Common Optimal Scaling for Customer Satisfaction Multidimensional Models

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Abstract: A singular algorithm of ALSOS's (Alternating Least Squares with Optimal Scaling) is presented. It allows to assign the same scaling to all variables measured on the same categorical level in a categorical regression. The algorithm is applied to a model for measurement and evaluation of Customer Satisfaction (CS). The results seem to support the use of multiplicative models like Cobb-Douglas's, to analyze how the overall satisfaction of goods or services customers is shaped

Keywords: ALSOS, Customer Satisfaction, Cobb-Douglas models.

1. Customer satisfaction and ALSOS methods.

The satisfaction degree of a customer about a good or a service, namely *Customer Satisfaction*, is a variable typically measured on an ordinal scale (for example: very dissatisfied, dissatisfied, neither satisfied nor dissatisfied, satisfied, very satisfied). Unfortunately this measurement scale doesn't always allow very meaningful analysis. Several methods can be used to overcome this limit: a very easy one replaces ordinal categories with their ranks. Such transformation is obviously arbitrary and can be considered acceptable only if categories are conceptually equidistanced. Other methods are based on distributional functions (Thurstone method, etc.), but they imply the choice of a distributional form. Optimal Scaling (OS) is instead a class of distribution free methods. OS methods allow to assign numerical values to categorical variables in a way which optimises an analysis model (see Boch, 1960 and Kruskall 1965). Among the most interesting OS methods, ALSOS permits the optimisation of a model using an algorithm based on Alternating Least Squares (ALS) and Optimal Scaling (OS) principles (see Young et all., 1976). In this paper a singular ALSOS method is proposed, allowing to obtain a common scaling for all evaluation model variables measured on the same ordinal scale. This does not normally happens with the standard ALSOS programs.

2. The algorithm.

Let $\mathscr{Y}$ the satisfaction degree of a customer about a good or service and $\mathscr{X}_1, \mathscr{X}_2, \ldots, \mathscr{X}_p$ the satisfaction degrees of some aspects of the good or service. All satisfactions are measured on a scale of k ordinal categories $c_1, c_2, \ldots, c_k$.

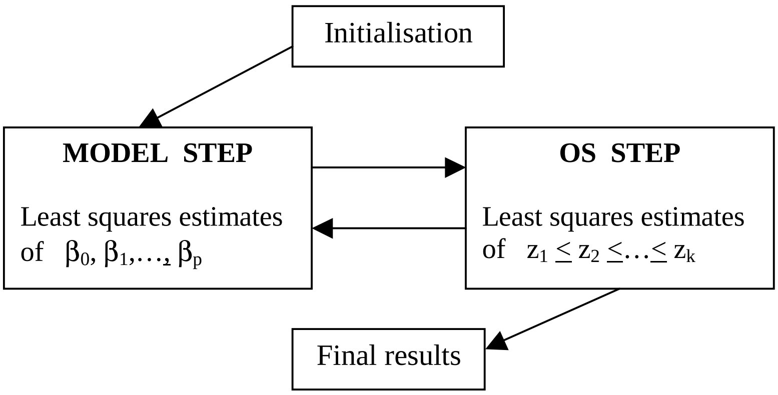
The algorithm aims at transforming the qualitative data into quantitative data by means of a common transformation $\mathcal{Z}$ in order to minimize the error ε of regression:

$$Y = \beta_0 + \beta_1 X_1 + ... + \beta_p X_p + \varepsilon \tag{1}$$

where $Y = \mathcal{Z}(\mathcal{Y})$, $X_1 = \mathcal{Z}(\mathcal{X}_1), \ldots, X_p = \mathcal{Z}(\mathcal{X}_p)$.
The transformation $\mathcal{Z}$ will define *k* ordered values $z_1 \leq z_2 \leq \ldots \leq z_k$ corresponding to the *k* ordered categories $c_1, c_2, \ldots, c_k$.

Following the approach of ALSOS, the algorithm consists of a iterative two-steps estimation process:

Figure 1: *The ALSOS algorithm.*



2.1 Scaling and model parameters estimates.

Assuming data are observed on n customers, the model (1) can be described in a dual matricial form, i.e. the classic form and the "scaling" one:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \tag{2}$$

$$\mathbf{Uz} = \mathbf{b_0} + \mathbf{Bz} + \boldsymbol{\varepsilon} \;\rightarrow\; (\mathbf{U}\text{-}\mathbf{B})\mathbf{z} = \mathbf{b_0} + \boldsymbol{\varepsilon} \tag{2bis}$$

where:
- **U** is a (*n* x *k*) matrix with $u_{i,h} = 1$ if the ith customer respond "c_h" about $\mathcal{Y}$, else 0;
- **z** is the vector of *k* scaling value: $z_1 \leq z_2 \leq \ldots \leq z_k$;
- $\mathbf{b_0}$ is a (*n* x *1*) vector with all elements like β_0;
- **B** is a (*n* x *k*) matrix with $b_{i,h} = \sum \beta_j$, summing only over the values of *j* corresponding to the $\mathcal{X}_j$s for which the i^{th} customer chose category c_h

