Empirical evidences about electricity hourly prices in some European markets

Object

The poster reports some empirical evidences coming out of the analysis of the 2007-2008 *electricity hourly prices* (*EHPs*) in 4 electricity European markets: Italy, Spain, Germany-Austria, Norway.

Background

- The study originates from a work about the economic risk analysis of a wind-power plant to be built in Italy.
- The annual gain distribution of the plant is drawn by means of *simulations* of a business year.
- A crucial point is the simulation of the EHPs of the electricity on the basis of a suitable time-series model.

What is known about EHPs?

1. They are affected by multiple seasonality, particularly by daily (24-hours) seasonality and weekly (168-hours) seasonality
2. The stationarised EHPs are a mean-reverting process
3. They are characterised by volatility clustering determined by conditional heteroscedasticity
4. They are characterized by spikes due to the difficulty of the electricity supply to match the demand

1. The daily and weekly seasonality

- Descriptive analysis of EHPs show that mean and standard deviation of EHPs change in the 24 hours and in the 7 days of week
- Then the daily and weekly seasonality make the EHPs process not stationary in mean and variance

1.1 Stationarity in variance

A common way to get the stationary in variance is the logarithmic transformation, **but**:

- This transformation can't be applied if the EPHs are equal to zero and determine negative spikes if the EPHs are quite zero
- Moreover the hourly standard deviation increases when the hourly mean is increasing, but not linearly

Then the squared root transformation is suggested: $p_t = \sqrt{EHP_t}$

1.2 Stationarity in mean

A way to get the stationarity in case of daily and weekly seasonality is the multiple difference:

$z_t = (1-B^{24})(1-B^{168})p_t$, **but** it might mean *over differencing*.

Indeed $(1-B^{168}) = (1-B^{24})(1+B^{24}+B^{48}+B^{72}+B^{96}+B^{120}+B^{144})$

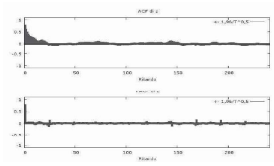
so that: $(1-B^{24})(1-B^{168}) = (1-B^{24})^2(1+B^{24}+B^{48}+B^{72}+B^{96}+B^{120}+B^{144})$

Then we suggest the *average seasonal difference* between daily and weekly difference:

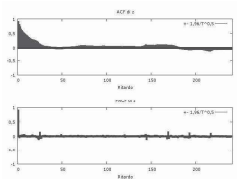
$$z_t = [0.5(1-B^{24})+0.5(1-B^{168})]p_t \qquad z_t = (1-0{,}5B^{24}-0{,}5B^{168})p_t$$

1.3 Correlogramms of stationarized EHPs (z_t)

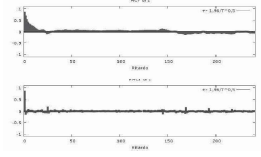
Italy:



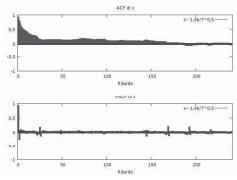
Germany and Austria:



Spain:



Norway:



2. The mean reversion property: an AR(1) component in the standardised EHPs

- In each of the 4 correlograms the ACF is decreasing; the PACF is very high at lag 1 and quite zero at the other lags
- Also the standardized EHPs might be shaped by an AR(1) model:

  $(1-\phi_1 B)z_t = \phi_0 + \varepsilon_t$ or $(1-\phi_1 B)(z_t - \mu) = \varepsilon_t$

  where ε_t is a process of non correlated errors and $\phi_1 < 1$ as consequence of stationarity

- That means z_t is a mean reverting process:

  $(1-B)z_{t+1} = (\phi_1 - 1)(z_t - \mu) + \varepsilon_{t+1}$

Since $(\phi_1 - 1) < 0$, every difference from the mean of process produces a correction toward the mean.

3. Conditional heteroschedasticity

- The volatility clustering of the innovations ε_t is frequently shaped by a GARCH(1,1) model:

  $\varepsilon_t = \sigma_t u_t$

  $\sigma_t^2 = \varpi + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \qquad u_t \sim WN(0, 1)$

- The square root transformation solves only partially the seasonal pattern of the variance: $Var(\varepsilon_t) \approx Var(\varepsilon_{t-24})$

- Therefore a way to manage the volatility could be 24 periodic GARCH(1,1): $\sigma_t^2 = \varpi_k + \alpha_{1k}\varepsilon_{t-1}^2 + \beta_{1k}\sigma_{t-1}^2$
  k=1, 2, ..., 24
- As an alternative, a more parsimonious modelling might be the following seasonal GARCH model:
  $\sigma_t^2 = \varpi + \alpha_1 \varepsilon_{t-1}^2 + \beta_{24}\sigma_{t-24}^2$ with α_1 quite 0 and β_{24} quite 1

4. The presence of spikes: non normal standardised innovations

- In original GARCH modelling u_t are assumed $WN(0, 1)$ Normal distributed.
- But spikes in EHPs make the innovations ε_t having tails so heavy that the Normal distribution about the standardised innovation u_t can not be assumed; better the standardised t-Student distribution.

