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Coalitional extreme desirability in finitely additive economies with asymmetric information

Anuj Bhowmik[a,*], Francesca Centrone[b], Anna Martellotti[c]

[a] Indira Gandhi Institute of Development Research, Gen. A.K. Vaidya Marg, Santosh Nagar, Goregaon (East), Mumbai 400 065, India
[b] Dipartimento di Studi per l'Economia e l'Impresa, Università del Piemonte Orientale, Via Perrone 18, 28100 Novara, Italy
[c] Dipartimento di Matematica e Informatica, Università di Perugia, Via Vanvitelli 1, 06123 Perugia, Italy

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ABSTRACT

We prove a coalitional core-Walras equivalence theorem for an asymmetric information exchange economy with a finitely additive measure space of agents, finitely many states of nature, and an infinite dimensional commodity space having the Radon–Nikodym property and whose positive cone has possibly empty interior. The result is based on a new cone condition, firstly developed in Centrone and Martellotti (2015), called coalitional extreme desirability. We also formulate a notion of incentive compatibility suitable for coalitional models and study it in relation to equilibria.

1. Introduction

Since the seminal paper of Radner (1968) a huge literature has grown in the area of Equilibrium Theory under Asymmetric Information, which allows for the possibility of having differently informed agents. From the mathematical point of view, the classical Arrow–Debreu exchange economy representation is thus enriched by taking into account the informational aspects; namely, if Ω is a set of states of the world, each agent is endowed with a probability measure on Ω representing the agent's prior beliefs, an *ex-ante* utility function which depends on the possible states of the world, an initial endowment which specifies the agent's resources in each state, and a partition of Ω which represents the agent's initial information. The notion of a Walras equilibrium, called *a Walras expectation equilibrium*, is adapted to include the aforesaid informational aspects. The second notion of our paper, the *core*, allows for the possibility of cooperation among agents and is usually associated with Edgeworth. It is well recognized that the asymmetric context gives rise to different possibilities of sharing information among members of coalitions and thus, accordingly, different notions of core have been developed (Wilson, 1978; Yannelis, 1991).

In individual models, both the cases of a finite and an infinite dimensional commodity space have been treated, with various degrees of generality; most of these models assume anyway a countably additive measure space of agents, and a finite-dimensional commodity space or a commodity space whose positive cone has a nonempty interior, in order to apply classical separation theorems to support optimal allocations with nonnegative prices, refer to Angeloni and Martins-da Rocha (2009), Einy et al. (2001), Graziano and Meo (2005). Only recently, Bhowmik (2013) has adapted Rustichini and Yannelis's (1991) additivity condition and extremely desirable commodity assumption[1] to the asymmetric information framework, in a way, to obtain a countably additive individualistic core-Walras equivalence theorem with an infinite dimensional commodity space, without assumptions on the positive cone.

Anyway, in the literature, Vind's (1964) model is well established, where the author proposed to replace the individualistic

* Corresponding author.
E-mail addresses: anujbhowmik@igidr.ac.in (A. Bhowmik), francesca.centrone@uniupo.it (F. Centrone), anna.martellotti@unipg.it (A. Martellotti).

[1] this condition is very well known in the literature, together with Mas Colell's properness (Mas-Colell, 1986) and Chichilnisky and Kalman's cone condition (Chichilnisky and Kalman, 1980), and is a widely used condition as it allows a separation argument; see also (Aliprantis et al., 2000) for a complete survey.

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representation of a complete information economy with a coalitional one: the rationale of this choice is the fact that only the bargaining power of coalitions matters. Hence, coalitions are used as primitives, preferences are defined directly on coalitions and allocations are countably additive measures. Later, Armstrong and Richter (1984) abandoned the countable additivity setting to assume finite additivity for the sake of realism, claiming that a countable union of coalitions need not be a coalition. Under this hypothesis and considering infinitely many commodities, we can also recall the work of Cheng (1991). Basile et al. (2009) then introduced asymmetric information into this framework, working with a Boolean algebra and a Euclidean space of commodities. As pointed out by the authors themselves another advantage of the f.a. approach is that it allows us to include the case of a countable set of asymmetrically informed agents, as in limit economies. We refer the reader to Basile et al. (2009) for further motivations.

The use of coalitional models in the presence of asymmetric information to prove core-Walras equivalence theorems, finds a justification in the criticism of Forges et al. (2002): they in fact claim that, being the notion of the core based on agents making agreements to trade among themselves and not through an anonymous market, this involves communication among agents, and "it is then unreasonable to impose the restriction that an agent cannot entertain a contract which varies with information he does not possess". Indeed, the main point which allows to overcome this criticism lies in the definition of the private core introduced in Basile et al. (2009), as in coalitional models private feasibility of allocations depends just on the information of the coalitions themselves (requiring measurability just with respect to information of the coalitions) regardless of the one available to single individuals.

Up to now, to our knowledge, there are no core-Walras equivalence results covering both the cases of a finitely additive coalitional model with an infinite dimensional commodity space. The aim of this work is to try to fill this gap, by introducing in the asymmetric context a Banach lattice as the commodity space, the notion of *coalitional extremely desirable commodity*, which is the extension of that in Centrone and Martellotti (2016) given for complete information. We point out that the use of properness conditions in asymmetric information models already appeared in the literature in the context of individualistic models (see for example Aliprantis et al., 2000; Angeloni and Martins-da Rocha, 2009).

In this paper we obtain a coalitional asymmetric core-Walras equivalence result in a framework whose commodity space is X_+, the positive cone of a Banach lattice X having the Radon–Nikodym property (see Diestel Jr. and Uhl (1977)) and feasibility is defined as free disposal; note that this allows for a great variety of infinite dimensional commodity spaces interesting for economics and finance, for example, all the L_p spaces for $p > 1$. The idea underlying our properness condition is linked to the familiar idea to see an economy with asymmetric information where uncertainty is captured by n states of nature and having X_+ as a commodity space, as a complete information economy having X_+^n as a commodity space. In individualistic models, when feasibility is defined with free disposal it is however well known that equilibria may not be incentive compatible and hence contracts may not be enforceable (see Glycopantis et al. (2003)). So this is the main motivation to try to give results assuming exact feasibility (see Angeloni and Martins-da Rocha (2009)). The same problem can thus be faced in the coalitional setting. Under suitable hypothesis, we are not only able to prove a core-Walras equivalence without free disposal, but we also propose a suitable notion of *coalitional incentive compatibility* and show that private core allocations are incentive compatible. We also point out that the introduction of asymmetric information and of the arising informational constraint made it necessary to adopt new techniques with respect to those in Centrone and Martellotti (2016).

The rest of the paper is organized as follows: Section 2 deals with the description of our model, some assumptions and the necessary concepts. In Section 3, we introduce the notion of *coalitional extremely desirable commodity* in the asymmetric information framework and prove some technical lemmas that play central roles in the proofs of our main results. We also compare our properness notion with the one of Aliprantis et al. (see Podczek and Yannelis (2008)). In Section 4, we present our coalitional core-Walras equivalence theorem under the free disposal feasibility condition. In Section 5 we face the question of exact feasibility and prove a core-Walras equivalence theorem under this assumption. Moreover, we define a suitable notion of coalitional incentive compatibility and prove that core allocations are incentive compatible. Section 6 is devoted to some asymmetric individualistic results, deriving from our coalitional ones in the spirit of comprehensiveness of Armstrong and Richter (1984). Lastly, we summarize and compare our results in Section 7.

2. Description of the coalitional model

A coalitional model of pure exchange economy $\mathscr{E}_C$ with asymmetric information is presented. The exogenous uncertainty is described by a measurable space $(\Omega, \mathscr{F})$, where $\Omega = \{\omega_1, \ldots, \omega_n\}$ is the set of states of nature containing n elements and $\mathscr{F}$ denotes the power set of Ω. The economy extends over two time periods $\tau = 0, 1$. Consumption takes place at $\tau = 1$. At $\tau = 0$, there is uncertainty over the states and agents make contracts that are contingent on the realized state at $\tau = 1$. Let X be a Banach lattice having the Radon–Nikodym property (RNP) and a quasi-interior point. The partial order on X is denoted by $\leq$ and the positive cone of X, given by $X_+ = \{x \in X : 0 \leq x\}$, represents the commodity space of $\mathscr{E}_C$. The symbol $0 < x$ means that x is a non-zero point of X_+.

Let the space of agents be a space $(I, \Sigma, \mathbb{P})$, where I is the set of agents with Σ an algebra on I and $\mathbb{P}$ a strongly non-atomic finitely additive (f.a.) probability measure on Σ, that is, for every $A \in \Sigma$ and $\varepsilon \in (0, 1)$ there is some $B \in \Sigma$ such that $B \subseteq A$ and $\mathbb{P}(B) = \varepsilon\mathbb{P}(A)$. Each element in Σ with positive probability is termed as a *coalition*, whose economic weight on the market is given by $\mathbb{P}$. If E and F are two coalitions, and $E \subseteq F$ then E is called a *sub-coalition* of F.

Analogously to Radner (1968), we assume that assignment of resources is state-contingent. By an *assignment*, we mean a function $\alpha : \Sigma \times \Omega \to X_+$ such that $\alpha(\cdot, \omega)$ is a f.a. measure of bounded variation on Σ, for each $\omega \in \Omega$. Moreover, each assignment α can be associated with the function $\bar{\alpha} : \Sigma \to (X_+)^n$ by letting $\bar{\alpha}(E) = (\alpha(E, \omega_1), \ldots, \alpha(E, \omega_n))$, where $(X_+)^n$ is the positive cone of the Banach lattice X^n, which is endowed with the point-wise algebraic operations, the point-wise order and the product norm. With slightly abuse of notation, we assume that $\leq$ also denotes a point-wise order on X^n. The only admissible assignments in our model are connected with some absolute continuity property. Recall that, given a Banach lattice Y and two vector measures $\mu : \Sigma \to X^k$ and $\nu : \Sigma \to Y$, μ is called *absolutely continuous with respect to* ν, denoted by $\mu \ll \nu$, if for every $\varepsilon > 0$ there is some $\delta > 0$ such that each $F \in \Sigma$ with $\|\nu(F)\|_Y < \delta$ implies $\|\mu(F)\|_{X^k} < \varepsilon$. Let

$$\mathscr{M} = \{\alpha : \Sigma \times \Omega \to X_+ : \alpha \text{ is an assignment and } \bar{\alpha} \ll \mathbb{P}\}.$$

Thus, an *allocation* is defined to be an element of $\mathscr{M}$. There is a special allocation, denoted by $e : \Sigma \times \Omega \to X_+$, such that $e(F, \omega)$ is the initial endowment of the coalition F if the state of nature ω occurs. We call such an allocation as *initial endowment allocation*.

