal{F}}_\mu\varphi) = *^{\mathcal{F}}(\mathrm{d}x^\mu)\star\partial^{\mathcal{F}}_\mu\varphi = *(\mathrm{d}x^\mu)\star\frac{1}{1-\frac{i}{\kappa}\partial_0}\partial_\mu\varphi\,, \tag{3.7}$$

henceforth

$$\begin{aligned}
\mathrm{d}\left(*^{\mathcal{F}}(\mathrm{d}\varphi)\right) &= \mathrm{d}(*(\mathrm{d}x^\mu))\star\frac{1}{1-\frac{i}{\kappa}\partial_0}\partial_\mu\varphi+(-1)^{n-1}*(\mathrm{d}x^\mu)\wedge_\star\mathrm{d}\left(\frac{1}{1-\frac{i}{\kappa}\partial_0}\partial_\mu\varphi\right)\\
&=\left(\frac{1}{(n-1)!}\partial_\rho(\sqrt{g}g^{\mu\nu})\varepsilon_{\nu\nu_1\ldots\nu_{n-1}}\mathrm{d}x^\rho\wedge\mathrm{d}x^{\nu_1}\ldots\wedge\mathrm{d}x^{\nu_{n-1}}\right)\star\frac{1}{1-\frac{i}{\kappa}\partial_0}\partial_\mu\varphi\\
&\quad+(-1)^{n-1}\left(\frac{1}{(n-1)!}\sqrt{g}g^{\mu\nu}\varepsilon_{\nu\nu_1\ldots\nu_{n-1}}\mathrm{d}x^{\nu_1}\ldots\wedge\mathrm{d}x^{\nu_{n-1}}\right)\wedge_\star\frac{1}{1-\frac{i}{\kappa}\partial_0}\partial_\rho\partial_\mu\varphi\,\mathrm{d}x^\rho\\
&=\left(\partial_\nu(\sqrt{g}g^{\mu\nu})\star\left(1-\frac{i}{\kappa}\partial_0\right)^{n-1}\partial_\mu\varphi+\sqrt{g}g^{\mu\nu}\star\left(1-\frac{i}{\kappa}\partial_0\right)^{n-2}\partial_\nu\partial_\mu\varphi\right)\star(\mathrm{d}x^1\wedge\mathrm{d}x^2\ldots\mathrm{d}x^n).
\end{aligned} \tag{3.8}$$

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Here in the last passage we have used that

$$\sqrt{g}g^{\mu\nu}\varepsilon_{\nu\nu_1\ldots\nu_{n-1}}\mathrm{d}x^\rho\wedge\mathrm{d}x^{\nu_1}\ldots\wedge\mathrm{d}x^{\nu_{n-1}}=\sqrt{g}g^{\mu\nu}\star\varepsilon_{\nu\nu_1\ldots\nu_{n-1}}\mathrm{d}x^\rho\wedge_\star\mathrm{d}x^{\nu_1}\ldots\wedge_\star\mathrm{d}x^{\nu_{n-1}},$$

then we have moved $\frac{1}{1-\frac{i}{\kappa}\partial_0}\partial_\mu\varphi$ to the left, and similarly for the other addend. Finally we rewrote $\mathrm{d}x^\rho\wedge\mathrm{d}x^{\nu_1}\ldots\wedge\mathrm{d}x^{\nu_{n-1}}=\varepsilon_{\rho\nu_1\ldots\nu_{n-1}}\mathrm{d}x^1\wedge\mathrm{d}x^2\ldots\wedge\mathrm{d}x^n$ and performed the usual epsilon tensor contractions.

From the invertibility of the Hodge $*^{\mathcal{F}}$ operator, (or directly moving in (3.8) the n-form $\mathrm{d}x^1\wedge\mathrm{d}x^2\ldots\mathrm{d}x^n$ to the left and then applying the Hodge $*^{\mathcal{F}}$ operator) we see that the n-dimensional wave equation $\Box^{\mathcal{F}}\varphi=*^{\mathcal{F}}\mathrm{d}*^{\mathcal{F}}\mathrm{d}\varphi=0$ in the presence of κ-noncommutative spacetime and with arbitrary curved metric is equivalent to $\mathrm{d}*^{\mathcal{F}}\mathrm{d}\varphi=0$ and to

$$\sqrt{g}g^{\mu\nu}\star\left(1-\frac{i}{\kappa}\partial_0\right)^{n-2}\partial_\nu\partial_\mu\varphi+\partial_\nu(\sqrt{g}g^{\mu\nu})\star\left(1-\frac{i}{\kappa}\partial_0\right)^{n-1}\partial_\mu\varphi=0\,. \tag{3.9}$$

We conclude this section observing that if we consider the usual Minkowski metric $g^{\mu\nu}=\eta^{\mu\nu}=diag(-1,1\ldots1)$ then (3.9) gives the wave equation in κ-Minkowski spacetime studied in [21]. This wave equation is equivalent to the wave equation constructed from the quadratic quantum Casimir operator $P^{\mu\mathcal{F}}P^{\mathcal{F}}_\mu$, see appendix A for the definition of the quantum momenta $P^{\mathcal{F}}_\mu$; it is also equivalent to the wave equation constructed from the canonical twist deformation $\Box\mapsto\Box^{\mathcal{F}}=\mathcal{D}(\Box)=\bar{\mathrm{f}}^\alpha(\Box)\bar{\mathrm{f}}_\alpha$ of the d'Alembert operator of usual Minkowski spacetime, see [21]. For massless fields this wave equation is also equivalent to the usual wave equation in commutative Minkowski spacetime; its physical interpretation leads to unmodified dispersion relations and to modified Einstein-Planck relation between energy and frequency [21].

4 Dispersion relations in κ-noncommutative cosmology

We study a theoretical model based on first principles leading to massless fields dispersion relations and focus on the case of gamma ray bursts and the time delay between high energy and low energy photons. The natural setting is that of a distant source that emits a gamma ray burst, emitter and observer in first approximation do not have peculiar velocities and can be considered at rest with respect to the usual comoving coordinate system (t,x^i) of FLRW cosmology, where $\mathrm{d}s^2=-\mathrm{d}t^2+a(t)^2\mathrm{d}x^2$.

In this section we study a model in 2 and in 4 dimensions and guided by the results obtained we extrapolate general considerations. We show that when the (t,x^i) coordinates become noncommutative the speed of propagation of a massless scalar field — i.e., neglecting spin, the speed of light — depends on the energy, on the cosmic time and on the expansion rate; hence nontrivial dispersion relations occur. These are due to the interaction between spacetime curvature and its noncommutativity, indeed, if the background is curved and commutative, or if it is flat and noncommutative, as we have shown in [21], there is no dispersion relation.

