Corrections on
SOME INSIGHTS ABOUT THE SMALL BALL PROBABILITY FACTORIZATION FOR HILBERT RANDOM ELEMENTS

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1. In defining the residual term $\mathcal{R}(x,\varepsilon,d)$ in equation (3.1) (and (3.5) in the Supplementary materials), the mean is conditioned to $(x_1,\dots,x_d)$ as follows:
$$\mathcal{R}(x,\varepsilon,d)=\mathbb{E}\left[(1-S)^{d/2}\,\mathbb{I}_{\{S\le 1\}}\middle|(x_1,\dots,x_d)\right].$$
This new definition leaves true all the remaining statements in the paper; in particular, the min–max Courant–Fisher principle guarantees that Lemma 1 (in the Supplementary Materials) and Proposition 1 hold true.

2. In the proof of Theorem 1 (in the Supplementary materials), the following modifications have to been made:

   – The density $f_d(\cdot)$ which appears at the top of page 2 (in lines 1, 3 and 6) is the conditional density $f_{d|s}(\cdot)$ to $S=s$;

   – The first two equations at the top of page 3 have to be modified as follows:
$$\left|\frac{\int_0^1(\varphi(s|x,\varepsilon,d)-f(x_1,\dots,x_d)I)\,dG(s)}{\int_0^1 f(x_1,\dots,x_d)IdG(s)}\right|\le$$
$$\le\left|\frac{\varepsilon^2}{2\lambda_d}\frac{C_2\int_0^1 f(x_1,\dots,x_d)IdG(s)}{\int_0^1 f(x_1,\dots,x_d)IdG(s)}\right|=C_2\frac{\varepsilon^2}{2\lambda_d}$$
and, supposing that S has positive density over $(0,1)$,
$$\int_0^1 f_{d|s}(x_1,\dots,x_d)I(s,\varepsilon,d)dG(s)=f_d(x_1,\dots,x_d)\int_0^1 I(s,\varepsilon,d)f_{S|d}(s)ds$$
$$=\frac{\varepsilon^d\pi^{d/2}}{\Gamma(d/2+1)}\mathbb{E}\left[(1-S)^{d/2}\,\mathbb{I}_{\{S\le 1\}}\middle|(x_1,\dots,x_d)\right]$$
where $f_{S|d}(s)$ is the conditional density of S to $(\theta_1,\dots,\theta_d)=(x_1,\dots,x_d)$.

3. The right hand side of Equation (5.3) must be multiplied by 2.