Similarly to Basile et al. (2009), a *preference relation* $\succ_F$ is defined on $\mathscr{M}$ for any coalition F. Intuitively, $\alpha \succ_F \beta$ expresses the idea that the members of the coalition F prefer what they get from α to what they get from β. Each coalition F is also associated with some *private information*, which is described by a $\mathscr{F}$-measurable partition $\mathscr{P}_F$ of Ω. The interpretation is that, if ω is the true state of nature, then coalition F cannot discriminate the states in the unique element $\mathscr{P}_F(\omega)$ of $\mathscr{P}_F$ containing ω. Let $\mathscr{F}_F$ be the σ-algebra generated by $\mathscr{P}_F$. The triple $(\mathscr{F}_F, \succ_F, e(F,\cdot))$ is called the *characteristics* of the coalition F. Thus, the economy can be described by

$$\mathscr{E}_C = \{(I,\Sigma,\mathbb{P}); X_+; (\Omega,\mathscr{F}); (\mathscr{F}_F, \succ_F, e(F,\cdot))_{F\in\Sigma}\}.$$

To relate the weight of coalitions to the commodities that they can trade on the market, we assume that e is equivalent to $\mathbb{P}$, that is, e and $\mathbb{P}$ are absolutely continuous with respect to each other. We now impose some restriction on the class of preferences. To this end, given an allocation $\alpha \in \mathscr{M}$ and a coalition F, define a vector measure $\bar{\alpha}_{|_F} : \Sigma \to (X_+)^n$ by letting $\bar{\alpha}_{|_F}(E) = \bar{\alpha}(E\cap F)$ for all $E \in \Sigma$. A *simple allocation* is any allocation s such that, for every $\omega \in \Omega$,

$$s(\cdot,\omega) = \sum_{i=1}^{q} y_i(\omega)\mathbb{P}_{|_{H_i}},$$

where $\{H_i\}_i$ is a decomposition of I such that each H_i is measurable. The following assumptions on preferences will be assumed implicitly throughout the rest of the paper:

[P.1] $\succ_F$ is irreflexive and transitive, for every $F \in \Sigma$;

[P.2] For any coalition F and $\alpha_1, \alpha_2 \in \mathscr{M}$ with $\alpha_1 \succ_F \alpha_2$, we must have $\alpha_1 \succ_G \alpha_2$ for all sub-coalitions G of F;

[P.3] If $\alpha_1 \succ_F \alpha_2$ and $\alpha_1 \succ_G \alpha_2$ for two coalitions F and G, then $\alpha_1 \succ_{F\cup G} \alpha_2$;

[P.4] For any $\alpha \in \mathscr{M}$ and any element $x \in (X_+)^n \setminus \{0\}$, we have $\alpha + x\mathbb{P} \succ_I \alpha$, where the allocation $x\mathbb{P} : \Sigma \times \Omega \to X_+$ is defined by $x\mathbb{P}(F,\omega) = x(\omega)\mathbb{P}(F)$;

[P.5] If $\alpha_1, \alpha_2, \alpha_3 \in \mathscr{M}$ and F is a coalition satisfying $\bar{\alpha}_{1_{|_F}} = \bar{\alpha}_{2_{|_F}}$, then the following double implications hold:

$$[\alpha_1 \succ_F \alpha_3 \Longleftrightarrow \alpha_2 \succ_F \alpha_3] \text{ and } [\alpha_3 \succ_F \alpha_1 \Longleftrightarrow \alpha_3 \succ_F \alpha_2].$$

Remark 2.1. Assumption [P.2] claims that as almost all members of F prefer what they get from α than what they get from β (refer to Debreu (1967) for a deterministic economy), they do the same under G. Assumption [P.3] is similar to Assumption (VI) in Debreu (1967). The monotonicity assumption is discussed in [P.4], which is analogous to the assumption (WM) in Centrone and Martellotti (2016). It is worth pointing out that our monotonicity assumption is weaker than the one in Armstrong and Richter (1984) for a deterministic economy. Lastly, Assumption [P.5] is termed as *selfish property* in the literature, and can be found in Armstrong and Richter (1984), Basile (1993), Basile et al. (2009).

The following assumption is referred to as *nested condition* in the literature for information sharing rules in individualistic economies (Allen, 2006; Bhowmik, 2015; Hervés-Beloso et al., 2014), where the initial private information of each agent is susceptible to be altered when the agent becomes a member of a coalition. This issue is particularly interesting in the individualistic economies as the information structures within coalitions have great influence on the set of allocations which can be attainable alternatives. The mechanism which associates information with agents within a coalition can be the result of an information sharing process among agents belonging to the same coalition, or on the contrary, it could be a consequence of some rules that prevent the use of information other than the common one.

[A.1] For all $E, F \in \Sigma$ with $E \subseteq F$, we have $\mathscr{F}_E \subseteq \mathscr{F}_F$.

Remark 2.2. The above assumption also appeared in a coalitional model of Basile et al. (2009), representing the intuitive idea that the state of information can never decrease if coalitions share their private information. To support this assumption, we assume that the information $\mathscr{F}_F$ of a coalition F is given by some information sharing rule that may depend on the initial private information of each of its members (Allen, 2006; Bhowmik, 2015; Hervés-Beloso et al., 2014). This formalization of information sharing may provide a connection between the information of a coalition and the information of each of its members. Some of these connections are given, for any coalition F, as follows: (i) $\mathscr{P}_F = \{\emptyset, \Omega\}$, which means that this information sharing rule associates to each coalition member the null information $\{\emptyset, \Omega\}$ which is independent of the initial information of agents; (ii) $\mathscr{P}_F$ is defined to be the join information of all agents in I; (iii) $\mathscr{P}_F$ is defined to be the common information of all agents in I; (iv) $\mathscr{P}_F$ is defined to be the join information of members of F only.

We shall assume throughout the rest of the paper that $\mathscr{P}_I = \{\{\omega_1\},\ldots,\{\omega_n\}\}$, i.e. the discrete partition of the set Ω; this is a reasonable assumption because, if two states are not distinguished by the whole grand coalition, no other coalition will distinguish between them, so they can be identified.

Let $\mathfrak{P} = \{\mathscr{Q}^1,\ldots,\mathscr{Q}^m\}$ denote the finite family of partitions of Ω such that for each $\mathscr{Q}^i \in \mathfrak{P}$ there is some coalition $F \in \Sigma$ satisfying $\mathscr{P}_F = \mathscr{Q}^i$. For each $\mathscr{Q}^i$, define

$$T_i = \{F \in \Sigma : \mathscr{P}_A = \mathscr{Q}^i \text{ for all non-empty measurable subset } A \text{ of } F\},$$

$$I_i = \bigcup\{F : F \in T_i\}.$$

We shall assume the following:

[A.2] $I_i \in \Sigma$ for each $\mathscr{Q}^i \in \mathfrak{P}$.

Observe that, the decomposition $\mathscr{P}_I$ is maximal in $\mathfrak{P}$ in the sense of refinements, i.e. no decomposition in $\mathfrak{P}$ refines it. For the sake of consistency in notation, let us denote it as $\overline{\mathscr{Q}} = \mathscr{P}_I$.

Set

$$\mathfrak{P}_0 = \{\mathscr{Q}^i \in \mathfrak{P} | \nexists \mathscr{Q}' \in \mathfrak{P} \text{ that is refined by } \mathscr{Q}^i\}.$$

As Ω is finite, $\mathfrak{P}_0$ is non-empty.

Let now $\mathscr{Q}_0 \in \mathfrak{P}_0$ be fixed; a finite sequence $\mathscr{Q}_0, \mathscr{Q}_1, \ldots, \mathscr{Q}_p \in \mathfrak{P}$ is a $\mathfrak{P}$-*chain*, if $\mathscr{Q}_{i+1}$ refines $\mathscr{Q}_i$, $i = 0,\ldots,p-1$.

A $\mathfrak{P}$-chain is *optimal* if there is no $\mathscr{Q} \in \mathfrak{P}$ refining $\mathscr{Q}_i$ and that is refined by $\mathscr{Q}_{i+1}$. In other words an optimal chain connects the worst state of information $\mathscr{Q}_0$ to the best $\overline{\mathscr{Q}}$ into finitely many steps, all representing real state of information in $\mathfrak{P}$, and without ignoring possible intermediate states of information; clearly, one can connect two different states of information, one refining the other, through many different optimal chains. For any $\mathscr{Q} \in \mathfrak{P}\backslash\mathfrak{P}_0$, define

$$L(\mathscr{Q}) = \{\mathscr{Q}' \in \mathfrak{P} : \mathscr{Q} \text{ refines } \mathscr{Q}'\}.$$

Analogously, for any $\mathscr{Q} \in \mathfrak{P} \setminus \overline{\mathscr{Q}}$, define

$$U(\mathscr{Q}) = \{\mathscr{Q}' \in \mathfrak{P} : \mathscr{Q}' \text{ refines } \mathscr{Q}\}.$$

Lemma 2.3. *Under the assumption* [A.1], *the following are satisfied;*

(i) *for every $\mathscr{Q}^i \in \mathfrak{P}_0$ and each non-empty $F \in \Sigma$ with $\mathscr{P}_F = \mathscr{Q}^i$ one finds that $F \in T_i$.*

(ii) *for every* $\mathscr{Q}^i \in \mathfrak{P} \setminus \mathfrak{P}_0$ *and each non-empty* $F \in \Sigma$ *with* $\mathscr{P}_F = \mathscr{Q}^i$ *one finds that* $G := F \setminus \bigcup\{I_j : \mathscr{Q}^j \in L(\mathscr{Q}^i)\} \in T_i$ *if* $G \neq \emptyset$.