We implement noncommutativity of FLRW spacetime $\mathrm{d}s^2=-\mathrm{d}t^2+a(t)^2\mathrm{d}x^2$ by prescribing the commutation relations of the comoving coordinates (t,x^i). Let's recall that t is the time measured by a clock in position x^i (a comoving observer in x^i), so that $\mathrm{d}t$ captures a local property of spacetime in region (t,x^i), while x^i is the rescaled distance so that velocity of light is $\mathrm{d}|x|/\mathrm{d}t=\frac{1}{a(t)}$. Changing perspective between metric and noncommutative structures, we can say that we implement curvature in κ-spacetime by identifying the coordinates of κ-spacetime as comoving coordinates.

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4.1 Scalar field in 2 dimensions

We first specialize (for simplicity) to the 2 dimensional FLRW spacetime with metric $g^{\mu\nu} = (-1, 1/a(t)^2)$, the wave equation in (3.9) then reads

$$a \star \partial_0^2 \varphi - a^{-1} \star \partial_x^2 \varphi + (\partial_0 a) \star \left(1 - \frac{i}{\kappa}\partial_0\right) \partial_0 \varphi = 0\,. \tag{4.1}$$

In order to solve this equation and study the corresponding dispersion relations we proceed in analogy with the commutative case, that is propaedeutical and reviewed in appendix B. We set $\varphi_k = \lambda(t) \star e^{-ikx} = \lambda(t)e^{-ikx}$ so that $\partial_x^2\varphi = -k^2\varphi$ and the equation simplifies to:

$$a \star \partial_0^2 \lambda + (\partial_0 a) \star \left(1 - \frac{i}{\kappa}\partial_0\right) \partial_0 \lambda + k^2 a^{-1} \star \lambda = 0\,. \tag{4.2}$$

We study this equation at the first order in the noncommutativity parameter $\frac{1}{\kappa}$; its expansion, using (2.3), explicitly reads:

$$a\, \partial_0^2 \lambda + \partial_0 a \left(1 - \frac{i}{\kappa}\partial_0\right) \partial_0 \lambda + k^2 a^{-1}\lambda - \frac{i}{\kappa} t \left(\partial_0 a\, \partial_0^3 \lambda + \partial_0^2 a\, \partial_0^2 \lambda + k^2 \partial_0 a^{-1} \partial_0 \lambda\right) = 0\,. \tag{4.3}$$

Since this equation is a deformation of the wave equation in commutative FLRW spacetime, following that case we change the time coordinate into conformal time η. We use the relations $\partial_0 = \frac{1}{a}\partial_\eta$; $\partial_0^2 = -\frac{a'}{a^3}\partial_\eta + \frac{1}{a^2}\partial_\eta^2$; $\partial_0^3 = \frac{1}{a}\left(\frac{3(a')^2}{a^4} - \frac{a''}{a^3}\right)\partial_\eta - \frac{3a'}{a^4}\partial_\eta^2 + \frac{1}{a^3}\partial_\eta^3$ and introduce the simplifying notation $s = \ln a$; $s' = \frac{a'}{a}$; $\frac{a''}{a} = s'' + (s')^2$, where the prime denotes the derivative $\partial/\partial\eta$. This results in:

$$\frac{1}{a}\lambda'' + \frac{k^2}{a}\lambda - \frac{i}{\kappa a^3} t(\eta) \left(\left(2(s')^3 - 2s's'' - k^2 s'\right)\lambda' + \left(s'' - 3\,(s')^2\right)\lambda'' + s'\lambda'''\right) - \frac{i}{\kappa a^2} s' \left(-s'\lambda' + \lambda''\right) = 0\,. \tag{4.4}$$

We substitute

$$\lambda = \exp\left(i\omega\eta + \frac{i}{\kappa}F\right) \tag{4.5}$$

and at zero-th order in the deformation parameter we obtain $(\omega^2 - k^2)\,\lambda = 0$, that we solve choosing $\omega = k$ (corresponding to a forward travelling wave), while at the first order in $\frac{1}{\kappa}$, using the zero-th order solution $\omega = k$, we obtain the differential equation for $F(\eta)$:

$$F'' + 2ikF' = \frac{ikt(\eta)}{a^2}\left(2\,(s')^3 - 2s's'' - 2k^2 s' + ik(s'' - 3\,(s')^2)\right) - \frac{ik}{a} s' \left(s' - ik\right)\,. \tag{4.6}$$

We aim at the expression of the group velocity for the wave

$$\varphi_k(x,t) = \lambda(t) \star e^{-ikx} = \lambda(t)e^{-ikx} = \exp\left(ik\eta + \frac{i}{\kappa}F\right) e^{-ikx} = e^{i(f_k(t) - kx)} \tag{4.7}$$

where the last equality defines $f_k(t) = \left(k\eta + \frac{1}{\kappa}F\right)(t)$. Recalling the group velocity expression $v_g = \frac{\partial x}{\partial t} = \frac{\partial}{\partial k}\frac{\partial f_k(t)}{\partial t}$, cf. (B.5) in appendix B, we see that we need to compute $\dot{F} = \partial F/\partial t$.

This is easily obtained from the differential equation (4.6) in the physical regime we are interested in: cosmic time related to large scale structure formation, and high frequency waves. There are three frequency parameters in the differential equation (4.6): $\omega = k$, t^{-1} and the Hubble parameter H; we obviously have $\omega \gg t^{-1}$ for the present cosmic time as well as the cosmic time of emission of the travelling γ-ray, typically at redshift $z = a^{-1} - 1$ below $z = 10$. Similarly $\omega \gg H \sim t^{-1}$.

