Which seasonality in Italian daily electricity prices? A study with state space models.

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Abstract The paper presents a study of seasonality in Italian daily electricity prices. In particular, it compares the ARIMA approach with the structural state space approach in the case of seasonal data. Unlike ARIMA modeling, the structural approach has enabled us to detect, in the prices under consideration, the presence of stochastic daily effects whose intensity is slowly decreasing over time. This dynamic of seasonality is the consequence of a more balanced consumption of electricity over the week. Some causes of this behavior will be discussed in the final considerations. Moreover, it will be proved that state space modeling allows the type of seasonality, stochastic or deterministic, to be tested more efficiently than when unit root tests are used.

Key words: Electricity prices, seasonal unit roots, HEGY test, structural space state models

1 Introduction

In the past twenty years, competitive wholesale markets of electricity have started in the OECD countries in the international context of the deregulation of energy markets. At the same time, an increasing number of studies on electricity prices have been published. Most of these studies have sought to identify good prediction models, and, for this reason, ARIMA modeling has been the most common methodology. Nevertheless, electricity prices present periodic patterns, seasonality in time series terminology, for which ARIMA modeling does not always seem the best approach.

The treatment of seasonality in the ARIMA framework is conceptually similar to the treatment of trends: like these, seasonality entails the non-stationarity of the process, and its non-stationary effect has to be removed before modeling the process. More specifically, if the seasonal effects are constant at corresponding times (e.g. every Sunday, every Monday,...,), the seasonality can be represented by a periodic linear function $s(t)$ (*deterministic seasonality*). In this case, the correct treatment consists in subtracting the seasonality, and then in modeling the non-seasonal prices using an ARIMA model:

$$\phi(B)\Delta[p_t - s(t)] = \theta(B)\varepsilon_t{}^1 \qquad (1)$$

On the other hand, if the seasonal effects are characterized by stochastic variability (*stochastic seasonality*), the correct treatment consists in applying the seasonal difference to the prices

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[1] Formally, model 1 is called the Reg-Arima model by some authors, ARMAX by others.

$\Delta_s p_t = p_t - p_{t-s}$, and then modeling the differences using an ARIMA model:

$$\phi(B)\Delta\Delta_s p_t = \theta(B)\varepsilon_t \tag{2}$$

The two treatments are not interchangeable. In fact, in the case of deterministic seasonality, the seasonal difference is not efficient because it introduces seasonal unit roots into the moving average part $\theta(B)$ of the ARIMA model; in the case of stochastic seasonality, the first treatment does not assure stationarity in the second moment of the data [6]. Hence, the correct application of ARIMA models to seasonal data requires first the identification of the type, stochastic or deterministic, of the seasonality present in the prices.

In many cases, statistical tests indicate the presence of deterministic seasonality, at least in the short run. For this reason, as well as for easiness reasons, many scholars ([1, 12, 9]) have opted for representing seasonality by means of periodic functions. This approach makes it possible to measure the seasonal effects, but it is based on the strong assumption that seasonal effects remain constant over time. On the other hand, the seasonal difference approach does not satisfy the need to understand and model the real dynamic of seasonality in electricity prices. Therefore, other scholars [10] turned to periodic ARIMA models, but this modeling requires numerous parameters when seasonality presents numerous periods (e.g. daily pattern). On the basis of these considerations, Structural State Space Models could be a solution for representing seasonality in a flexible way, but using few parameters. The paper illustrates some structural space state models that yielded interesting findings about seasonality in Italian daily electricity prices. More specifically, the paper is organized as follows. The next section illustrates some items about deterministic and stochastic seasonality, and the most common test for checking seasonality is presented. In Section 3, an analysis of the Italian daily electricity prices is discussed, comparing the ARIMA approach with the structural (space state) approach. Final considerations are made in the last Section.

2 Deterministic and stochastic seasonality

Seasonality can be viewed as a periodic component s_t of a seasonal process y_t that makes the process non-stationary:

$$y_t = y_t^{ns} + s_t \tag{3}$$

The remaining part $y_t^{ns} = y_t - s_t$ is the non-seasonal process and is generally assumed stochastic, but s_t can be either deterministic or stochastic.