Proof. The proof of statement (i) is straightforward. To prove (ii) choose a partition $\mathscr{Q}^i \in \mathfrak{P} \setminus \mathfrak{P}_0$ and a non-empty element $F \in \Sigma$ with $\mathscr{P}_F = \mathscr{Q}^i$ such that $L(\mathscr{Q}^i) \subseteq \mathfrak{P}_0$. Define

$$G = F \setminus \bigcup\{I_j : \mathscr{Q}^j \in L(\mathscr{Q}^i)\}.$$

Assume that $G \neq \emptyset$. Choose a non-empty element $H \in \Sigma$ such that $H \subseteq G$. By [A.1] again, either $\mathscr{P}_H = \mathscr{Q}^i$ or $\mathscr{P}_H = \mathscr{Q}^k$ for some $\mathscr{Q}^k \in L(\mathscr{Q}^i)$. But, if $\mathscr{P}_H = \mathscr{Q}^k$ for some $\mathscr{Q}^k \in L(\mathscr{Q}^i)$, by (i), one concludes $H \subseteq I_k$, whereas $H \subseteq G$ and $G \cap I_k = \emptyset$. Hence, $\mathscr{P}_H = \mathscr{Q}^i$ for every non-empty measurable subset H of G, i.e. $G \in T_i$. Thus, the statement is true for every $\mathscr{Q}^i \in \mathfrak{P} \setminus \mathfrak{P}_0$ and all non-empty $F \in \Sigma$ such that $\mathscr{P}_F = \mathscr{Q}^i$ and $L(\mathscr{Q}^i) \subseteq \mathfrak{P}_0$. Let $\mathscr{Q}^i \in \mathfrak{P} \setminus \mathfrak{P}_0$ and assume that the statements (i) and (ii) are satisfied for all $\mathscr{Q}^r \in L(\mathscr{Q}^i)$. Let F be a non-empty measurable set such that $\mathscr{P}_F = \mathscr{Q}^i$. Define

$$G = F \setminus \bigcup\{I_r : \mathscr{Q}^r \in L(\mathscr{Q}^i)\}.$$

Let $G \neq \emptyset$ and take a non-empty subset H of G. By [A.1] again, either $\mathscr{P}_H = \mathscr{Q}^i$ or $\mathscr{P}_H = \mathscr{Q}^j$ for some $\mathscr{Q}^j \in L(\mathscr{Q}^i)$. Assume that $\mathscr{P}_H = \mathscr{Q}^j$ for some $\mathscr{Q}^j \in L(\mathscr{Q}^i)$. Then either $\mathscr{P}_H = \mathscr{Q}^j \in \mathfrak{P}_0$, in which case $H \subseteq I_j$, or $\mathscr{P}_H = \mathscr{Q}^j \in \mathfrak{P} \setminus \mathfrak{P}_0$, in which case

$$A := H \setminus \bigcup\{I_s : \mathscr{Q}^s \in L(\mathscr{Q}^j)\} \in T_j,$$

if $A \neq \emptyset$. So, $A \subseteq I_j$ if $A \neq \emptyset$. Thus, in either case,

$$H \subseteq \bigcup\{I_s : \mathscr{Q}^s \in L(\mathscr{Q}^j)\} \bigcup I_j.$$

Since $\mathscr{Q}^j \in L(\mathscr{Q}^i)$ and $L(\mathscr{Q}^j) \subseteq L(\mathscr{Q}^i)$, one must have

$$H \subseteq \bigcup\{I_s : \mathscr{Q}^s \in L(\mathscr{Q}^i)\},$$

which leads to a contradiction as $H \subseteq G$. Hence, our supposition that $\mathscr{P}_H = \mathscr{Q}^j$ for some $\mathscr{Q}^j \in L(\mathscr{Q}^i)$ is wrong. Thus, $\mathscr{P}_H = \mathscr{Q}^i$. Therefore, $G \in T_i$. By the principle of Mathematical Induction, the proof is completed. □

Corollary 2.4. *Given* $\mathfrak{P} = \{\mathscr{Q}^1, \dots, \mathscr{Q}^m\}$ *and assumptions* [A.1]–[A.2], *the grand coalition decomposes into finitely many pairwise disjoint coalitions* $I_1, \dots, I_m$ *such that* $\mathscr{P}_{I_i} = \mathscr{Q}^i$, *for every* $i = 1, \dots, m$, *and* $\mathscr{P}_E = \mathscr{Q}^i$ *for every non-empty measurable set* $E \subseteq I_i$, $i = 1, \dots, m$.

Proof. From Lemma 2.3 and the fact that for every $\mathscr{Q}^i \in \mathfrak{P}$, there is a coalition F such that $\mathscr{P}_F = \mathscr{Q}^i$ it is clear that $T_i \neq \emptyset$. Applying Lemma 2.3 to the grand coalition I and [A.2], we obtain the decomposition of I,

$$I = \bigcup\{I_i : 1 \leq i \leq m\},$$

into the finitely many pairwise disjoint sub-coalitions $I_1, \dots, I_m$, fulfilling the assertion. □

Similarly to Basile et al. (2009), we now restrict the set of consumption bundles that are informationally attainable for any coalition F, that is, the coalition F cannot consume different amounts on events that it cannot distinguish. Thus, the *consumption set* of a coalition F is the set of such restricted consumption bundles, which can be formally defined as

$$\mathscr{X}_F = \left\{x \in X_+^n : x \text{ is } \mathscr{F}_F\text{-measurable}\right\}.$$

An allocation α is said to be *privately feasible for a coalition* F whenever $\alpha(E, \cdot) \in \mathscr{X}_E$ for each coalition $E \subseteq F$. It means that any privately feasible allocation for a coalition F requires not only that the coalition F is able to distinguish what it consumes but also requires all sub-coalitions of it do the same thing. We denote the set of privately feasible allocations for a coalition F by $\mathscr{M}_F$. In the case when $F = I$, then we simply say $\mathscr{M}_I$ as the set of privately feasible allocations. We assume that e is privately feasible. An allocation α is termed as *physically feasible for a coalition* F if $\alpha(F, \omega) \leq e(F, \omega)$ for all $\omega \in \Omega$. In particular, physically feasible allocations for I are simply referred to as physically feasible allocations. Finally, we say that an allocation is *feasible for a coalition* F if it is privately as well as physically feasible for F, and the set of such allocations is denoted by $\mathscr{Y}_F$. Without any confusion, feasibility for I will be termed as *feasibility*.

Definition 2.5. An allocation α is *privately blocked by a coalition* F if there is an allocation $\beta \in \mathscr{Y}_F$ such that $\beta \succ_F \alpha$. The *private core* of $\mathscr{E}_C$, denoted by $\mathscr{PC}(\mathscr{E}_C)$, is the set of feasible allocations which are not privately blocked by any coalition.

A *price system* is a non-zero function $\pi : \Omega \to X_+^*$, where X_+^* is the positive cone of the norm-dual X^* of X. The *budget set* of a coalition F with respect to a price system π is defined by

$$\mathscr{B}(F, \pi) = \left\{\alpha \in \mathscr{M}_F : \sum_{i=1}^{n} \pi(\omega_i)\alpha(F, \omega_i) \leq \sum_{i=1}^{n} \pi(\omega_i)e(F, \omega_i)\right\}.$$

Analogously to the private core, the definition of Walras equilibrium also takes into account the information structure.

Definition 2.6. A *Walrasian expectations equilibrium* of $\mathscr{E}_C$ is a pair (α, π), where α is a feasible allocation and π is a price system, such that

(i) $\alpha \in \mathscr{B}(F, \pi)$ for each coalition $F \in \Sigma$;

(ii) $\sum_{i=1}^{n} \pi(\omega_i)\alpha(I, \omega_i) = \sum_{i=1}^{n} \pi(\omega_i)e(I, \omega_i)$;

(iii) for every coalition F and $\beta \in \mathscr{M}_F$, $\beta \succ_F \alpha \Longrightarrow \beta \notin \mathscr{B}(F, \pi)$.

In this case, α is termed as *Walrasian expectations allocation* and the set of such allocations is denoted by $\mathscr{W}(\mathscr{E}_C)$.

3. Some technical results

In this section, we establish some technical lemmas for later use.

Lemma 3.1. *Under assumptions* [A.1]–[A.2], *if* $\alpha \in \mathscr{M}_F$ *for some coalition* F, *then for each* $\varepsilon > 0$ *there exists a simple allocation* $s \in \mathscr{M}_F$ *such that* $\|\bar{\alpha} - \bar{s}\| < \varepsilon$.

Proof. Let us fix a listing order for Ω, say $\Omega = \{\omega_1, \dots, \omega_n\}$. For each $\mathscr{Q}^i \in \mathfrak{P}_0$ denote by Ω_i the subset of its real distinguished states formed by exactly those elements from each member of $\mathscr{Q}^i$ whose index is the lowest among all the elements in the same member of $\mathscr{Q}^i$. Let now F, α and ε be fixed. Set

$$J = \{k : 1 \leq k \leq m \text{ and } \mathbb{P}(F \cap I_k) > 0\},$$

where I_k is defined in [A.2]. Hence the information partition of each $F_j := F \cap I_j$ is $\mathscr{Q}^j$ for $j \in J$ and $F = \bigcup_{j \in J}(F \cap I_j)$.

Suppose for simplicity that $1 \in J$, and set $\Omega_1 := \{\omega_{i_1}, \dots, \omega_{i_m}\}$, where $1 = i_1 < i_2 < \dots < i_m$.

Since $\alpha \in \mathscr{M}_F$, we have that $\alpha_{|F_1} \in \mathscr{M}_{F_1}$. For any index ℓ such that $\omega_\ell \notin \Omega_1$ necessarily $i_h < \ell < i_{h+1}$ for one $1 \leq h \leq m-1$ and $\alpha_{|F_1}(\cdot, \omega_\ell) = \alpha_{|F_1}(\cdot, \omega_{i_h})$. In other words only the components of $\alpha_{|F_1}$ relative to states in Ω_1 can be different in

F_1. Then according to the approximate Radon–Nikodym Theorem (Diestel Jr. and Uhl, 1977; Uhl, 1967), we can choose precisely m simple approximations $\{s_1^1, \ldots, s_m^1\} \subseteq \mathscr{M}$ in F_1 such that

$$\left|\alpha_{|F_1}(\cdot, \omega_{i_h}) - s_h^1\right| < \frac{\varepsilon}{2n|J|},$$

and note that each s_i^1 also approximates the identical components of $\alpha_{|F_1}$ as well. Thus we can rearrange them into an n-dimensional simple function $\sigma_1 \in \mathscr{M}_{F_1}$.[2]

Thus

$$\left\|\overline{\alpha}_{|F_1} - \sigma_1\right\| \leq \sum_{\omega \in \Omega_1} \left|\alpha_{|F_1}(\cdot, \omega) - s_\omega^1\right| < |\Omega_1| \frac{\varepsilon}{2n|J|} < \frac{\varepsilon}{2|J|}.$$

Finally choose any simple approximation s_0 of $\bar{\alpha}_{|I \setminus F_1}$ such that

$$\left\|\bar{\alpha}_{|I \setminus F_1} - s_0\right\| < \frac{\varepsilon}{2}.$$

We set now $s = \sum_{j \in J} \sigma_{j|F_j} + s_{0|I \setminus F}$, so

$$\|\bar{\alpha} - \bar{s}\| \leq \sum_{j \in J} \left\|\bar{\alpha}_{|F_j} - \sigma_j\right\| + \left\|\bar{\alpha}_{|I \setminus F} - s_0\right\| < \varepsilon$$

which completes the proof. □

For a fixed allocation $\alpha \in \mathscr{M}_F$ and a coalition F, let

$$\mathscr{K} = \bigcup_{F \in \Sigma,\ \mathbb{P}(F) > 0} \{\bar{\gamma}(F) - \bar{e}(F) : \gamma \in \mathscr{M}_F, \gamma \succ_F \alpha\}.$$

Our next technical results and main theorems require some continuity-like assumptions. Here, we employ an assumption similar to that in Centrone and Martellotti (2016).