In this regime (4.6) simplifies to $2ikF' = -\frac{2ik^3ts'}{a^2}$, i.e.[1]

$$\dot{F} = -\frac{k^2 t\dot{a}}{a^3}\,. \tag{4.8}$$

Hence the group velocity, at the first order in the $\frac{1}{\kappa}$ deformation, results

$$v_g = \frac{\partial x}{\partial t} = \frac{\partial}{\partial k}\frac{\partial f_k(t)}{\partial t} = \frac{1}{a} + \frac{1}{\kappa}\frac{\partial \dot{F}}{\partial k} = \frac{1}{a}\left(1 - \frac{2}{\kappa}\frac{kt\dot{a}}{a^2}\right) = \frac{1}{a}\left(1 - \frac{2}{\kappa}\frac{\omega t\dot{a}}{a^2}\right). \tag{4.9}$$

In the last equality we have expressed the group velocity in terms of the frequency ω; this last expression is easily seen to hold also if at zero-th order in $\frac{1}{\kappa}$ we consider the backward travelling wave solution $\omega = -k$. Taking into account the $\frac{1}{a}$ factor due to the comoving coordinates and inserting the flat spacetime speed of light c we see that κ-spacetime noncommutativity in the presence of a FLRW metric leads to a physical velocity of massless scalar 2d particles $v_{ph} = v_g a$ given by

$$v_{ph} = c\left(1 - \frac{2}{\kappa}\frac{\omega t\dot{a}}{a^2}\right). \tag{4.10}$$

4.2 Scalar field in 4 dimensions

In the 2 dimensional commutative case the scalar field equation $\Box\varphi = *\,\mathrm{d} * \mathrm{d}\,\varphi = 0$ describes a minimal coupling to the curvature (there is no term $R\varphi$ proportional to the scalar curvature R); this is also a conformal coupling (if φ is a solution with metric $g_{\mu\nu}$ it is also a solution with conformally rescaled metric $\Omega(t,x)g_{\mu\nu}$). In the 4d case, as in the 2d case, and since electromagnetism couples conformally to the metric, we consider a scalar field conformally coupled to gravity, hence we add to the 4d noncommutative scalar field equation (3.9) a term that is proportional to the commutative scalar curvature,

$$\sqrt{g}g^{\mu\nu} \star \left(1 - \frac{i}{\kappa}\partial_0\right)^2 \partial_\nu\partial_\mu\varphi + \partial_\nu(\sqrt{g}g^{\mu\nu}) \star \left(1 - \frac{i}{\kappa}\partial_0\right)^3 \partial_\mu\varphi - \frac{1}{6}(\sqrt{g}R) \star \varphi = 0\,. \tag{4.11}$$

A rigorous derivation of the last addend requires a study of noncommutative gravity coupled to noncommutative scalar fields. The expression chosen reduces to the usual conformal coupling in the commutative limit (cf. e.g. [31]) and allows to write the wave equation as an operator $\star$-acting on φ. Other possibilities, like $-\frac{1}{6}\sqrt{g}\star R\star\varphi$, can be considered, but we will see that they do not affect the scalar field dispersion relations in the regime we are interested in.

[1] An easy consistency check is to compute F'' from (4.8) and explicitly see that it is negligible with respect to kF'. It is also instructive to consider, as an example, a single fluid evolutionary scenario with the normalized scale factor $a(t) = (t/t_0)^\alpha, \alpha = \frac{2}{3(1+w)}$, where w is a barotropic factor ($w = \frac{1}{3}, \alpha = 1/2$ — radiation, $w = 0, \alpha = 2/3$ — dust/Dark Matter dominated era) and t_0 denotes the age of our Universe. Due to the normalization condition the scale factor is of order 1, i.e. $a(t) \sim 1$, and $t_0 \sim H_0^{-1} \sim 10^{18}$ s. Thus $\frac{t}{a^2} \sim H_0^{-1}$. Similarly, in the conformal time, $a(\eta) = (\eta/\eta_0)^\beta, \beta = \frac{2}{1+3w}$ and the function s' is of the order of H_0 while s'' is at most few orders of magnitude bigger than H_0^2 (for the low redshifts z of the gamma ray bursts events of interest). Now, because of the extremely small value of the Hubble constant H_0 with respect to the frequency $\omega = k$, the only term which survives this approximation on the r.h.s. of (4.6) is $\frac{k^3ts'}{a^2} \sim k^2$, while, on the l.h.s., it is $2ikF'$.

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Similarly to the 2d case, we set $\varphi = \lambda e^{-i(k_x x + k_y y + k_z z)} = \lambda \star e^{-i(k_x x+k_y y+k_z z)}$, insert the values of $g^{\mu\nu} = \mathrm{diag}(-1, a^2, a^2, a^2)$ and $\sqrt{g} = a^3$ in (4.11) and obtain

$$a^3 \star \left(1 - \frac{i}{\kappa}\partial_0\right)^2 \partial_0^2 \lambda + k^2 a \star \left(1 - \frac{i}{\kappa}\partial_0\right)^2 \lambda + (\partial_0 a^3) \star \left(1 - \frac{i}{\kappa}\partial_0\right)^3 \partial_0 \lambda + \frac{1}{6}(a^3 R) \star \lambda = 0, \tag{4.12}$$

where $k = \sqrt{k_x^2 + k_y^2 + k_z^2}$. We next expand the powers of $\left(1 - \frac{i}{\kappa}\partial_0\right)$ and the $\star$-product at leading order in $\frac{1}{\kappa}$,

$$\begin{aligned} a^3\partial_0^2\lambda + k^2 a\lambda + (\partial_0 a^3)\partial_0\lambda + \frac{1}{6}a^3 R\lambda - \frac{i}{\kappa}\left(2a^3\partial_0^3\lambda + 2k^2 a\partial_0\lambda + 3(\partial_0 a^3)\partial_0^2\lambda\right) + \\ - \frac{it}{\kappa}\left((\partial_0 a^3)\partial_0^3\lambda + k^2(\partial_0 a)\partial_0\lambda + (\partial_0^2 a^3)\partial_0^2\lambda + \frac{1}{6}\partial_0(a^3 R)\partial_0\lambda\right) = 0\,. \end{aligned} \tag{4.13}$$

Setting

$$\lambda = a^{-1} e^{i\omega\eta + \frac{i}{\kappa}F(t)} = a^{-1}e^{ik\eta}\left(1 + \frac{i}{\kappa}F(t)\right) + \mathcal{O}\left(\frac{1}{\kappa^2}\right),$$

where η is the conformal time coordinate, we observe that, since $a^{-1}e^{i\omega\eta}$ with $\omega = k$ is the classical solution to the 4d scalar field conformally coupled to FLRW cosmology (cf. appendix B), the sum of the first four addends vanishes as long as $\partial_0 = \frac{1}{a}\partial_\eta$ does not hit $F(t)$. We are then left with terms at leading order in the noncommutativity deformation $\frac{1}{\kappa}$ and here, as in the 2d case, using the physical regime $\omega = k \gg H \sim t^{-1}$, we consider only the leading order in k, given by ∂_0 hitting $e^{ik\eta}$, for example the first addend just gives $\frac{i}{\kappa}2a^3\partial_0(a^{-1}e^{ik\eta})\partial_0 F$. Equation (4.13) thus becomes

$$\begin{aligned} 2a^3\partial_0\left(a^{-1}e^{ik\eta}\right)\partial_0 F - 2a^3\left(\frac{1}{a}\partial_\eta\right)^3(a^{-1}e^{ik\eta}) - 2k^2\partial_\eta(a^{-1}e^{ik\eta}) + \\ - t\left((\partial_0 a^3)\left(\frac{1}{a}\partial_\eta\right)^3(a^{-1}e^{ik\eta}) + k^2(\partial_0 a)\frac{1}{a}\partial_\eta(a^{-1}e^{ik\eta})\right) = 0\,. \end{aligned}$$