Deterministic seasonality can be represented by periodic functions of time (having s periods) like the following ones:

$$s_t = \sum_{j=1}^{s} \gamma_j d_{j,t} \text{ with } \sum_{j=1}^{s} \gamma_j = 0 \tag{4}$$

$$\text{or } s_t = \sum_{j=1}^{[s/2]} A_j cos(\omega_j t - \phi_j) \tag{5}$$

In equation 4, the parameter γ_j represents the seasonal effect in the *j-th* period ($d_{j,t}$ is a dummy variable indicating the period). In equation 5, seasonality is viewed as the sum of $[s/2]$[2] harmonic functions each of them having angular frequency $\omega_j = j2\pi/s$; $j = 1, 2,..., [s/2]$. Deterministic seasonality satisfies the following relation:

[2] $[s/2] = s/2$ for s even, and $[s/2] = (s-1)/2$ for s odd

$$S(B)s_t = 0 \tag{6}$$

where $S(B) = 1 + B + B^2 + ... + B^{s-1}$ is *the seasonal summation operator* based on the backward operator B [3]. In the case of stochastic seasonality, the relation 6 becomes:

$$S(B)s_t = w_t \tag{7}$$

where w_t is a *zero-mean* stochastic process (stationary or integrated). Now seasonality can be viewed as the sum of of $[s/2]$ *stochastic harmonic paths* $h_{j,t}$:

$$\gamma_j(B)h_{j,t} = w_{j,t} \tag{8}$$

where

$$\gamma_j(B) = (1 - e^{i\omega_j}B)(1 - e^{-i\omega_j}B) \text{ if } 0 < \omega_j < \pi \tag{9}$$

$$\gamma_j(B) = (1 + B) \text{ if } \omega_j = \pi \tag{10}$$

and $w_{j,t}$ is a *zero-mean* stochastic process (stationary or integrated).
Since each seasonal operator $\gamma_j(B)$ is a polynomial with unit roots, each stochastic harmonic path implies the presence of one or two (complex and conjugate) unit roots in the process (more exactly, in the autoregressive representation of the process) and vice-versa. Finally, since:

$$\Delta_s = \Delta S(B) = \Delta \prod_{j=1}^{[s/2]} \gamma_j(B) \tag{11}$$

the application of the filter Δ_s to a seasonal process y_t makes the process stationary, removing a stochastic trend (eventually present in the non-seasonal data) and $[s/2]$ stochastic harmonic paths present in seasonality.

2.1 HEGY test

A very common methodology used to test for non-stationarity due to seasonality is the procedure developed by Hylleberg, Engle, Granger, and Yoo [8], and known as the HEGY test. This test was originally devised for quarterly seasonality, but it has also been extended for weekly seasonality in daily data by Rubia [13].

Under the null hypothesis, the HEGY test assumes that the relevant variable is *seasonally integrated*. This means, in the case of daily electricity prices (p_t), that the weekly difference $\Delta_7 p_t$ is assumed to be a stationary process.

Since:

$$\Delta_7 = (1 - B) \prod_{j=1}^{3} (1 - e^{i\omega_j}B)(1 - e^{-i\omega_j}B) \tag{12}$$

($\omega_j = 2\pi/7, 4\pi/7, 6\pi/7$), the null hypothesis of the HEGY test entails the presence in the process of seven unit roots: one at zero frequency (corresponding to a stochastic trend) and three pairs of complex unit roots corresponding to three stochastic harmonic paths with frequencies $2\pi/7$, $4\pi/7$, $6\pi/7$.
The test consists in checking the presence in the process of the unit roots; in this sense it can be

[3] $S(B)s_t = s_t + s_{t-1} + ... + s_{t-s+1}$

viewed as an extension of the Dickey-Fuller tests [2]. Like these tests, the HEGY test is based on an auxiliary regression:

$$\Delta_7 p_t = \alpha + \sum_{s=2}^{7} \gamma_s d_{s,t} + \sum_{r=1}^{7} \alpha_r z_{r,t-1} + \sum_{j=1}^{p} \phi_j \Delta_7 p_{t-j} + \varepsilon_t{}^4 \tag{13}$$

where $d_{s,t}$ is a zero/one dummy variable corresponding to the *s-th* day of the week, and each regressor $z_{r,t}$ is obtained by filtering the process p_t so that:

- it will be orthogonal to the other regressors;
- it will include only one root of the seven roots included in p_t.