Resuming all observations, the following modelling for EHPs is proposed:

$$\begin{cases} p_t = \sqrt{EHP_t} \\ (1-\phi_1 B)(1-0.5B^{24}-0.5B^{168})p_t = \phi_0 + \sigma_t u_t \\ \sigma_t^2 = \varpi + \alpha_1 \varepsilon_{t-1}^2 + \beta_{24}\sigma_{t-24}^2 \end{cases} \qquad (1)$$

where u_t are i.i.d. as a *standardised t-Studend* with suitable *degrees of freedom*

5. Results

5.1 Model parameters

The modelling (1) has been estimated for Italy, Spain, Germany-Austria, and Norway using the 17544 EHPs of 2008 and 2009 :

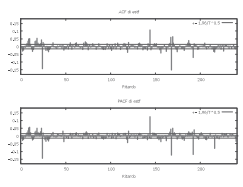
parameter	ITALY	SPAIN	GERMANY	NORWAY
phi0 (1)	-0,0030	0,0002	-0,0006	-0,0020
phi1	0,7854	0,8541	0,8951	0,9076
phi1 bis	-	-	-	0,2254
omega	0,1109	0,0028	0,0081	-
alfa1	0,2129	0,1871	0,2066	0,1755
beta24	0,3850	0,7648	0,7172	0,8245
mean EHP	87,02	50,72	52,57	39,88
RMSE	9,63	3,23	4,63	1,96
RMRSE (2)	12,3%	10,7%	13,4%	9,0%

(1) Only the estimates of ϕ_0 are not significantly different from zero

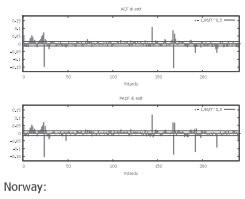
(2) Root Mean Relative Square Error = $\sqrt{Mean\left[(EHP - E\hat{H}P)/E\hat{H}P\right]}$

5.2 Correlogramms of standardized innovations

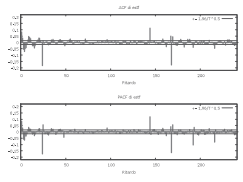
Italy:



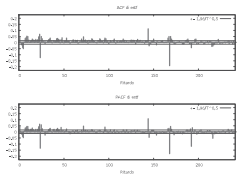
Germany and Austria:



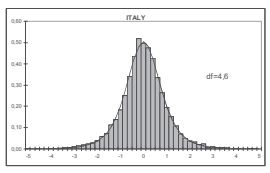
Spain:

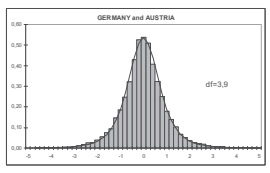


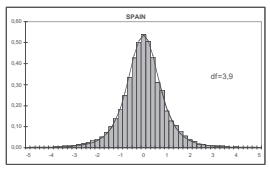
Norway:

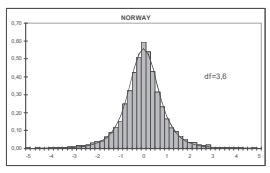


5.3 Distribution of standardized innovations vs standard *t-Student*









6. Some Considerations

- On the basis of the modelling (1) Spain and Germany (with Austria) are very similar; Italy is characterized by more mean-reversion (ϕ_1-1) and a lower persistency in volatility (β_{24}).
- Norwegian market is a bit different:
  - the fit improves significantly adding another AR(1) component (on the whole an AR(2))
  - the innovations are properly shaped as an IGARCH
- Generally the autocorrelations of the standardized innovations are quite null, except for some lags (24, 144, 168) where, anyway, the autocorrelation remain low (around 0,15 or less)
- It means that the modelling (1) can be improved adding AR and/or MA components with those lags. As example, the models:

  $(1-\phi_1 B)(1-0{,}5B^{24}-0{,}5B^{168})p_t = \phi_0 + (1-\phi_{168}B^{168})\sigma_t u_t$

  $(1-\phi_1 B)(1-\phi_{24}B^{24}-\phi_{168}B^{168}-\phi_{336}B^{336})p_t = \phi_0 + \sigma_t u_t$

  have innovations more similar to a WN than model (1) has. Nevertheless these models don't reduce significantly (in economic sense) the RMSE
- The modelling (1) shapes the most of the autocorrelations in the EHPs processes!

7. References

Bunn D.W. (2004), *Modelling price in competitive electricity markets*, Wiley & Sons.

Bosco B., Parisio L. and Pelagatti M. (2007), Deregulated Wholesale Electricity Prices in Italy: An Empirical Analysis, *International Advanced Economic Research*, 13.

Gianfreda A., Grossi L. (2010), Fractional Integration models for Italian electricity zonal prices, *Proceedings of 45th Scientific Meeting of the Italian Statistical Society,* University of Padua.

Koopman S.J, Oooms M. and Carnero M.A. (2007), Periodic seasonal Reg-ARFIMA-GARCH Models for daily Electricity Prices, *Journal of the American Statistical Association*, 102, 477.