Again, the time derivatives give non-negligible terms only if they hit $e^{ik\eta}$ rather than the conformal factor a; it follows that the second and third addend are proportional to $a^{-1}k^3$ and cancel out, so that the equation simplifies to $2aik\dot{F} - t(-2\frac{\dot{a}}{a^2}ik^3) = 0$, hence we obtain

$$\dot{F} = -\frac{k^2 t\dot{a}}{a^3}\,. \tag{4.14}$$

This is the same equation as the 2d case one (4.8). Hence from the expression of the group velocity of a wave packet in 4d, $\boldsymbol{v}_g = \frac{\partial \boldsymbol{x}}{\partial t} = \frac{\partial}{\partial t}\nabla(ik\eta + \frac{i}{\kappa}F)$, (cf. appendix B) the modulus v_g of the group velocity of the 4d scalar field at first order in the $\frac{1}{\kappa}$ deformation is given in (4.9),

$$v_g = \frac{1}{a}\left(1 - \frac{2}{\kappa}\frac{\omega t\dot{a}}{a^2}\right). \tag{4.15}$$

Taking into account the $\frac{1}{a}$ factor due to the comoving coordinates we arrive at a physical velocity of massless scalar 4d particles $v_{ph} = v_g a$ given by

$$v_{ph} = c\left(1 - \frac{2}{\kappa}\frac{\omega t\dot{a}}{a^2}\right) \tag{4.16}$$

as in the 2d case (cf. (4.10)). As usual we define the energy where classical Lorentz violation (or better in our case Lorentz deformation) is manifested $E_{LV} := |\kappa|\hbar$. We extrapolate the 2d and 4d results of scalar fields conformally coupled to curved spacetime to spin one fields and hence assume that also photons in κ-noncommutative FLRW spacetime have the same dispersion relations (4.16). The variation of the physical speed of light v_{ph} with respect to the usual one c (of photons in flat spacetime, or of low energetic photons) is then given by

$$|1 - v_{ph}/c| \;\sim\; \frac{E_{ph}}{E_{LV}}\frac{2t\dot{a}}{a^2} \quad . \tag{4.17}$$

We finally come to the other scalar curvature couplings in (4.11), like $-\frac{1}{6}\sqrt{g}\star R\star\varphi$, $-\frac{1}{6}\sqrt{g}R\left(1-\frac{i}{\kappa}\partial_0\right)\star\varphi$, or $-\frac{1}{6}\sqrt{g}R\left(1-\frac{i}{\kappa}\partial_0\right)^2\star\varphi$. In all these cases the first order in $\frac{1}{\kappa}$ of these terms, in the regime $\omega = k \gg H \sim t^{-1}$ is always negligible with respect to those proportional to $k\dot{F}$ or k^3, therefore the group velocity and dispersion relations results (4.15)–(4.17) are independent from the ambiguities in the coupling to the scalar curvature.

4.3 Physical considerations

We begin listing a few comments:

- The combined effects of noncommutativity and gravity affect the velocity of light by a term linearly dependent on the frequency ω, the cosmic time t, the Hubble parameter $H = \dot{a}/a$ and inversely proportional to the scale factor. We have $v_{ph} < c$ for $\frac{1}{\kappa}$ a positive time (as it is usually considered, and in an expansion phase of the universe $\dot{a} > 0$). If on the other hand we consider the commutation relations $t\star x - x\star t = -|\frac{1}{\kappa}|ix$ then $v_{ph} > c$ and the opposite conclusions hold. In flat spacetime ($\dot{a} = 0$) as well as in commutative spacetime ($\kappa \to \infty$) there are no modified dispersion relations.

- This result offers an explicit cosmological correction to the usually considered models, which assume, as the leading power for the correction to the light speed, the expression $v_{ph} \sim c\left(1 - \frac{E_{ph}}{E_{LV}}\right)$ [2]. It is actually interesting to estimate the fractional variation (4.17) of the speed of light, that in terms of the redshift reads

$$\delta v/c \equiv |1 - v_{ph}/c| \sim 2(1+z)tH\, E_{ph}/E_{LV}\, . \tag{4.18}$$

  For example, the most energetic photons detected by Fermi-LAT from GRB 080916C have measured energy $E_{ph} = 13.2\,\mathrm{GeV}$ (cf. e.g. [8]). Assuming that κ is the Planck mass, the Lorentz deformation scale corresponds to the Planck energy scale $E_{LV} = 1.22\times 10^{19}\,\mathrm{GeV}$ and we obtain the fractional decrease in velocity $\delta v/c = 2.15\times 10^{-18}$ for these energetic photons at time of detection ($z = 0$, $tH = t_0H_0 = 13.29\,\mathrm{Gyr} \times 73\,\frac{\mathrm{km/s}}{\mathrm{Mpc}}$, according to ΛCDM model). These same photons at time of emission, i.e., at redshift $z = 4.35$, had energy $(1+z)\times 13.2\,\mathrm{GeV}$ and fractional decrease in velocity $\delta v/c = 41.6\times 10^{-18}$.