For example, $z_{1,t}$ includes only the unit root having zero frequency (stochastic trend), but not the seasonal roots; $z_{2,t}$ and $z_{3,t}$ include only the seasonal roots having frequency $2\pi/7$, and so on (see [13] for more details).
The number p of lags of the dependent variable in the auxiliary regression (augmentation) has to be chosen to avoid serial correlation in the error term ε_t.

If $\Delta_7 p_t$ is a stationary process, all roots have been removed, and the coefficients αs are not significant. As in the augmented unit root test of Dickey and Fuller (ADF), the null hypothesis $\alpha_1 = 0$ is accepted against the alternative hypothesis $\alpha_1 < 0$ on the basis of a non-standard t-statistic. In regard to the seasonal roots, the test should be performed on each couple of roots having the same frequency. Indeed, only the hypothesis $\alpha_{2j} = \alpha_{2j+1} = 0$ ($k = 1,2,3$) means the absence in $\Delta_7 p_t$ (i.e. the presence in p_t) of an harmonic path with frequency $2\pi j/7$. This assumption can be tested by a joint F-test; the distribution of each statistic F_j is not standard, but the critical values are reported in [11]. In conclusion: if some hypothesis $\alpha_{2j} = \alpha_{2j+1} = 0$ is not rejected, the seasonality should be stochastic; if all the hypotheses $\alpha_{2j} = \alpha_{2j+1} = 0$ are rejected and some coefficient γ_s is significant, the seasonality should be deterministic.

3 Analysis of the Italian daily electricity prices

The HEGY test was performed on the 2008-2011 Italian daily PUN[5] (more specifically the log-PUN). As reported in Table 1, none of the null hypotheses (H_0) was significant at 1% level. Nevertheless, the absence of a stochastic trend was not confirmed by the ADF test on the same data. This might mean that the prices process is nearly a stochastic trend, but also that the process is not homogeneous over the whole period. Indeed, after performing the HEGY test on the sub-periods 2008-09 and 2010-11, it can be noted that the statistic t concerning the presence of a stochastic trend gives different signals: the 2008-09 daily prices seem to include a stochastic trend, whereas the 2010-11 daily prices do not. Such deductions were confirmed by performing the ADF test on the data (Table 1). The absence of mean-reversion in the first period is a particular case and should be related to the high variation of the oil prices in the same period. On the other hand, seasonality remains non stochastic in both periods (absence of seasonal roots). According to these findings, the 2008-09 daily log-PUN was represented by a Reg-ARIMA model, but the 2010-11 Log-PUN by a Reg-ARMA model: more specifically, a Reg-IMA(1,2) for the first period and a Reg-AR(7) for the second one. In both cases the regression was the following:

[4] This is a standard version of the HEGY test for daily data, but it can be extended to include trends. Nevertheless, in this case, there is no reason for doing so.

[5] The PUN is the National Single Price in the Italian electricity market (IPEX). The PUN series are downloadable from the web site of the Energy Markets Manager: http://www.mercatoelettrico.org

Table 1 HEGY and ADF tests

H_0	stat.	2008-11 (sign.)	2008-09 (sig.)	2010-11 (sign.)
$\alpha_1 = 0$	t	-3,739 ***	-1,572	-3,742 ***
$\alpha_2 = \alpha_3 = 0$	F_1	163,127 ***	65,480 ***	84,874 ***
$\alpha_4 = \alpha_5 = 0$	F_2	175,639 ***	66,215 ***	90,891 ***
$\alpha_6 = \alpha_7 = 0$	F_3	232,567 ***	103,114 ***	93,018 ***
ADF test	τ	-2,288	-1,167	-3,371 **

$$p_t = \gamma_{Mon} d_{Mon,t} + \ldots + \gamma_{Sat} d_{Sat,t} + p_t^{ns} \tag{14}$$