The steps of algoritm:

1) *Initialisation*: an arbitrary $\hat{\mathbf{z}}_t$ (t=0) is chosen

2) *Model step*: $\hat{\boldsymbol{\beta}}_t$ is estimated by classic estimator: $\hat{\boldsymbol{\beta}} = (\mathbf{X'X})^{-1}\mathbf{X'}\hat{\mathbf{z}}$

3) *OS step*: $\hat{\mathbf{z}}_{t+1}$ is estimated by solving the linear problem:

$$\begin{cases} min\left[(\mathbf{U}-\mathbf{B})\mathbf{z}-\mathbf{b_0}\right]'\left[(\mathbf{U}-\mathbf{B})\mathbf{z}-\mathbf{b_0}\right] \\ \mathbf{V}\mathbf{z}=\mathbf{v} \end{cases}$$

where $\mathbf{v}=\begin{bmatrix} z_{\min} \\ z_{\max} \end{bmatrix}$ $\mathbf{V}=\begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 0 & \dots & 1 \end{bmatrix}$ [1]

4) *Control step* if $|\hat{\mathbf{z}}_{t+1} - \hat{\mathbf{z}}_t| < \xi$[2] go to next step, else t=t+1 and go to step 2)

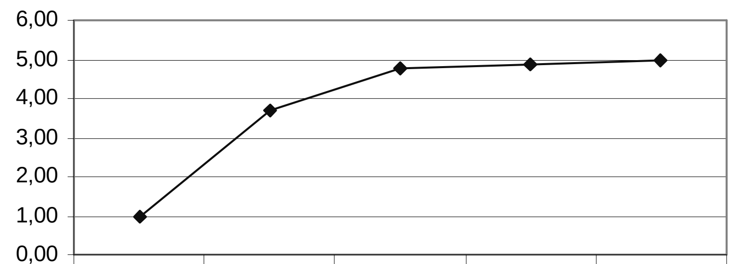
5) *Final Results* the last $\hat{\mathbf{z}}_t$ and $\hat{\boldsymbol{\beta}}_t$ are the final results.

The convergence is guaranteed because the sum of squares errors (SSE) decreases at every step and round. There is one hooker: the ALSOS procedure does not guarantee convergence on the globally least squares solution. Nevertheless every final scaling is better (in terms of SSE) than an initial, supposedly good scaling.

3. Moltiplicative Models for CS.

The proposed algorithm was applied to a survay on CS in a Piedmont ASL (Local Sanitary Firm) obtaining the following results:

Figure 2: *The optimal scaling in a sanitary case*



	very dissatisfied	dissatisfied	neutral	satisfied	very satisfied
scaling	1,00	3,68	4,76	4,88	5,00

This result contrasts with idea of conceptual equidistance among categories. Nevertheless it is possible to partially recover equidistance with an exponential transformation like:

$$\tilde{\mathbf{z}}' = \left[a^{Z_1} \quad a^{Z_2} \quad \dots \quad a^{Z_k}\right] \tag{3}$$

where a is a suitable basis. For model (1), it means:

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[1] there are no constrains for the monotonicity of transformation: they are normally not needed, but there are no problems to include them.

[2] ξ is a suitable convergence level.

$$a^{Y} = a^{\beta_0+\beta_1 X_1+...+\beta_p X_p+\varepsilon} \qquad \text{so:} \qquad \widetilde{Y} = \widetilde{\beta}_0 \cdot \widetilde{X}_1^{\beta_1} \cdot ... \cdot \widetilde{X}_p^{\beta_p} \cdot \widetilde{\varepsilon} \tag{4}$$

where parameter/variable with " ~ " correspond to " $a^{\text{parameter/variable}}$ ".
The algorithm is based on the linear regression so that it presents the features of ALSOS programs, but it doesn't mean that the better relation between dependent and independent variables has to be necessary linear: this could be also multiplicative, like a Cobb-Douglas function. In this case, the final scaling should be got after a suitable exponential transformation.

4. Conclusions.

The algorithm has the typical features of ALSOS programs: free distribution method and convergence of estimates obtained by analytic functions. It also ensures a common scaling for all data measured on a same ordinal scale, whereas ALSOS programs included in the most popular statistic software do not. In fact these programs, as a general approach, assign different scaling to every qualitative variable, whether it is measured on a common scale or not. However the same values should be assigned to same categories, if the scaling gives a metric significance to the measurement of qualitative data (see Chirico 2005).
The application of algorithm in a CS evaluation study has pointed out that the relation between the overall satisfaction and its factors seems to be formalized better by multiplicative models, like Cobb-Douglas ones. In other words: the overall satisfaction and its factors are conceptually comparable to overall utility and its factors (goods) in the marginal consumer theory[3].
At present, further studies on how to get the final scaling in a multiplicative model are being carried on.

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[3] The Cobb-Douglas' function was originally proposed like a production function, but subsequently it was also used to confirm the marginal consumer theory.