- We can also study the time lag Δt between the arrival of a low energetic and a high energetic photon emitted simultaneously during a gamma ray burst. Following [17] we observe that the comoving distance between the gamma ray burst and the observer is the same for both photons; for the high energy photon it reads $\int_{t_{em}}^{t_0+\Delta t} v_g\, dt$, where v_g is given by (4.9), while for the low energy one it reduces to $\int_{t_{em}}^{t_0} \frac{c}{a}\, dt$. Equating these distances, and considering only first order corrections we obtain that the time delay Δt is given by

$$\Delta t = \frac{2E_{ph}}{E_{LV}} \int_{t_{em}}^{t_0} \frac{t\dot{a}}{a^3} dt = \frac{2E_{ph}}{E_{LV}} \int_0^z t\,(1+z')dz'\,. \tag{4.19}$$

For the range of redshifts we are interested into (up to $z \sim 10$) we can use the analytic solution $a(t) = (1+z)^{-1} = (\frac{\Omega_m}{\Omega_\Lambda})^{1/3} \sinh^{2/3}(t/t_\Lambda)$, $t_\Lambda = \frac{2}{3H_0\sqrt{\Omega_\Lambda}}$ and obtain the time lag

$$\Delta t = 2\frac{E_{ph}}{E_{LV}} t_\Lambda \int_0^z \operatorname{arcsinh}\sqrt{\frac{\Omega_\Lambda}{\Omega_m}(1+z')^{-3}}\,(1+z')dz'\,. \tag{4.20}$$

This differs from $\Delta' t = \frac{E_{ph}}{E_{LV}} \frac{1}{H_0} \int_0^z \frac{(1+z')dz'}{\sqrt{\Omega_m(1+z)^3+\Omega_\Lambda}}$, which is the typical time lag considered in the literature for the correction to the dispersion relations induced by a linear Lorentz invariance violation [17]. Our model gives a time lag that is ~ 3 times this latter (we use $\Omega_m = 0.27$ for the matter density parameter and $\Omega_\Lambda = 0.73$ for the cosmological constant density).

- The proposed expression for the group velocity v_g has been derived from a 2 as well as from a 4 dimensional wave equation and well describes the potentialities of the noncommutative theory developed from first principles: because of specific quantitative predictions like (4.17)–(4.20) it can be used to constrain the noncommutativity parameter κ and test the model. For example the GRB data analyses in [7, 8] estimate a Lorentz violation energy at the scales 3.6×10^{17} GeV and $\sim 10^{18}$ GeV respectively. Taking into account the factor ~ 3 due to (4.20), we obtain $\hbar\kappa \sim$ few 10^{18} GeV, i.e., very close to Planck Energy.

  It would be interesting to further investigate the phenomenological implications of the present model, and to consider spin one massless fields. Indeed in the comparison with GRBs data we have extrapolated that the speed of light is the same as that of massless scalar fields conformally coupled to gravity. This is supported by the independence of our results from the change of dimension and the fact that the wave equation for the scalar field in 2d can be seen as that of the scalar gauge potential for electromagnetism in 2d, indeed in the commutative case, by defining the 1-form $F = \mathrm{d}\varphi$, we have that locally $\mathrm{d} * \mathrm{d}\varphi = 0$ is equivalent to $\mathrm{d} * F = 0$ and $\mathrm{d}F = 0$.

- In the present work, as a first approximation, we have considered a commutative gravity background, hence noncommutativity affects only propagation of light. In a noncommutative theory of gravity consistently coupled to light, see e.g. [29], one could consider the backreaction effects of turning on noncommutativity also on the gravitational field.

The result that the combined effects of noncommutativity and curvature produce modified dispersion relations is expected to be a general feature of wave equations in noncommutative curved spacetime. The interaction between noncommutativity and curvature responsible for the modified dispersion relations can be traced back to the fact that the vector fields composing the twist are not Killing vector fields for the metric. Indeed Killing vector fields trivially act on the metric and therefore the corresponding twist acts also trivially on the metric or on the Hodge $*$-operator. This leads to wave equations that are undeformed. The same conclusion holds more in general if the twist is composed by affine Killing vector fields, i.e. vector fields X such that $X(g) = \lambda g$ with λ a constant. This is so because the equations for massless particles are independent from rescalings of the metric. Explicitly, $*^{\mathcal{F}} = \mathcal{D}(*) = \bar{\mathrm{f}}^\alpha(*)\bar{\mathrm{f}}_\alpha$ is proportional to $*$ if $\bar{\mathrm{f}}^\alpha$ contains only affine Killing vector fields.

An example of this is provided by the wave equation in κ-Minkowski spacetime, indeed the dilatation D present in the Jordanian twist is an affine Killing vector field (while the time translation P_0 is Killing) for the usual Minkowski metric, and indeed that wave equation is undeformed [21]. These observations parallel those for the gravity field equations considered in [30]. On the basis of these observations, while modified dispersion relations are a generic feature of curved and noncommutative spacetimes, one can concoct examples of flat spacetimes with modified dispersion relations (considering vector fields in the twist that are neither Killing nor affine Killing) as well as examples of curved and noncommutative spacetimes with unmodified dispersion relations (provided the curved spacetime metric admits affine Killing vector fields).

We do not have arguments to support a direct correlation between the large scale structure of spacetime, given by the metric via a cosmological solution to Einstein equations, and the quantum spacetime structure possibly due to local quantum gravity effects. Actually in general relativity different cosmological solutions are compatible with the (classical) local spacetime structure, hence it is natural to assume, as a first approximation, that the metric structure and the noncommutative one are uncorrelated, and therefore to consider twists with no (affine) Killing vector fields for the metric. Flat spacetime on the other hand captures local properties of spacetime structure and quantum gravity effects in this background might result in a noncommutative spacetime structure where noncommutativity and the Minkowski metric are compatible. The examples of κ-Minkowski spacetime (that has Killing and affine Killing vector fields) and of κ-FLRW spacetime (that is without such vector fields) are according to these general expectations.

In conclusion, it seems reasonable to first consider noncommutativity of flat Minkowski spacetime, test it against experimental constraints and then extend the model to curved noncommutative spacetime assuming nontrivial interaction between noncommutativity and curvature. Among the three main kinds of noncommutativity, defined by the deformation parameter being massles, of mass dimension one or two: the canonical noncommutativity $x^\mu \star x^\nu - x^\nu \star x^\mu = i\theta^{\mu\nu}$, the Lie algebra-type noncommutativity $x^\mu \star x^\nu - x^\nu \star x^\mu = iC^{\mu\nu}{}_\rho x^\rho$, and the quadratic one $x^\mu \star x^\nu = \Lambda^{\mu\nu}{}_{\rho\sigma} x^\rho \star x^\sigma$, the κ-deformed cosmological spacetime we consider is of the appealing Lie algebra type that has dimensionful deformation parameter and it is obtained by requiring space commutativity and isotropy. The methods developed in this paper however are canonically derived from twist differential geometry and can be applied to other noncommutative and curved spacetime structures (e.g. with space anisotropy or with canonical noncommutativity $x^\mu \star x^\nu - x^\nu \star x^\mu = i\theta^{\mu\nu}$) in order to provide further phenomenological models. These models would give different dispersion relations depending on the gravitational background considered. Once more data of time of flight of GRBs photons becomes available, these different models could be tested by comparing the time lags predictions relative to different GRB sources: close by versus distant sources, so to test the argument of the integral in the time lag relation (4.20) and, in order to test isotropy, sources in different directions.