[A.3] Let F be a coalition and $\alpha, \beta \in \mathscr{M}_F$ be such that $\beta \succ_F \alpha$. For every $\tau > 0$, we can find some $\rho(\tau) > 0$ such that for every simple allocation $s \in \mathscr{M}_F$ with $\|\bar{s} - \bar{\beta}\| < \rho(\tau)$ there exists a coalition $F_0 = F_0(s, \tau) \subseteq F$ with $\mathbb{P}(F \setminus F_0) < \tau$ and $s \succ_{F_0} \alpha$.

Given this assumption, our proof for the next lemma exactly follows analogous arguments of the final step of Lemma 3.3 in Centrone and Martellotti (2016), taking into account Lemma 3.1. Thus, we skip the formal proof for this result.

Lemma 3.2. *Suppose that $\mathscr{E}_C$ satisfies* [A.1] *and* [A.3]. *Then the set $\overline{\mathscr{K}}$ is convex, where $\overline{\mathscr{K}}$ denotes the norm-closure of $\mathscr{K}$ in X^n.*

It is well-known that an affirmative answer to the classical core-Walras equivalence result in a framework of a Banach lattice as the commodity space cannot be obtained without any “properness-like”assumption (refer to Rustichini and Yannelis (1991)). In our model, we suitably extend the *extremely desirable commodity* assumption of Centrone and Martellotti (2016).

[A.4] There exist some $u \in X_+^n$ and an open, convex, solid neighborhood U of 0 in X^n such that the following two conditions are satisfied: (i) $U^c \cap X_+^n$ is convex, where U^c is the complement of U in X^n; (ii) If $y \in X_+^n$ and $z \in \overline{(y + C_u)} \cap X_+^n$, then $z\mathbb{P} \succ_I y\mathbb{P}$, where

$$C_u = \bigcup \{t(u + U) : t > 0\}.$$

Observe that since $\mathscr{P}_I$ is the finest partition of Ω the allocations $z\mathbb{P}$ and $y\mathbb{P}$ are both in $\mathscr{M}_I$.

In the rest of the paper, we shall refer to (u, U) as a *properness pair*. To prove our next result, given (u, U) is a properness pair, we now find other possible properness pairs (w, U). Observe first that, if $u \leq \hat{u}$ then $(\hat{u}, U)$ is a properness pair as well. Indeed, let $y \in X_+^n$ and $z \in \overline{(y + C_{\hat{u}})} \cap X_+^n$. Pick an $\varepsilon > 0$. It follows that $B(z, \varepsilon) \cap (y + t(\hat{u} + U)) \neq \emptyset$ for some $t > 0$, where $B(z, \varepsilon)$ denotes the open ball in X^n centered at z with radius ε. Thus,

$$B(z, \varepsilon) \cap (y + t(\hat{u} - u) + t(u + U)) \neq \emptyset.$$

Consequently, $z \in \overline{(y + t(\hat{u} - u) + C_u)} \cap X_+^n$. So, $z\mathbb{P} \succ_I (y + t(\hat{u} - u))\mathbb{P}$. By [P.1] and [P.4], we conclude $z\mathbb{P} \succ_I y\mathbb{P}$. As a result, we can replace the original extremely desirable commodity $u = (u_1, \ldots, u_n)$ with $w = (w_0, \ldots, w_0)$, where

$$w_0 = \sum_{i=1}^{n} u_i,$$

so that $w\mathbb{P} \succ_I u\mathbb{P}$ and the allocation $w\mathbb{P}_{|F} \in \mathscr{M}_F$, for each coalition $F \in \Sigma$. Henceforth, the vector w will be used instead of u in the extreme desirability assumption.

One may want to compare the properness condition expressed in [A.4] with the ATY-properness proposed in Podczek and Yannelis (2008) for a **finite** set of agents. In the coalitional language, ATY-properness would appear as follows:

For each coalition F the preference $\succ_F$ is *ATY-proper at* $x \in \mathscr{X}_F$ if there exists a convex set $\widetilde{P}_F(x)$ with a non-empty interior and such that $\text{int}\widetilde{P}_F(x) \cap \mathscr{X}_F \neq \emptyset$ and $z\mathbb{P} \succ_F x\mathbb{P}$ for all $z \in \widetilde{P}_F(x) \cap \mathscr{X}_F$. It is then clear that, under assumptions [A.1] and [A.4], for each coalition F, $\succ_F$ would be ATY-proper at each $x \in \mathscr{X}_F$ with $\widetilde{P}_F(x) = x + C_w$.

Define

$$K = \bigcup \left\{t\left(w + \frac{1}{n}U\right) : t > 0\right\}.$$

As $\frac{1}{n}U \subseteq U$, we must have $K \subseteq C_w$. Let $y \in X_+^n$ and $z \in \overline{(y + K)} \cap X_+^n$. It follows that $z \in \overline{(y + C_w)} \cap X_+^n$. Consequently, $z\mathbb{P} \succ_I y\mathbb{P}$.

Our next result follows the lines of the last part of the proof of Theorem in Rustichini and Yannelis (1991). However, since some of the steps of their original prof need to be adapted to the present situation, we shall include the whole proof for the sake of comprehension.

Lemma 3.3. *Assume that* [A.1]–[A.4] *hold. If α is a private core allocation, then $\overline{\mathscr{K}} \cap (-K) = \emptyset$.*

Proof. Since $-K$ is open, it is enough to prove that $\mathscr{K} \cap (-K)$ is empty. Assume $\mathscr{K} \cap (-K) \neq \emptyset$ and that

$$\zeta = \bar{\gamma}(F) - \bar{e}(F) \in -K$$

for some coalition F. Pick an $\varepsilon > 0$ such that $\zeta + B(0, \varepsilon) \subset -K$, where $B(0, \varepsilon)$ is the open ball in X^n centered at the origin and radius ε. By the absolute continuity of γ and e with respect to $\mathbb{P}$, there exists some $\delta > 0$ such that for all $E \in \Sigma$, $\|\overline{\gamma}(E)\|, \|\overline{e}(E)\| < \frac{\varepsilon}{7}$ whenever $\mathbb{P}(E) < \delta$.

Corresponding to $\delta = \delta\left(\frac{\varepsilon}{7}\right)$ choose $\rho = \rho(\delta)$ according to assumption [A.3]. Choose now, by means of Lemmas 2.3 and 3.1, a simple allocation s, defined by $s(\cdot, \omega) = \sum_{i=1}^{r} y_i(\omega)\mathbb{P}_{|F_i}$ for all $\omega \in \Omega$, where $\{F_i : 1 \leq i \leq r\}$ is a decomposition of F, such that

(i) $s \in \mathscr{M}_F$;
(ii) $\|\bar{\gamma} - \bar{s}\| < \min\left\{\frac{\varepsilon}{7}, \rho\right\}$.
(iii) for each $1 \leq i \leq r$, there is some $1 \leq j \leq m$ such that $F_i \in T_j$.

[2] For example, if $\Omega = \{\omega_1, \omega_2, \omega_3\}$ and $\mathscr{Q}_1 = \{\{\omega_1, \omega_2\}, \{\omega_3\}\}$ whence $\Omega_1 = \{\omega_1, \omega_3\}$ then, given the pair (s_1^1, s_2^1), we shall consider $\sigma_1 = (s_1^1, s_1^1, s_2^1)$.

In the light of [A.3], we can find some coalition $F_0 \subseteq F$ with $\mathbb{P}(F \setminus F_0) < \delta$, and $s \succ_{F_0} \alpha$. Since s is simple, $s \in \mathscr{M}_F$, $F_0 \subseteq F$, we must have $s \in \mathscr{M}_{F_0}$.

Put $\zeta_0 = \bar{s}(F_0) - \bar{e}(F_0)$, then

$$\begin{aligned}\|\zeta_0 - \zeta\| &< \|\bar{s}(F_0) - \bar{\gamma}(F_0)\| + \|\bar{\gamma}(F \setminus F_0)\| + \|\bar{e}(F \setminus F_0)\| \\ &< \frac{3\varepsilon}{7}.\end{aligned}$$

We assume that $\mathbb{P}(F_i) = \xi$ for all $1 \leq i \leq r$[3]. Since $\zeta_0 \in -K$, there exists some $t > 0$ such that

$$\bar{s}(F_0) - \bar{e}(F_0) \in -t\left(w + \frac{1}{n}U\right),$$

whence $\bar{s}(F_0) - \bar{e}(F_0) = -t(w + v_0)$ for some $v_0 \in \frac{1}{n}U$. By setting $z = \frac{t}{\xi}w$ and $v = -\frac{t}{\xi}v_0 \in \frac{t}{\xi n}U$, we have

$$\sum_{i=1}^{r} y_i + z - v = \frac{\bar{e}(F_0)}{\xi} \in X_+^n.$$

Since $\sum_{i=1}^{r} y_i + z \in X_+^n$ and $\frac{t}{\xi n}U$ is solid, we have $v^+ \in \frac{t}{\xi n}U$ and $v^+ \leq \sum_{i=1}^{r} y_i + z$. For any r-tuple $\sigma = (\sigma_1, \ldots, \sigma_r)$ of non-negative real numbers with $\sum_{i=1}^{r} \sigma_i = 1$, we have

$$v^+ \leq \sum_{i=1}^{r}(y_i + \sigma_i z).$$

By the Riesz-decomposition property, we obtain a finite set $\{v_1^\sigma, \ldots, v_r^\sigma\} \subseteq X_+^n$ such that

$$v^+ = \sum_{i=1}^{r} v_i^\sigma \text{ and } v_i^\sigma \leq y_i + \sigma_i z$$

for all $1 \leq i \leq r$. Define $\Lambda_k = \{i : F_i \subseteq I_k\}$ for all $1 \leq k \leq m$. Thus, $y_i + \sigma_i z$ is $\mathscr{Q}^k$-measurable if $i \in \Lambda_k$. For $i \in \Lambda_k$, define the function $d_i^\sigma : \Omega \to X_+$ by letting

$$d_i^\sigma(\omega) = \max\{v_i^\sigma(\omega') : \omega' \in \mathscr{Q}^k(\omega)\},$$

for all $\omega \in \Omega$. So, d_i^σ is $\mathscr{Q}^k$-measurable and $d_i^\sigma \leq y_i + \sigma_i z$ for all $i \in \Lambda_k$.