The models parameters and their significance are reported in Table 2.
Since the analyzed data are log-prices, each daily coefficient (lower part of the table) indicates the average per-cent difference between the corresponding daily price and the Sunday price, which is obviously the lowest price. Indeed, the consumption of electricity is generally lowest on Sundays. To be noted is that the daily effects are lower in the second period. This result may mean that there was a structural break in the seasonality as a consequence of a structural break in the daily demands or in the daily supplies of electricity. On the other hand, seasonality may have had fluctuations of slowly decreasing intensity in the period 2008-2011 as a consequence of slow changes in the daily demands and/or daily supplies of electricity.
In order to gain better understanding of the dynamics of seasonality in the electricity prices, we analyzed the prices by means of state space models.

Table 2 Models parameters

param.	model 2008-09 value/sign.	model 2010-11 value/sign.
const	-0,001	4,136 ***
AR1	-	0,351 ***
AR2	-	0,116 ***
AR3	-	0,084 **
AR4	-	0,134 ***
AR5	-	0,088 **
AR6	-	0,038
AR7	-	0,067 *
MA1	-0,498 ***	-
MA2	-0,237 ***	-
Mon	0,148 ***	0,076 ***
Tue	0,173 ***	0,093 ***
Wed	0,190 ***	0,091 ***
Thu	0,169 ***	0,097 ***
Fri	0,151 ***	0,080 ***
Sat	0,103 ***	0,072 ***

3.1 State Space analysis of electricity prices

First, the following model was performed on the 2008-2011 daily log-PUN:

$$p_t = m_t + s_t + \varepsilon_t \tag{15}$$
$$m_{t+1} = m_t + b_t + \varepsilon_{1,t} \tag{16}$$
$$b_{t+1} = b_t + \varepsilon_{2,t} \tag{17}$$
$$s_{t+1} = -s_t - s_{t-1} - \ldots - s_{t-5} + \varepsilon_{3,t} \tag{18}$$

where m_t is the non-seasonal level of the log-PUN p_t; b_t is the slope and s_t is the seasonality (daily effect). The disturbance factors ε_t, $\varepsilon_{1,t}$, $\varepsilon_{2,t}$ and $\varepsilon_{3,t}$ are *white noises* with variances σ^2, σ_1^2, σ_2^2 and σ_3^2.
This model, also known as the *local linear trend model with seasonal effect* [3], is a common starting state space model for seasonal data. Equation 18 is a particular case of assumption 7 (w_t is assumed to be a white noise) and entails stochastic seasonality. This assumption permits seasonality to change in the period 2008-2010 according to the findings in Table 1. The estimation results of this model are reported in the second column of Table 3. To be noted is that the estimate of the standard deviation of ε_2 is zero, which means the slope of the trend b_t can be assumed to be non stochastic; moreover, the estimate of b_t converges to zero. For these reasons, a seasonal model without slope (without b_t in equation 16 and without equation 17), also known as the *local level with seasonal effect*, shows better indices of fit (third column). The consideration of the diagram of the smoothed seasonality

Table 3 Three state space models for the log-prices

Parameters	local trend with seasonal effect	local level with seasonal effect	local level with decreasing seas. eff.
σ	0,0856 ***	0,0858 ***	0,0858 ***
σ_1	0,0401 ***	0,0397 ***	0,0397 ***
σ_2	0,0000	no	no
σ_3	0,0034 ***	0,0034 ***	0,0029 ***
α	no	no	0,9923
AIC	-2.314,61	-2.346,51	-2.365,15
SBC	-2.293,11	-2.330,37	-2.343,64