5 Conclusions

One of the mostly studied possible phenomenological effects of quantum gravity is the modifications in wave dispersion. While in [21] we have applied the general framework of noncommutativity arising from twist deformation to the study of κ-Minkowski spacetime providing a fresh look on modifications in dispersion relations, here, with the same canonical differential geometry methods following from twist deformation, we have focused on the Friedman-Lemaitre-Robertson-Walker case. While in flat κ-Minkowski spacetime there are no modified dispersion relations (but there are modified Einstein-Planck relations) here in curved spacetime we find modified dispersion relations and are able to obtain an actual correction to the group velocity. This is due to the interactions between noncommutativity and curvature of the spacetime. Even though the presented result concerns massless scalar fields it shows the potential of this geometrical framework in applications to quantum gravity phenomenology. The natural next step is to investigate the modified dispersion relations for noncommutative electromagnetism in a FLRW cosmological background; but equally interesting would be the study of the dispersion relations in black holes or other curved and noncommutative backgrounds which can be implemented in the framework here proposed.

A Quantum vector fields and infinitesimal translations

A vector field $u = u^\mu \partial_\mu$ is uniquely defined as a derivation on the algebra $A = C^\infty(\mathbb{R}^n)$, i.e., a differential operator on A that satisfies the Leibniz rule. Similarly, vector fields on the noncommutative algebra $A_\star = C^\infty_\star(\mathbb{R}^n)$ are braided derivations (deformed derivations) that satisfy the braided Leibniz rule. The braiding is related with the so-called universal R-matrix $\mathcal{R}$ with inverse $\mathcal{R}^{-1} = \mathcal{F}\mathcal{F}_{21}^{-1}$, where $\mathcal{F}_{21}^{-1} = \bar{\mathrm{f}}_\alpha \otimes \bar{\mathrm{f}}^\alpha$ (i.e., we have flipped the two factors in the tensor product). The notation for the universal R-matrix is analogous to the one for the twist: $\mathcal{R}^{-1} = \bar{R}^\alpha \otimes \bar{R}_\alpha$. The algebra $A_\star = C^\infty_\star(\mathbb{R}^n)$ is noncommutative with the noncommutativity controlled by the R-matrix, indeed we have

$$f \star h = \bar{R}^\alpha(h) \star \bar{R}_\alpha(f)$$

as it is easily seen from the definition of the $\star$-product in (2.2). We say that the algebra $A_\star$ is braided commutative, the braiding being provided by the R-matrix.

There is a one-to-one correspondence between vector fields on the commutative algebra A and on the noncommutative algebra $A_\star$. Any vector field on $A_\star$ can be written as

$$u^{\mathcal{F}} = \mathcal{D}(u) \tag{A.1}$$

where u is a vector field on A. Here

$$\mathcal{D}(u) := \bar{\mathrm{f}}^\alpha(u)\bar{\mathrm{f}}_\alpha\,, \tag{A.2}$$

and the expression $\bar{\mathrm{f}}^\alpha(u)$ denotes the (iterated) Lie derivative action of the vector field D entering the twist, on the vector field u. Explicitly, $D(u) = [D, u]$, $D^2(u) = [D, [D, u]]$, and iteratively $D^{p+1}(u) = [D, [D^p(u)]]$.

The vector field $u^{\mathcal{F}} = \mathcal{D}(u)$ satisfies the braided (deformed) Leibniz rule

$$u^{\mathcal{F}}(f \star h) = u^{\mathcal{F}}(f) \star h + \bar{R}^\alpha(f) \star \left(\bar{R}_\alpha(u)\right)^{\mathcal{F}}(h)\,. \tag{A.3}$$

Furthermore, vector fields on $A_\star$ form a braided Lie algebra. The braided commutator (Lie bracket) is defined by

$$\left[u^{\mathcal{F}}, v^{\mathcal{F}}\right]_{\mathcal{F}} = u^{\mathcal{F}} v^{\mathcal{F}} - \left(\bar{R}^\alpha(v)\right)^{\mathcal{F}} \left(\bar{R}_\alpha(u)\right)^{\mathcal{F}} \tag{A.4}$$

and it is again a braided vector field. Moreover, it is braided-antisymmetric and satisfies the braided-Jacobi identity

$$\left[u^{\mathcal{F}}, v^{\mathcal{F}}\right]_{\mathcal{F}} = -\left[\left(\bar{R}^{\alpha}(v)\right)^{\mathcal{F}}, \left(\bar{R}_{\alpha}(u)\right)^{\mathcal{F}}\right]_{\mathcal{F}} \tag{A.5}$$

$$\left[u^{\mathcal{F}}, \left[v^{\mathcal{F}}, z^{\mathcal{F}}\right]_{\mathcal{F}}\right]_{\mathcal{F}} = \left[\left[u^{\mathcal{F}}, v^{\mathcal{F}}\right]_{\mathcal{F}}, z^{\mathcal{F}}\right]_{\mathcal{F}} + \left[\left(\bar{R}^{\alpha}(v)\right)^{\mathcal{F}}, \left[\left(\bar{R}_{\alpha}(u)\right)^{\mathcal{F}}, z^{\mathcal{F}}\right]_{\mathcal{F}}\right]_{\mathcal{F}}, \tag{A.6}$$

for a proof we refer to eq. (3.7) (3.9), (3.10) in [26]; indeed the braided Lie algebras presented here and there are isomorphic via $\mathcal{D}^{-1}$ (cf. also [27, section 7 and section 8] and [21]).

In particular, infinitesimal translations $P_\mu^{\mathcal{F}}$ are given by

$$P_\mu^{\mathcal{F}} = \mathcal{D}(P_\mu) = P_\mu \frac{1}{1 + \frac{1}{\kappa} P_0}, \tag{A.7}$$

satisfy the braided Leibniz rule (A.3), that explicitly reads $P_\mu^{\mathcal{F}}(f \star h) = P_\mu^{\mathcal{F}}(f) \star h + e^{-\sigma}(f) \star P_\mu^{\mathcal{F}}(h)$, and have vanishing braided commutator, $[P_\mu^{\mathcal{F}}, P_\nu^{\mathcal{F}}]_{\mathcal{F}} = 0$.