Given any $1 \leq i \leq r$, put

$$\delta_i^\sigma = \operatorname{dist}\left(y_i + \sigma_i z - d_i^\sigma, (y_i + C_w) \cap X_+^n\right),$$

and consider the continuous function $f : \Delta^r \to \Delta^r$ defined by

$$f(\sigma) = \left(\frac{\sigma_1 + \delta_1^\sigma}{1 + \sum_{j=1}^{r} \delta_j^\sigma}, \ldots, \frac{\sigma_m + \delta_r^\sigma}{1 + \sum_{j=1}^{r} \delta_j^\sigma}\right),$$

where Δ^r denotes the $(r-1)$-dimensional simplex. By Brouwer's fixed point theorem, one obtains a $\sigma^* = (\sigma_1^*, \ldots, \sigma_r^*) \in \Delta^r$ satisfying $\delta_i^{\sigma^*} = \sigma_i^* \sum_{j=1}^{r} \delta_j^{\sigma^*}$ for all $1 \leq i \leq r$. The rest of the proof is decomposed into two sub-cases.

Sub-case 1. $\delta_i^{\sigma^*} = 0$ for all $1 \leq i \leq r$. In this sub-case,

$$y_i + \sigma_i^* z - d_i^{\sigma^*} \in \overline{(y_i + C_w) \cap X_+^n} \subseteq \overline{(y_i + C_w)} \cap X_+^n$$

for all $1 \leq i \leq r$. By (ii) of [A.4], we obtain $(y_i + \sigma_i^* z - d_i^{\sigma^*})\mathbb{P} \succ_I y_i\mathbb{P}$ for all $1 \leq i \leq r$. Thus, it follows from [P.2] that $(y_i + \sigma_i^* z - d_i^{\sigma^*})\mathbb{P} \succ_{F_i} y_i\mathbb{P}$ for all $1 \leq i \leq r$. Consequently, applying [P.3], we have

$$\sum_{i=1}^{r}(y_i + \sigma_i^* z - d_i^{\sigma^*})\mathbb{P}_{|F_i} \succ_{F_0} \sum_{i=1}^{r} y_i\mathbb{P}_{|F_i} = s \succ_{F_0} \alpha.$$

Define s_1 by letting

$$s_1(\cdot, \omega) = \sum_{i=1}^{r}(y_i(\omega) + \sigma_i^* z(\omega) - d_i^{\sigma^*}(\omega))\mathbb{P}_{|F_i} + \alpha(\cdot, \omega)_{|I \setminus F}$$

for all $\omega \in \Omega$. Then $s_1 \succ_{F_0} \alpha$ and

$$\bar{s}_1(F_0) \leq \xi\left(\sum_{i=1}^{m} y_i + z - v^+\right) \leq \xi\left(\sum_{i=1}^{m} y_i + z - v\right) = \bar{e}(F_0).$$

Hence, $\alpha \notin \mathscr{PC}(\mathscr{E}_C)$, which is a contradiction.

Sub-case 2. $\sum_{j=1}^{r} \delta_j^{\sigma^*} > 0$. In this sub-case, $\delta_i^{\sigma^*} = 0$ if and only if $\sigma_i^* = 0$. Define $J = \{i : \delta_i^{\sigma^*} = 0\}$. Let $J \neq \emptyset$ and pick an $i \in J$. Then

$$y_i - d_i^{\sigma^*} \in \overline{(y_i + C_w)} \cap X_+^n.$$

From (ii) of [A.4], we conclude

$$(y_i - d_i^{\sigma^*})\mathbb{P} \succ_I y_i\mathbb{P},$$

which is a contradiction. Thus, $J = \emptyset$ and

$$y_i + \sigma_i^* z - d_i^{\sigma^*} \notin (y_i + C_w) \cap X_+^n.$$

for all $1 \leq i \leq r$. Consequently,

$$y_i + \frac{\sigma_i^* t}{\xi} w - d_i^{\sigma^*} \notin y_i + C_w,$$

which further implies that $d_i^{\sigma^*} \notin \frac{\sigma_i^* t}{\xi}U$ and so, by (i) of [A.4], $\sum_{i=1}^{r} d_i^{\sigma^*} \notin \frac{t}{\xi}U$. Note that $d_i^{\sigma^*} \leq \sum_{\omega \in \Omega} v_i^{\sigma^*}(\omega)\mathbf{1}_\Omega$, where $\mathbf{1}_\Omega = (1, \ldots, 1)$, and so

$$\sum_{i=1}^{r} d_i^{\sigma^*} \leq \sum_{\omega \in \Omega} v^+(\omega)\mathbf{1}_\Omega.$$

Since, $\sum_{\omega \in \Omega} v^+(\omega)\mathbf{1}_\Omega \in \frac{t}{\xi}U$ and $\frac{t}{\xi}U$ is solid, we must have $\sum_{i=1}^{r} d_i^{\sigma^*} \in \frac{t}{\xi}U$, which is a contradiction. □

Remark 3.4. Both Lemmas 3.2 and 3.3 slightly generalize the analogous versions in Centrone and Martellotti (2016), in that assumption [A.3] above is more general than the continuity assumption there. Indeed in Centrone and Martellotti (2016) the "large" subcoalition F_0 was assumed to depend simply upon β and τ, that is a form of uniform continuity with respect to the simple approximations of β was supposed.

4. Coalitional core-walras equivalence

In this section, we provide core-Walras equivalence theorems for the model described in Section 2. To obtain the first main result, we use the following assumption on the initial endowment.

[A.5] The following two conditions are satisfied:

(i) $e(I, \omega)$ is a quasi-interior point of X_+ for all $\omega \in \Omega$;
(ii) For every privately feasible allocation α and every partition $\{F_1, F_2\}$ of I, where F_1 and F_2 are coalitions, there exists

[3] Otherwise, it follows from Lemma 3.1 in Centrone and Martellotti (2016) that there are a subset $E_0 \subseteq F_0$ with $\mathbb{P}(F_0 \setminus E_0) < \delta$ and a decomposition $\{E_1, \ldots, E_k\}$ of E_0 with $\mathbb{P}(E_i) = \xi$ for all $1 \leq i \leq k$. If $\zeta' = \bar{s}(E_0) - \bar{e}(E_0)$, then

$$\begin{aligned}\|\zeta - \zeta'\| &< \|\zeta - \zeta_0\| + \|\zeta_0 - \zeta'\| \\ &< \frac{3\varepsilon}{7} + \|\bar{s}(F_0) - \bar{\gamma}(F_0)\| + \|\bar{\gamma}(F_0 \setminus E_0)\| \\ &\quad + \|\bar{s}(E_0) - \bar{\gamma}(E_0)\| + \|\bar{e}(F_0 \setminus E_0)\| \\ &< \varepsilon.\end{aligned}$$

As a result, $\zeta' = \bar{s}(E_0) - \bar{e}(E_0) \in -K$ with $\bar{s} \succ_{E_0}^n \bar{\alpha}$.

some $\beta \in \mathscr{M}_{F_2}$ such that $\beta \succ_{F_2} \alpha$ and

$$\beta(F_2, \omega) \leq e(F_1, \omega) + \alpha(F_2, \omega)$$

for all $\omega \in \Omega$.

The second condition is known as *irreducibility assumption* (Basile et al., 2009). We shall also require a further continuity condition on coalitional preferences, namely

[A.6] Let F be a coalition and $\alpha, \beta \in \mathscr{M}_F$ be such that $\beta \succ_F \alpha$. For every $\tau > 0$, there exist an $\varepsilon \in (0, 1)$ and a coalition $F_0 = F_0(\tau) \subset F$ such that $\mathbb{P}(F \setminus F_0) < \tau$ and $\varepsilon\beta \succ_{F_0} \alpha$.

We are now ready to state our first core-Walras equivalence Theorem.

Theorem 4.1. *Suppose that $\mathscr{E}_C$ satisfies* [A.1]–[A.6]*, and let α be a private core allocation. Then there exists an equilibrium price for α.*

Proof. Applying Lemmas 3.2 and 3.3 together with the separation theorem, we can find an n-tuple $p = (p_1, \ldots, p_n) \in (X_+^*)^n$ that separates $\overline{\mathscr{K}}$ and $-K$. As usual, this would yield that $px \geq 0$ for every $x \in \mathscr{K}$. Define $\pi : \Omega \to X_+^*$ by letting $\pi(\omega_i) = p_i$ for all $1 \leq i \leq n$. To show that (α, π) is a competitive equilibrium of $\mathscr{E}_C$, we need to verify conditions (i)–(iii) of Definition 2.6. By invoking arguments similar to those of Basile et al. (2009), items (i) and (ii) of Definition 2.6 can be proved. Thus, we now turn to prove assertion (iii). Observe first that (i) and (ii) together imply $p\bar{\alpha} = p\bar{e}$ on Σ. Suppose (iii) is not true and that there are a coalition E and an allocation $\beta \in \mathscr{M}_E$ such that $\beta \succ_E \alpha$ and $p[\bar{\beta}(E)] = p[\bar{e}(E)]$.[4] The rest of the proof is decomposed in the following three cases:

Case 1. $p[\bar{\beta}(E)] > 0$. Then there exists some $\tau > 0$ such that for each sub-coalition F of E with $\mathbb{P}(E \setminus F) < \tau$, we have $p[\bar{\beta}(F)] > 0$. From [A.6] there is a $\varepsilon \in (0, 1)$ and a subcoalition $F \subset E$ with $\mathbb{P}(E \setminus F) < \tau$ and such that $\varepsilon\beta \succ_F \alpha$. Then the allocation $\gamma = \varepsilon\beta_{|_F} + \beta_{|_{I\setminus F}}$ is in $\mathscr{M}_E$ and $\gamma \succ_E \alpha$; therefore $\overline{\gamma}(E) - \bar{e}(E) \in \mathscr{K}$, whence

$$\begin{aligned} p[\bar{e}(E)] &\leq p[\overline{\gamma}(E)] = \varepsilon p[\overline{\beta}(F)] + p[\overline{\beta}(E \setminus F)] \\ &< p[\overline{\beta}(F)] + p[\overline{\beta}(E \setminus F)] = p[\overline{\beta}(E)] = p[\bar{e}(E)], \end{aligned}$$

which is impossible.

Case 2. $p[\bar{\beta}(E)] = 0$. In this case, $p[\bar{e}(E)] = 0$. The rest of the proof is decomposed into following two sub-cases:

Sub-case 1. There exists an allocation γ such that $\gamma \in \mathscr{M}_{I\setminus E}$, $\gamma \succ_{I\setminus E} \alpha$ and $p[\bar{\gamma}(I \setminus E)] = p[\bar{e}(I \setminus E)]$. Applying (i) of [A.5], we have $p[\bar{e}(I \setminus E)] = p[\bar{e}(I)] > 0$. Thus, we would again fall in the contradiction determined by occurrence of *Case 1* with $I \setminus E$ in the role of E.