(Figure 1) shows that the daily effects tend to decrease in the period 2008-2011 (in this case, the daily effects should be viewed as the percentage deviations, positive on working days and negative at weekends, from the trend of prices). According to this evidence, the standard local level model was modified by the following seasonal state equation:

$$s_{t+1} = -\alpha(s_t + s_{t-1} + \ldots + s_{t-5}) + \varepsilon_{3,t} \tag{19}$$

where $0 < \alpha < 1$ so that the daily effects can tend to decrease. On performing the new model on the 2008-2011 log-PUN, the value of alpha resulted equal to 0,9923 (Table 3, fourth column); the standard deviation of the disturbance on seasonality was equal to 0,0029 (less than $0,3\%$). The values of the Akaike (AIC) and Schwarz (SBC) indices are less than in previous models, denoting an improvement in fit. These findings prove that the daily effects were very slowly decreasing in the period 2008-2011; indeed, so slowly decreasing and so little varying that they could be viewed as constant in a short period. For this reason, the HEGY test, which is not a particularly powerful test, detected deterministic seasonality (Table 1).

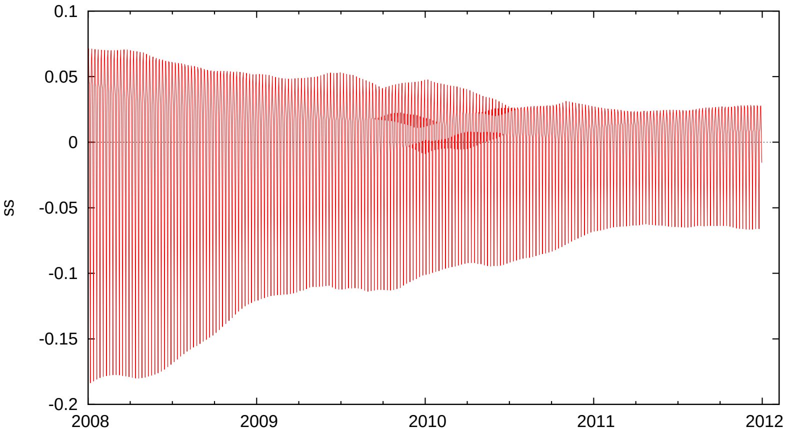

Fig. 1 Seasonality in the period 2008-2011

4 Final considerations

The analysis described in the previous sections has shown that the daily effects (i.e. seasonality) on daily wholesale electricity prices exhibited slowly decreasing intensity in the period 2008-11 in Italy. We reiterate that the daily effects can be viewed as deviations from the trend of the prices due to the days of the week. A reduction of the daily effects means a reduction of the differences among the daily prices. Some causes regarding the demand and the supply of electricity can be highlighted. In regard to the demand, a more balanced consumption of electricity over the week has been noted in recent years. One reason is certainly that more and more families have subscribed contracts of domestic electricity provision which make electricity consumption cheaper in the evenings and at weekends. Moreover, the difficulties of the Italian economy in recent years have caused a reduction in electricity consumption on working days.
In regard to the supply, the entry into the market of several small electricity producers has made the supply of electricity more flexible.

Regarding the methodology, Structural State Space Models seem to be a more powerful tool than the HEGY test for detecting the type of seasonality. From the state sequence of the seasonal components, it is possible to gain a first view on the kind of seasonality affecting the data. Moreover, the significance test on the standard deviation of the disturbance in the seasonal component makes it possible to check whether or not seasonality is stochastic. More specifically, if the standard deviation is not significant, the seasonality should be assumed to be deterministic; otherwise it should be assumed stochastic. Although these models are not usually employed for electricity prices, they have interesting features for the analysis and prediction of electricity prices. As known, State Space modeling can include ARIMA modeling, but it allows easier modeling of periodic components compared with the latter. Moreover, Structural State Space models can represent electricity prices according to an economic or behavioral theory.

This study has not dealt with volatility clustering, a well-known feature/problem of electricity prices. As known, the GARCH models (in all versions) are typically used to model volatility clustering. Although such modeling is generally associated with ARIMA modeling, conditional heteroscedasticity can be considered in structural framework as well ([7]).

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