In order to obtain (A.7) use that $\mathcal{F}^{-1} = \exp(-iD \otimes -\sigma)$ and that $-iD$ on momenta acts as the identity operator: $-iD(P_\mu) = [-iD, P_\mu] = P_\mu$, hence $\mathcal{F}^{-1} = \exp(-iD \otimes -\sigma) = \exp(1 \otimes -\sigma)$ if the first leg of $\mathcal{F}^{-1}$ acts on P_μ. The other expressions are computed in [21, section 3.2, 3.3].

As a consistency check of the differential geometry presented we can verify that in the equality $\mathrm{d}f = \mathrm{d}x^\mu \star \partial_\mu^{\mathcal{F}} f$, obtained in (2.8) the quantum partial derivative $\partial_\mu^{\mathcal{F}} f = \frac{1}{1 - \frac{i}{\kappa}\partial_0}\partial_\mu$ is exactly $\partial_\mu^{\mathcal{F}} = \mathcal{D}(\partial_\mu)$. This confirms the interpretation of $\partial_\mu^{\mathcal{F}} = iP_\mu^{\mathcal{F}}$ as infinitesimal translations.

B Group velocity in FLRW cosmology

We here briefly review the study of the wave equation in 2 and 4 dimensions for a massless scalar field conformally coupled to the Friedman-Lemaitre-Robertson-Walker (FLRW) spacetime (see e.g. [31], (section 5.2), and [32] (section 6.1)) and in particular derive the group velocity v_g of a wave packet in these spacetimes. This study is propaedeutical to the noncommutative one in section 4. The FLRW metric using comoving coordinates is given by

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\,\mathrm{d}x^i \mathrm{d}x^i \tag{B.1}$$

where $a(t)$ is the cosmological expansion factor (scale factor). Correspondingly, the wave equation in 2 dimensions $\Box\varphi = 0$ explicitly reads

$$\Box\varphi = \left(-\partial_t^2 - a^{-1}\dot{a}\partial_t + a^{-2}\partial_x^2\right)\varphi = 0 \tag{B.2}$$

where $\dot{a} = \frac{\partial a}{\partial t}$. The solution of this linear differential equation can be found with the method of separation of variables, we set

$$\varphi_k(x,t) = \lambda_k(t)\, e^{-ikx} = e^{i[f_k(t) - kx]} \tag{B.3}$$

and observe that φ_k solves the wave equation if $\lambda_k(t)$ satisfies $(\partial_t^2 + a^{-1}\dot{a}\partial_t + a^{-2}k^2)\lambda_k(t) = 0$. Under the change of variables $t \to \eta = \int \frac{1}{a}\mathrm{d}t$, so that $\mathrm{d}\eta = \frac{1}{a}\mathrm{d}t$ (η is the so-called conformal time because the metric becomes conformally flat $g = a^2(-\mathrm{d}\eta^2 + \mathrm{d}x^2)$), this latter equation becomes the usual harmonic oscillator equation $(\partial_\eta^2 + k^2)\lambda = 0$, hence we have the forward travelling wave solution $\varphi_k(x,t) = e^{i[k\eta(t)-kx]}$ (as well as the backward travelling solution $\tilde{\varphi}_k(x,t) = e^{-i[k\eta(t)+kx]}$). The spacetime points of constant phase of this wave satisfy $k\eta(t) - kx = const.$, hence the phase velocity is $v_p = \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}\eta(t)}{\mathrm{d}t} = \frac{1}{a}$.

A more physical measure of the field propagation speed is given by the group velocity. We derive its expression for a wave packet that is a superposition of waves of the kind (B.3), with general time dependence $f_k(t)$; these are waves that are harmonic in space but not necessarily in time. We hence consider the wave packet $\varphi = \frac{1}{2\pi}\int_{k-\delta k}^{k+\delta k} a_{\tilde{k}}\varphi_{\tilde{k}}\mathrm{d}\tilde{k}$ that is peaked around a given value k, i.e., $\delta k/k \ll 1$. As usual we rewrite the wave packet factorising the wave $\varphi_k = e^{i[f_k(t)-kx]}$, so that

$$\begin{aligned}\varphi &= \frac{1}{2\pi}\int_{k-\delta k}^{k+\delta k} a_{\tilde{k}} e^{i[f_{\tilde{k}}(t)-\tilde{k}x]}\mathrm{d}\tilde{k} \\ &= \frac{1}{2\pi}\left(\int_{k-\delta k}^{k+\delta k} a_{\tilde{k}} e^{i[f_{\tilde{k}}(t)-f_k(t)-(\tilde{k}-k)x]}\,\mathrm{d}\tilde{k}\right) e^{i[f_k(t)-kx]} \\ &\simeq \frac{1}{2\pi}\left(\int_{k-\delta k}^{k+\delta k} a_{\tilde{k}} e^{i(\frac{\partial f_k(t)}{\partial k}-x)(\tilde{k}-k)}\mathrm{d}\tilde{k}\right) e^{i[f_k(t)-kx]}\,. \end{aligned} \tag{B.4}$$

We observe that the integral has a phase that is slowly varying in space with respect to the phase of the wave $e^{i[f_k(t)-kx]}$, i.e., the wave packet φ can be described as the wave $A_k(x,t)e^{i[f_k(t)-kx]}$ with the amplitude $A_k(x,t) = \frac{1}{2\pi}\int_{k-\delta k}^{k+\delta k} a_{\tilde{k}} e^{i(\frac{\partial f_{\tilde{k}}(t)}{\partial k}-x)(\tilde{k}-k)}\mathrm{d}\tilde{k}$ that is slowly varying in space. The points in spacetime where this amplitude is constant are determined by the condition $\frac{\partial f_k(t)}{\partial k} - x = const.$; it then follows that the shape of the amplitude moves with the velocity

$$v_g = \frac{\partial x}{\partial t} = \frac{\partial}{\partial t}\frac{\partial f_k(t)}{\partial k} = \frac{\partial}{\partial k}\frac{\partial f_k(t)}{\partial t} \tag{B.5}$$

that is by definition the group velocity of the wave packet φ peaked around the wave number k.