Sub-case 2. There does not exist any allocation γ such that $\gamma \in \mathscr{M}_{I\setminus E}$, $\gamma \succ_{I\setminus E} \alpha$ and $p[\bar{\gamma}(I\setminus E)] = p[\bar{e}(I\setminus E)]$. In this sub-case, setting $F_1 = E$, $F_2 = I \setminus E$, by (ii) of [A.5], there should be some $\gamma_* \in \mathscr{M}_{F_2}$ with $\gamma_* \succ_{F_2} \alpha$ and

$$\gamma_*(F_2) \leq \bar{e}(F_1) + \bar{\alpha}(F_2).$$

This, along with the fact that $px \geq 0$ for all $x \in \mathscr{K}$, yields $p[\bar{e}(F_1)] + p[\bar{\alpha}(F_2)] \geq p[\bar{\gamma}_*(F_2)]$ and $p[\bar{\gamma}_*(F_2)] > p[\bar{e}(F_2)]$. But, $p[\bar{e}(F_2)] = p[\bar{\alpha}(F_2)]$. Thus,

$$p[\bar{e}(F_1)] + p[\bar{e}(F_2)] = p[\bar{e}(F_1)] + p[\bar{\alpha}(F_2)] \geq p[\bar{\gamma}_*(F_2)] > p[\bar{e}(F_2)]$$

whence $p[\bar{e}(F_1)] = p[\bar{e}(E)] > 0$, which leads to a contradiction. □

5. Exact feasibility

In this section, we discuss the core-Walras equivalence theorem in the case of exact feasibility and discuss the issue of incentive compatibility.

[4] By [P.2] and [P.4], we ignore the case $p[\bar{\beta}(E)] < p[\bar{e}(E)]$.

5.1. Equivalence theorem

The physical feasibility of an allocation for any coalition in most of asymmetric information frameworks in the literature (also in our model) is expressed in terms of an inequality while the feasibility of an allocation in a complete information economy is expressed by means of an equality. Towards this direction, the question has been raised by some authors (for instance, Angeloni and Martins-da Rocha (2009), Einy et al. (2001)) whether free disposal is necessary in the definition of physical feasibility in order to obtain core-Walras equivalence theorems. We now show that a core-Walras equivalence theorem can be established under the exact feasibility condition in the presence of an additional assumption.

Assume that [A.4] is satisfied, i.e., the economy under consideration has a properness pair (u, U). Define

$$\tilde{w}_0 = \sum_{i=1}^{n} \left(u_i + m \sum_{j=1}^{m} e(I_j, \omega_i) \right).$$

Since (u, U) is a properness pair, we can also conclude that $(\tilde{w}, U)$ is a properness pair, where $\tilde{w} = (\tilde{w}_0, \ldots, \tilde{w}_0)$. To introduce our additional assumption, we first recall Proposition 3.1 in Martellotti (2007).

Proposition 5.1. *If (v, V) is a properness pair, then there exist a positive functional $x^* \in (X_+^*)^n$ and $c > 0$ such that $V \cap X_+^n = G^-(x^*, c) \cap X_+^n$ and $V^c \cap X_+^n = F^+(x^*, c) \cap X_+^n$, where $G^-(x^*, c)$ (respectively $F^+(x^*, c)$) denotes the open lower (resp. closed upper) half space determined by the hyperplane $\{x \in X^n : x^*(x) = c\}$.*

As a consequence of Proposition 5.1 and the fact that $(\tilde{w}, U)$ is a properness pair, there are some positive functional $x^* \in (X_+^*)^n$ and $c > 0$ such that (a) $x^*(x) < c$ for all $x \in U \cap X_+^n$; and (b) $x^*(x) \geq c$ for all $x \in U^c \cap X_+^n$. Since $(\tilde{w}, U)$ is a properness pair, we have $\tilde{w} \in U^c$. Therefore, $x^*(\tilde{w}) \geq c$.

Lemma 5.2. *Suppose that $\mathscr{E}_C$ satisfies* [A.1]–[A.3]*. Assume that $(\tilde{w}, \lambda U)$ remains a properness pair, where $\lambda = \dfrac{x^*(\tilde{w})}{c}$. If α is a private core allocation, then $\overline{\mathscr{K}} \cap (-C) = \emptyset$, where*

$$C = \bigcup \{t(\tilde{w} + \lambda U) : t > 0\}.$$

Proof. Similarly to Lemma 3.3, it is enough to prove that $\mathscr{K} \cap (-C)$ is empty. For each $1 \leq i \leq m$, let

$$\mathscr{K}_i = \bigcup_{F\in\Sigma,\ \mathbb{P}(F)>0} \{\bar{\gamma}(F \cap I_i) - \bar{e}(F \cap I_i) : \gamma \in \mathscr{M}_F, \gamma \succ_F \alpha\}.$$

Claim 1. $\mathscr{K}_i \cap (-C) = \emptyset$ for all $1 \leq i \leq m$ implies $\mathscr{K} \cap (-C) = \emptyset$. The claim can be done if we show that $\bar{\gamma}(E) - \bar{e}(E) \in \mathscr{K} \cap (-C)$ for some coalition E implies $\bar{\gamma}(E \cap I_j) - \bar{e}(E \cap I_j) \in -C$ for some j. Thus, we assume that $\bar{\gamma}(E) - \bar{e}(E) = -t(\tilde{w} + v)$ for some $v \in \lambda U$, $t > 0$. By putting $\bar{\beta} = \bar{\gamma} + \dfrac{tv^+}{\mathbb{P}(E)}\mathbb{P}$, we have $\bar{\beta}(E) - \bar{e}(E) + t\tilde{w} = tv^-$, which is equivalent to $\bar{\beta}(E) - \bar{e}(E) + \tilde{w} = tv^- + (1-t)\tilde{w}$. Clearly, $\bar{\beta}(E) - \bar{e}(E) \in \mathscr{K}$ and $v^- \in \lambda U$. Since $(\tilde{w}, \lambda U)$ is a properness pair, as a consequence of Proposition 5.1, we have $z \in \lambda U \cap X_+^n$ implies $x^*(z) < \lambda c = x^*(\tilde{w})$. Thus, $tv^- + (1-t)\tilde{w} \in X_+^n$ and $x^*(tv^- + (1-t)\tilde{w}) < \lambda c$. As a result, $\bar{\beta}(E) - \bar{e}(E) + \tilde{w} \in \lambda U \cap X_+^n$. If $\mathscr{K}_i \cap (-C) = \emptyset$ for all $1 \leq i \leq m$, then we have

$$\bar{\beta}(E \cap I_i) - \bar{e}(E \cap I_i) + \frac{1}{m}\tilde{w} \in \left(\frac{\lambda U}{m}\right)^c.$$

It follows from the definition of $\tilde{w}$ that

$$\bar{\beta}(E \cap I_i) - \bar{e}(E \cap I_i) + \frac{1}{m}\tilde{w} \in X_+^n.$$

The convexity of $\left(\frac{\lambda U}{m}\right)^c \cap X_+^n$ implies

$$\bar{\beta}(E) - \bar{e}(E) + \tilde{w} = \sum_{i=1}^{m} \left[\bar{\beta}(E \cap I_i) - \bar{e}(E \cap I_i) + \frac{1}{m}\tilde{w}\right] \notin \lambda U,$$

which is a contradiction.

Claim 2. $\mathscr{K}_i \cap (-C) = \emptyset$ *for all* $1 \le i \le m$. This can be proven by invoking arguments similar to those in the proof of Lemma 3.3 and using the monotonicity of $\succ_I$.[5] □

Remark 5.3. As the extremely desirable commodity bundle w is larger than the extremely desirable bundle u, the extra assumption in the previous Lemma says that w remains extremely desirable when added to a bundle y even if one subtracts something relatively "large", namely almost at the level of hyperplane determined by means of Proposition 5.1. This assumption is employed to demonstrate that Lemma 3.1 (and hence Theorem 4.1) can be obtained in a framework without free disposal assumption. However, in the absence of this assumption, a slightly different approach has been used in Lemma 3.1 under free disposal assumption. Note that such an approach is not applicable for the case when the feasibility is defined to be exact (without free disposal).

5.2. Incentive compatibility of core allocations

In this subsection, following Angeloni and Martins-da Rocha (2009) and Koutsougeras and Yannelis (1993), we introduce the concept of (weak) coalitional incentive compatibility of an allocation and show that the private core allocations are (weakly) coalitional incentive compatible.

The private core, as well as a Walras expectations equilibrium, is defined to be as ex-ante solution concepts corresponding to actions taken at $\tau = 0$, that is, before the resolution of uncertainty. In this period, all signed contracts specify deliveries and receipts of commodities contingent on the realized state at $\tau = 1$. However, if the execution of some contract in some state ω at $\tau = 1$ would lead to an inferior consumption bundle compared to the initial endowment for some coalition, then the coalition may have an incentive to refuse to admit that ω has been realized, even though the coalition knows that ω is a true state and that this is known by its complementary coalition too. Thus, in order to address the issue of execution (or enforcement) of contracts at $\tau = 1$, we assume that there is an intermediary (a "government institution" or a "market institution") who can verify the occurrence of the state ω and is responsible for the execution of contracts at $\tau = 1$. The aforesaid assumption is reasonable if we are in the symmetric information framework. But one cannot assume that the intermediary knows the true state in the asymmetric information framework. Otherwise, each coalition anticipates such knowledge of intermediary at $\tau = 0$ and then, trusting the enforcement capabilities of the intermediary at $\tau = 1$, coalitions might sign at $\tau = 0$, non-measurable contracts with respect to the private information, even though they may not be able by themselves to identify the true state at $\tau = 1$.