We now proceed to compute the group velocity v_g for waves satisfying the wave equation (B.2) of 2-dimensional FLRW spacetime; in this case $f_k(t) = k\eta$ and hence

$$v_g = \frac{1}{a} \tag{B.6}$$

equals the phase velocity. The fact that this velocity does not equal the speed of light c but (inserting c that was previously set equal to 1) is c/a should not be a surprise because the comoving reference frame is not a free falling reference frame. These results agree also with the kinematic light cone condition $\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}x^2 = 0$.

In 4 dimensions a massless scalar field conformally coupled to gravity satisfies the wave equation

$$\left(\Box - \frac{1}{6}R\right)\varphi = 0 \qquad \text{i.e.} \qquad \left(a\partial_t^2 + 3\dot{a}\partial_t - a^{-1}\partial_{x^i}^2 + \frac{1}{6}aR\right)\varphi = 0 \tag{B.7}$$

where R is the scalar curvature. The solution of this linear differential equation can be found as before with the method of separation of variables by setting $\varphi_{\boldsymbol{k}}(\boldsymbol{x},t) = \lambda_{\boldsymbol{k}}(t)\,e^{-i\boldsymbol{kx}}$ and then by considering conformal time $\eta(t)$. Recalling that $R = 6(\frac{\ddot{a}}{a^2} + \frac{\dot{a}^2}{a^2}) = 6\frac{a''}{a^3}$, the wave equation is equivalent to $\lambda''_{\boldsymbol{k}} + 2\frac{a'}{a}\lambda'_{\boldsymbol{k}} + k^2\lambda_{\boldsymbol{k}} + \frac{a''}{a}\lambda_{\boldsymbol{k}} = 0$, with $k = \sqrt{k_x^2 + k_y^2 + k_z^2}$.

If we further set $\lambda_{\boldsymbol{k}} = a^{-1}\chi_{\boldsymbol{k}}$ the equation is solved iff $\chi_{\boldsymbol{k}}$ satisfies the harmonic oscillator equation $\chi'' + k^2\chi = 0$. Hence the wave equation (B.7) is solved by $\varphi_{\boldsymbol{k}}(\boldsymbol{x}, t) = a^{-1}e^{i(\omega\eta(t)-\boldsymbol{kx})}$ with $\omega = k$.

The group velocity for a superposition of waves of the kind $\varphi_{\boldsymbol{k}}(\boldsymbol{x}, t) = a^{-1}e^{i[f_{\boldsymbol{k}}(t)-\boldsymbol{kx}]}$ is obtained, similarly to the 2 dimensional case, by considering a narrow wave packet centered around a 3-vector $\boldsymbol{k}$,

$$\begin{aligned}\varphi &= \frac{1}{(2\pi)^3}\int_{\boldsymbol{k}-\boldsymbol{\delta k}}^{\boldsymbol{k}+\boldsymbol{\delta k}} a_{\tilde{\boldsymbol{k}}} a^{-1} e^{i[f_{\tilde{\boldsymbol{k}}}(t)-\tilde{\boldsymbol{k}}\boldsymbol{x}]} \mathrm{d}\tilde{\boldsymbol{k}} \\ &= \frac{1}{(2\pi)^3}\left(\int_{\boldsymbol{k}-\boldsymbol{\delta k}}^{\boldsymbol{k}+\boldsymbol{\delta k}} a_{\tilde{\boldsymbol{k}}} e^{i[f_{\tilde{\boldsymbol{k}}}(t)-f_{\boldsymbol{k}}(t)-(\tilde{\boldsymbol{k}}-\boldsymbol{k})\boldsymbol{x}]}\,\mathrm{d}\tilde{\boldsymbol{k}}\right) a^{-1} e^{i[f_{\boldsymbol{k}}(t)-\boldsymbol{kx}]} \\ &\simeq \frac{1}{(2\pi)^3}\left(\int_{\boldsymbol{k}-\boldsymbol{\delta k}}^{\boldsymbol{k}+\boldsymbol{\delta k}} a_{\tilde{\boldsymbol{k}}} e^{i(\nabla f_{\boldsymbol{k}}(t)-\boldsymbol{x})(\tilde{\boldsymbol{k}}-\boldsymbol{k})}\mathrm{d}\tilde{\boldsymbol{k}}\right) a^{-1} e^{i[f_{\boldsymbol{k}}(t)-\boldsymbol{kx}]}\,.\end{aligned} \tag{B.8}$$

The slowly varying phase $A_{\boldsymbol{k}}(\boldsymbol{x}, t) = \frac{a^{-1}}{(2\pi)^3}\int_{\boldsymbol{k}-\boldsymbol{\delta k}}^{\boldsymbol{k}+\boldsymbol{\delta k}} a_{\tilde{\boldsymbol{k}}} e^{i(\nabla f_{\boldsymbol{k}}(t)-\boldsymbol{x})(\tilde{\boldsymbol{k}}-\boldsymbol{k})}\mathrm{d}\tilde{\boldsymbol{k}}$ of this wave packet has modulus square $A_{\boldsymbol{k}}A^*_{\boldsymbol{k}}(\boldsymbol{x}, t) = \frac{a^{-2}}{(2\pi)^6}\int_{\boldsymbol{k}-\boldsymbol{\delta k}}^{\boldsymbol{k}+\boldsymbol{\delta k}}\int_{\boldsymbol{k}-\boldsymbol{\delta k}}^{\boldsymbol{k}+\boldsymbol{\delta k}} a_{\tilde{\boldsymbol{k}}} a^*_{\hat{\boldsymbol{k}}} \cos((\nabla f_{\boldsymbol{k}}(t) - \boldsymbol{x})(\tilde{\boldsymbol{k}} - \hat{\boldsymbol{k}}))\mathrm{d}\tilde{\boldsymbol{k}}\mathrm{d}\hat{\boldsymbol{k}}$ that is maximal when the cosine is maximal, hence for $\nabla f_{\boldsymbol{k}}(t) - \boldsymbol{x} = 0$, leading to the group velocity

$$\boldsymbol{v}_g = \frac{\partial \boldsymbol{x}}{\partial t} = \frac{\partial}{\partial t}\nabla f_{\boldsymbol{k}}\,.$$

We have seen that massless scalar fields conformally coupled to 4 dimensional FLRW-spacetime have waves $\varphi_{\boldsymbol{k}}(\boldsymbol{x}, t) = a^{-1}e^{i(\omega\eta(t)-\boldsymbol{kx})}$ with $\omega = k$ and hence the resulting group velocity has modulus $v_g = \frac{c}{a}$ (we have restored the velocity of light c). Considering a free falling reference frame, rather than a comoving one, this becomes the velocity of light c as expected.

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