We thus assume that the intermediary has an incomplete information about the true state. Given this and the assumption that different coalitions may have different information, coalitions may have incentives to misreport the private information and claim the net trade corresponding to the false information reported by them. To clarify this, suppose that some state ω_0 is realized and $\succ_E^{\omega}$ is an ex-post preference of coalition E at an arbitrary state ω, that is $\succ_E^{\omega}$ is a preference relation on the set $\mathscr{M}^{\omega} = \{\alpha(\cdot, \omega) : \Sigma \to X^+ : \alpha \in \mathscr{M}\}$.[6] Then each coalition E only knows that the state of nature belongs to $\mathscr{P}_E(\omega_0)$, but does not know exactly the true state of nature. Now, given a feasible allocation α (that is, a contract at $\tau = 0$), if $e(E, \omega) + \alpha(E, \omega_E) - e(E, \omega_E) \in X_+$ and $e(\cdot, \omega) + \alpha(\cdot, \omega_E) - e(\cdot, \omega_E) \succ_E^{\omega} \alpha(\cdot, \omega)$ for all $\omega \in \mathscr{P}_E(\omega_0)$[7] and some state $\omega_E \notin \mathscr{P}_E(\omega_0)$, the coalition E will gain by reporting the state ω_E when the true state is ω_0. Suppose now that two coalitions E and $F = I \setminus E$ misreport that the true states are ω_E and ω_F, respectively, when the actual true state is ω_0. If $\alpha(E, \omega_E) + \alpha(F, \omega_F) \neq e(E, \omega_E) + e(F, \omega_F)$, then the intermediary cannot execute the contract. In order to avoid such false information, we assume that there is a legal procedure that any coalition can use to prove that either the coalition itself is not misreporting or the complementary coalition is reporting a false information, and that this procedure is costly and the cost of the procedure should be paid by the coalition reporting a false information. Then we can conclude that a coalition will not misreport unless it is sure that the misreport cannot be detected by any other coalition. Therefore, a coalition $E \subseteq I_i$ cannot lie whenever $\mu(E) < \mu(I_i)$ because $I_i \setminus E$ has the same information as the coalition E. So, if some coalition E lies then it must be the union of some I_i's. Moreover, when ω_0 is a true state, a coalition E will have an incentive to misreport by announcing the state ω' if (i) the complementary coalition cannot discern ω' and $\mathscr{P}_E(\omega_0)$, that is, $\{\omega'\} \cup \mathscr{P}_E(\omega_0) \subseteq \mathscr{P}_{I \setminus E}(\omega_0)$; and (ii) $e(E, \omega) + \alpha(E, \omega') - e(E, \omega') \in X_+$ and $e(E, \omega) + \alpha(E, \omega') - e(E, \omega') \succ_E^{\omega} \alpha(E, \omega)$ for all $\omega \in \mathscr{P}_E(\omega_0)$. This motivates us to define the concept of coalitional incentive compatibility.

Definition 5.4. A physically feasible allocation α is called *coalitionally incentive compatible* if it is not possible to find a coalition E and two states ω_0, ω' such that

(i) $\{\omega'\} \cup \mathscr{P}_E(\omega_0) \subseteq \mathscr{P}_{I \setminus E}(\omega_0)$;
(ii) $e(E, \omega) + \alpha(E, \omega') - e(E, \omega') \in X_+$ for all $\omega \in \mathscr{P}_E(\omega_0)$; and
(iii) $e(E, \omega) + \alpha(E, \omega') - e(E, \omega') \succ_E^{\omega} \alpha(E, \omega)$ for all $\omega \in \mathscr{P}_E(\omega_0)$.

Thus, a physically feasible allocation is coalitionally incentive compatible if it is not possible to find a coalition E and two states ω_0 and ω' such that the complementary coalition cannot discern ω' and $\mathscr{P}_E(\omega_0)$ and the coalition E is better off by announcing ω' whenever ω_0 is the true state. We now state and prove the coalitional incentive compatibility of feasible allocations, where the physical feasibility is defined as exact. This result, in particular, implies that any private core allocation is coalitionally incentive compatible.

Theorem 5.5. *Every feasible allocation is coalitionally incentive compatible.*

Proof. Suppose, by contradiction, that a feasible allocation α is not coalitionally incentive compatible, namely there exist a coalition E and two states ω_0 and ω' such that conditions (i)–(iii) in Definition 5.4 are satisfied. It follows from (i) that $\alpha(I \setminus E, \omega) = \alpha(I \setminus E, \omega')$ and $e(I \setminus E, \omega) = e(I \setminus E, \omega')$ for all $\omega \in \mathscr{P}_E(\omega_0)$. The feasibility of α implies that $\alpha(I, \omega) - e(I, \omega) = 0 = \alpha(I, \omega') - e(I, \omega')$ for all $\omega \in \mathscr{P}_E(\omega_0)$, which further implies $\alpha(I, \omega) - \alpha(I, \omega')$

[5] The monotonicity assumption will be used to obtain exact physically feasible allocation from s_1 to block a private core allocation.

[6] Given $\alpha, \beta \in \mathscr{M}$, we assume that $\alpha(\cdot, \omega) \succeq_E^{\omega} \beta(\cdot, \omega)$ for all $\omega \in \Omega$ and $\alpha(\cdot, \omega') \succ_E^{\omega'} \beta(\cdot, \omega')$ for some $\omega' \in \Omega$ imply $\alpha \succ_E \beta$, where $\alpha(\cdot, \omega) \succeq_E^{\omega} \beta(\cdot, \omega)$ means that for every sub-coalition F of E, $\beta(\cdot, \omega)$ is not preferred to $\alpha(\cdot, \omega)$ by $\succ_F^{\omega}$.

[7] Since e and α are $\mathscr{F}_E$-measurable, it is equivalent to $e(E, \omega_0) + \alpha(E, \omega_E) - e(E, \omega_E) \in X_+$ and $e(\cdot, \omega_0) + \alpha(\cdot, \omega_E) - e(\cdot, \omega_E) \succ_E^{\omega_0} \alpha(\cdot, \omega_0)$ if $\succ_F^{\omega} = \succ_F^{\omega'}$ for all $\omega, \omega' \in \Omega$ and all coalitions F.

$= e(I,\omega) - e(I,\omega')$ for all $\omega \in \mathscr{P}_E(\omega_0)$. Since α and e are both finitely additive, we must have $\alpha(E,\omega) - \alpha(E,\omega') = e(E,\omega) - e(E,\omega')$ for all $\omega \in \mathscr{P}_E(\omega_0)$. Hence, $e(E,\omega) + \alpha(E,\omega') - e(E,\omega') = \alpha(E,\omega)$ for all $\omega \in \mathscr{P}_E(\omega_0)$, which contradicts (iii). □

6. Individual core-Walras results

In this section, we derive individualistic core-Walras results in an economy with asymmetric information, from the equivalences stated in Theorem 4.1. We can express an individualistic economic model as follows

$$\mathscr{E}_I = \{(I,\Sigma,\mathbb{P}); X_+; (\Omega,\mathscr{F}); (\mathscr{F}_t, U_t, \eta(t,\cdot), \mathbb{P}_t)_{t\in I}\},$$

where (i) $(I,\Sigma,\mathbb{P})$ is a measure space of agents where $\mathbb{P}$ is a non-atomic countably additive measure on the σ-algebra Σ; (ii) X_+ and $(\Omega,\mathscr{F})$ are the same as in $\mathscr{E}_C$; (iii) $\mathscr{F}_t$ is the σ-algebra generated by a partition $\mathscr{P}_t \subseteq \mathscr{F}$ of Ω representing the *private information* of agent t; (iv) $U_t : \Omega \times X_+ \to \mathbb{R}$ is the *state-dependent utility function* of agent t, representing the (ex post) preference of agent t; (v) $\eta(t,\cdot) : \Omega \to X_+$ is the *initial endowment density* of agent t; and (vi) $\mathbb{P}_t$ is a probability measure on $\mathscr{F}$, representing the *prior belief* of agent t.

The *ex ante expected utility* of an agent t for $x : \Omega \to X_+$ is defined by $V_t(x) = \sum_{\omega\in\Omega} U_t(\omega, x(\omega))\mathbb{P}_t(\omega)$ and the *consumption set* of an agent t is defined by

$$\mathscr{X}_t = \left\{x \in X_+^n : x \text{ is } \mathscr{F}_t\text{-measurable}\right\}.$$

An *allocation* in $\mathscr{E}_I$ is a function $f : I \times \Omega \to X_+$ such that $f(\cdot,\omega)$ is Bochner integrable for all $\omega \in \Omega$. It is said to be *privately feasible* whenever $f(t,\cdot) \in \mathscr{X}_t$ $\mathbb{P}$-a.e., and *physically feasible* if $\int_I f(\cdot,\omega)d\mathbb{P} \le \int_I \eta(\cdot,\omega)d\mathbb{P}$ for all $\omega \in \Omega$. Furthermore, we say that an allocation is *feasible* if it is privately as well as physically feasible. An allocation f is *privately blocked by a coalition F* if there is an allocation g such that $g(t,\cdot) \in \mathscr{X}_t$ and $V_t(g(t,\cdot)) > V_t(f(t,\cdot))$ for all $t \in S$, and $\int_S f(\cdot,\omega)d\mathbb{P} \le \int_S \eta(\cdot,\omega)d\mathbb{P}$ for all $\omega \in \Omega$. The *private core* of $\mathscr{E}_I$, denoted by $\mathscr{PC}(\mathscr{E}_I)$, is the set of feasible allocations which are not privately blocked by any coalition. A *Walras expectations equilibrium* of $\mathscr{E}_I$ is a pair (f,π), where f is a feasible allocation and π is a price system, such that $f(t,\cdot)$ maximizes $\mathscr{B}(t,\pi)$ $\mathbb{P}$-a.e.[8] and

$$\sum_{i=1}^{n} \pi(\omega_i) \int_I f(\cdot,\omega_i)d\mathbb{P} = \sum_{i=1}^{n} \pi(\omega_i) \int_I \eta(\cdot,\omega_i)d\mathbb{P}.$$

We assume that X is separable. Let $\{\mathscr{Q}_1,\ldots,\mathscr{Q}_k\}$ be a collections of partitions of Ω such that $I_i = \{t \in I : \mathscr{P}_t = \mathscr{Q}_i\}$ is measurable and $\mathbb{P}(I_i) > 0$ for all $1 \le i \le k$. We assume that $I = \bigcup\{I_i : 1 \le i \le k\}$. Consider a function $\varphi : (I,\Sigma,\mathbb{P}) \to \Delta^n$ defined by $\varphi(t) = \mathbb{P}_t$ for all $t \in I$, where Δ^n denotes the $(n-1)$-simplex of $\mathbb{R}^n$. The function φ is assumed to be measurable, where Δ^n is endowed with the Borel structure. For each $\omega \in \Omega$, define a function $\psi_\omega : I \times X_+ \to \mathbb{R}$ by $\psi_\omega(t,x) = U_t(\omega,x)$. For each $\omega \in \Omega$, the function ψ_ω is assumed to be Carathéodory, that is, $\psi_\omega(\cdot,x)$ is measurable for all $x \in X_+$ and $\psi_\omega(t,\cdot)$ is norm-continuous for all $t \in I$. The following assumptions will also be used

[B.1] For all $(t,\omega) \in I \times \Omega$, $U_t(\omega, x+y) > U_t(\omega,x)$ for all $x, y \in X_+$ with $y > 0$.

[B.2] There exist some $u \in (X_+)^n$ and an open, convex, solid neighborhood U of 0 in X^n such that (i) $U^c \cap (X_+)^n$ is convex; and (ii) $y \in (X_+)^n$ and $z \in \overline{(y + C_u)} \cap (X_+)^n$ implies $V_t(z) > V_t(y)$ $\mathbb{P}$-a.e., where $C_u = \bigcup\{t(u+U) : t > 0\}$.

[B.3] η is a privately feasible allocation such that $\int_I \eta(\cdot,\omega)d\mathbb{P}$ is a quasi-interior point of X_+ for all $\omega \in \Omega$; and for every privately feasible allocation f and every partition $\{F_1,F_2\}$ of I, where F_i is a coalition, there exists an allocation g such that $g(t,\cdot) \in \mathscr{X}_t$ and $V_t(g(t,\cdot)) > V_t(f(t,\cdot))$ $\mathbb{P}$-a.e. on F_2, and $\int_{F_2} g(\cdot,\omega)d\mathbb{P} \le \int_{F_1} \eta(\cdot,\omega)d\mathbb{P} + \int_{F_2} f(\cdot,\omega)d\mathbb{P}$ for all $\omega \in \Omega$.

For each allocation f in $\mathscr{E}_I$, we are associating an allocation $\Xi[f]$ in $\mathscr{E}_C$ by letting $\Xi[f](E,\omega) = \int_E f(\cdot,\omega)d\mathbb{P}$. For each $F \in \Sigma$, we define $\mathscr{P}_F$ to be the smallest partition that refines each $\mathscr{P}_t$ for all $t \in F$. Thus, the individualistic economy $\mathscr{E}_I$ corresponds to the coalitional economy $\mathscr{E}_C$ given by

$$\mathscr{E}_C = \{(I,\Sigma,\mathbb{P}); X_+; (\Omega,\mathscr{F}); (\mathscr{F}_F, \succ_F, e(F,\cdot))_{F\in\Sigma}\},$$

where $e(F,\cdot) = \Xi[\eta](F,\cdot)$; $\mathscr{F}_F$ is the σ-algebra generated by $\mathscr{P}_F$; and the coalitional preference $\succ_F$ is defined by letting $\alpha \succ_F \beta$ if and only if $V_t(a(t,\cdot)) > V_t(b(t,\cdot))$ $\mathbb{P}$-a.e. on F, where $a(\cdot,\omega)$ and $b(\cdot,\omega)$ are Radon–Nikodym derivatives of $\alpha(\cdot,\omega)$ and $\beta(\cdot,\omega)$, respectively, for each $\omega \in \Omega$ with respect to $\mathbb{P}$. Analogously, one can define $\alpha(\cdot,\omega) \succ_F \beta(\cdot,\omega)$ if and only if $U_t(\omega, a(t,\omega)) > U_t(\omega, b(t,\omega))$ $\mathbb{P}$-a.e. on F. Since Proposition 4.2 of Basile et al. (2009) can be easily extended to this new framework, we have the following core-Walras equivalence theorem.

Theorem 6.1. *Let* [B.1]–[B.3] *be satisfied for the economy* $\mathscr{E}_I$. *A feasible private allocation* f *belongs to the private core of* $\mathscr{E}_I$ *if and only if it is a Walras expectations allocation of* $\mathscr{E}_I$.

By taking an assumption similar that of [A.7] and the results and techniques in Sections 4 and 5, one can obtain a similar individualistic core-Walras equivalence theorem without free disposal, in the sense of Angeloni and Martins-da Rocha (2009). This would then contain Theorem 5.1 in Angeloni and Martins-da Rocha (2009) as a corollary when preferences are represented by continuous and monotone utilities. Indeed, all the hypotheses of Theorem 6.1 are either explicitly assumed, or stated as part of the standard definition for the economy in Angeloni and Martins-da Rocha (2009) in a framework of an Euclidean space as the commodity space. Note that our core-Walras equivalence theorems in coalitional as well as the individualistic models are first attempts to the literature for the case of exact feasibility and an infinite dimensional commodity space.

Let $\omega \in \Omega$ and let $\mathscr{Q}_i(\omega)$ be the atomic event in $\mathscr{Q}_i$ which contains ω.[9] If $J \subseteq \{1,\ldots,k\}$ then we denote by $\mathscr{Q}_J$ the meet of $\mathscr{Q}_i$ for all $i \in J$ which is interpreted as the common knowledge information of the coalition whose members' types belong to $\{\mathscr{Q}_i : i \in J\}$. For any coalition E, we define $\mathscr{P}_E(\omega)$ to be equal to $\mathscr{Q}_{I(E)}(\omega)$, where $\mathscr{Q}_{I(E)}(\omega)$ is the atomic element of $\mathscr{Q}_{I(E)}$ and $I(E) = \{j \in \{1,\ldots k\} : \mathbb{P}(E \cap I_j) > 0\}$.

We now recall the definition of coalitional incentive compatibility for individualistic models given by Koutsougeras and Yannelis (Koutsougeras and Yannelis, 1993), as stated in Angeloni and Martins da Rocha (Angeloni and Martins-da Rocha, 2009) for the case of economies with a continuum of agents.

Definition 6.2. A feasible allocation f is said to be *coalitionally incentive compatible in* $\mathscr{E}_I$ if it is not possible to find a coalition E and two states ω_0, ω' such that

(i) $\{\omega'\} \cup \mathscr{P}_E(\omega_0) \subseteq \mathscr{P}_t(\omega_0)$, $\mathbb{P}$-a.e. on $I \setminus E$;
(ii) for every $\widetilde{\omega} \in \mathscr{P}_E(\omega_0)$, $e(t,\widetilde{\omega}) + f(t,\omega') - e(t,\omega') \in X_+$, $\mathbb{P}$-a.e. on E;
(iii) for every $\widetilde{\omega} \in \mathscr{P}_E(\omega_0)$, $U_t(\widetilde{\omega}, e(t,\widetilde{\omega}) + f(t,\omega') - e(t,\omega')) > U_t(\widetilde{\omega}, f(t,\widetilde{\omega}))$, $\mathbb{P}$-a.e. on E.

[8] $\mathscr{B}(t,\pi)$ denotes the ex ante budget set of agent t at price π.

[9] Recall that we assume that there exists a collection $\{\mathscr{Q}_1,\ldots,\mathscr{Q}_k\}$ of partitions of Ω such that $I_j = \{t \in I : \mathscr{P}_t = \mathscr{Q}_j\}$ is measurable and $\mathbb{P}(I_j) > 0$ for all $1 \le j \le k$.

Condition (i) says that almost all members of $I \setminus E$ cannot distinguish ω' and any element of $\mathscr{P}_E(\omega_0)$. Conditions (ii) and (iii) together imply that almost all members of E are better off by misreporting the state ω' whenever the true state is ω_0.

It is known from Angeloni and Martins-da Rocha (2009) and Koutsougeras and Yannelis (1993) that any feasible allocation that belongs to the private core of $\mathscr{E}_I$ is coalitionally incentive compatible in $\mathscr{E}_I$ where the physical feasibility is defined to be exact. Thus, it would be interesting to obtain the above result as a corollary of Theorem 5.5. As X has the RNP, it is true that f is in the private core of $\mathscr{E}_I$ if and only if $\Xi[f]$ belongs to the private core of $\mathscr{E}_C$ (indeed, Lemma 4.1 and Point 1. of Proposition 4.2 in Basile et al. (2009) are straightforwardly adapted). However, at this moment, it is unclear to us the link between the two concepts of incentive compatibility for the individualistic and coalitional case. In particular, to derive the result of Angeloni and Martins-da Rocha (2009) and Koutsougeras and Yannelis (1993) from Theorem 5.5 one should prove that the incentive compatibility of $\Xi[f]$ implies the one of f: as this does not appear to be an immediate task, it will be the subject of further work.

7. Concluding remarks

Remark 7.1. Our first concern will be that of comparing Theorem 6.1 with some of the core-Walras equivalence result for economies with asymmetric information already existing in the literature. Most of the results are given under the assumption that the commodity space coincides with the Euclidean space $\mathbb{R}^\ell$ for any given $\ell \geq 1$. In this case, clearly X is a separable Banach lattice having the RNP, and [B.2] is a default.

We begin our overview with Basile et al. (2009), where the commodity space is $\mathbb{R}^\ell$. The coalitional results in Basile et al. (2009) (i.e. Theorem 3.2 and Theorem 3.7) cannot be derived from our coalitional equivalences for two main reasons: (a) we are assuming a different form of continuity, and (b) we need assumption [A.2]. Nevertheless, when one turns to the individual formulation, Theorem 4.3 in Basile et al. (2009) can be proven via Theorem 4.1. In fact, the private feasibility of η in [B.3], although not explicitly stated, is mentioned as an implicit assumption (and needed to have the condition (A.4) of Basile et al. (2009) fulfilled). All the other conditions in Theorem 4.3 of the aforementioned paper either coincide or imply those of Theorem 6.1.

Secondly, our individualistic result (i.e. Theorem 6.1) is not a direct extension of the core-Walras equivalence results in Bhowmik (2013), Evren and Hüsseinov (2008) as the commodity spaces in Bhowmik (2013), Evren and Hüsseinov (2008) are not necessarily satisfying the RNP.

Remark 7.2. We conclude this paper with a list of possible directions of further investigations, and problems where the setting that we propose here (X has the RNP and preferences satisfy the transitivity and the properness-like assumption) could enlarge the class of economies in which previous results can be extended:

- Different types of the core are considered by several authors, both in the finite dimensional (Allen, 2006; Einy et al., 2001) and in infinite dimensional (Bhowmik, 2015; Graziano and Meo, 2005) commodity spaces; it would be interesting to investigate whether the results obtained for these cores in the mentioned papers can be extended under the properness-like assumption to a Banach lattice X having the RNP.

- A huge variety of papers focus on the *existence* results in the framework of differential information (Angeloni and Martellotti, 2004; Einy et al., 2001; Podczek and Yannelis, 2008). Do the assumptions proposed in our model provide extra tools to prove the existence of an equilibrium?

- Another problem one may think of, concerns of the transitivity assumption. Transitivity is standard in models making use of utility functions, and therefore also in coalitional models à la Radner stemming from such individual economies. Although it is quite a reasonable assumption, it is a fact that there exists literature not assuming it in the coalitional setting; hence, the one presented here is not the most general model. Therefore, it could be quite interesting to examine whether one can weaken the transitivity assumption, for instance directly requiring that the extremely desirable commodity is measurable with respect to any partition in $\mathfrak{P}_0$.

- A final problem to mention is the necessity of assumption [A.2] in the coalitional setting. We have not been able to provide a counterexample in this direction so far; and it could be in fact true that one could move from an economy where [A.2] does not hold to the finer economy where the σ-algebra of coalitions is enlarged somehow to the one generated by Σ and $\{I_i : 1 \leq i \leq m\}$. Perhaps a suitable extension of the probability $\mathbb{P}$ would provide a way to derive equivalence results in more general situations than those proved in this paper.

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