Epistemic Uncertainty Propagation in a Weibull Environment for a Two-Core System-on-Chip

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***Abstract*—Epistemic uncertainty analysis accounts for inaccurate input parameters and evaluates how such uncertainty propagates to output measures. In this work we will focus on Weibull distributions, in particular the one related to the reliability of multi-core systems-on-chips. We focus on a two-core system in which, when one of the cores fails, the other can take care of all the systems' tasks. However, this results in an increased operational temperature that leads to a reduced lifetime. We first discuss the epistemic uncertainty distribution that we expect when we estimate the scale parameter of a Weibull distribution from a limited set of samples. We then extend the procedure to consider the epistemic uncertainty in the two core system-on-chip when we can measure the failure times of the first and of the second core.**

***Index Terms*—Epistemic uncertainty, Uncertainty propagation, Weibull distribution, Min/Max Weibull distributions, Multi-Core System-on-Chip**

I. Introduction

The modeling process of a technological system can be affected by many sources of uncertainty, that are usually categorised as either *aleatory* or *epistemic* [9]. The *aleatory uncertainty* is due to the inherent variability or unpredictable knowledge of the system behavior (e.g. failure and repair times, arrival and service times, etc...), the *epistemic uncertainty* results from lack-of-knowledge or some level of ignorance about the parameter values characterizing the system behavior [11].

The aleatory uncertainty is commonly considered by resorting to stochastic modeling techniques that are well documented in safety [5] and dependability analysis [19]. The appropriate incorporation and presentation of the epistemic uncertainty is now widely recognized in safety and risk assessment as surveyed in the opening paper [4] of a special issue devoted to the topic of epistemic uncertainty [7]. On the other hand, the epistemic uncertainty has generally received a moderate attention in the area of dependability in the past, even if the appropriate incorporation of uncertainty into the dependability analyses of complex systems is a topic of growing interest [21]. The challenge is to model the uncertainty in the input parameters, via subjective evaluation or experimental measures, and to propagate such uncertainty to the output measures, given the underlying stochastic model of the system. The propagation of the uncertainty in a model has been generally studied by resorting to a simulation approach [1], [10], [14]. A closed-form analytic approach is developed in [12], where it is assumed that the probability distribution of the uncertainty that affects the measures can be inferred from the problem, and that measures are available to estimate the value of the input parameters. Then, the uncertainty is propagated along the model to evaluate how the input uncertainty influences the output quantities, and how the confidence intervals of the output quantities are affected by the number of samples available for the input parameters. The underlying stochastic model in [12] is a Homogeneous Continuous Time Markov Chain (HCTMC), so that the failure times, are bounded to be exponential and the input parameters are constant failure rates. In [17], the same approach is followed, but it is extended to Non-Homogeneous Continuous Time Markov Chains (NHCTMC) with time-dependent failure rates and the system structure is represented by a Fault tree. Further in [17] a sensitivity analysis is carried out in conjunction with uncertainty propagation [3] showing a correlation between the two methods of investigation. Sensitivity analysis provides an assessment of the contributions of individual input parameters to the total variability in the outcomes, and can be evaluated by simulation [3] or analytically [17].

The present paper applies the uncertainty propagation approach to a multi-core system-on-chip whose reliability characteristics have been the object of several studies [2], [6], [18]. The development of the uncertainty propagation model is particularly challenging because the lifetime of any single core is Weibull distributed and is influenced by the operating conditions of all the other cores. The time dependent rate of wear caused by the progressive aging is primarily related to the core temperature and utilization level. Hence, the present paper extends the approach in [12] in two directions: *i)* - The components are subject to wear and the time to failures are Weibull distributed and *ii)* - the time to failure of any component depends on the load and on the working conditions of the other components. In this preliminary work we restrict our analysis to a two-core system.

II. Reliability of a two-core system-on-chip

The increasing shrinking in transistor dimension in IC's causes the devices to operate at high temperatures making them more exposed to ageing and wear-out phenomena (such as time-dependent dielectric breakdown, thermal cycling, and electromigration), that are exponentially dependent on the temperature. High operating temperatures have a detrimental effect on the device lifetime [16], [20].

Multi-core systems are formed by a matrix of cores integrated on a single chip and the total load is shared among the cores. The temperature of each core, aside from the technological properties of the chip, depends on the power consumption (i.e., caused by the execution of the load) and on the heat exchanged with the adjacent cores. According to [8], the time to failure of each core is Weibull distributed so that its reliability can be written as:

$$R(t,T) \,=\, e^{-\left(\frac{t}{\lambda(T)}\right)^{\beta}} \tag{1}$$

where β is the temperature-independent shape parameter and $\lambda(T)$ the scale parameter. In Eq. (1) we have explicitly included the dependence on the operating temperature T of the core. The MTTF of a single core at a temperature T is:

$$MTTF(T) = \lambda(T)\,\Gamma\left(1 \,+\, \frac{1}{\beta}\right) \tag{2}$$

Since we know that the degrading mechanisms in IC's are thermally accelerated according to the Arrhenius law [8], the $MTTF(T)$ and consequently the scale parameter $\lambda(T)$ can be considered inversely proportional to the Arrhenius exponential:

$$MTTF(T) \sim Z\, e^{\frac{E_a}{kT}} \qquad \lambda(T) \sim Z\,\frac{e^{\frac{E_a}{kT}}}{\Gamma\left(1 \,+\, \frac{1}{\beta}\right)} \tag{3}$$

where Z is a proportionality constant that depends on the process and on the aging mechanism, E_a is the activation energy of the thermally accelerated mechanism, k the Boltzmann constant and T the absolute temperature in K.

Since the focus of this study is on the epistemic uncertainty propagation, we restrict our analysis to a two-core system. The two cores are statistically identical and share initially the same load, thus at $t = 0$ they have the same (low) temperature T_L. When the first core fails, the remaining core takes all the load and its operating temperature raises to a value $T_H > T_L$. From the literature, we assume that the shape parameter does not depend on the temperature and has a value $\beta = 2$. Since we are mainly concerned with the acceleration mechanism rather than the absolute values, we normalize $\lambda(T_L) = 1$ and we derive $\lambda(T_H)$ from Eq. (3). Assuming an activation energy of $E_a = 0.48\,eV$ (typical for electromigration in Al films), $T_L = 323\,K$ and a temperature increment $T_H - T_L = 10\ K$ we obtain $\lambda(T_H) \simeq 0.6$.

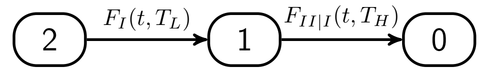

Fig. 1. State space of the model

The state-space of the model is reported in Fig. 1. The labels inside the states represent the number of operating cores. The transition time from state 2 to state 1 represents the time to failure of the first core that fails and its distribution $F_I(t,T_L)$ is given by the minimum between two Weibull distributions at a temperature T_L (1).

$$F_I(t,T_L) \,=\, 1 \,-\, e^{-2\left(\frac{t}{\lambda(T_L)}\right)^{\beta}} \tag{4}$$

which is again Weibull distributed with scale parameters $\sqrt[\beta]{2}\lambda(T_L)$. $F_{II|I}(t,T_H)$ is the failure distribution of the second component given that the first has already failed: notice that the core operating temperature, in this case, is T_H. The second core must account for the ageing it has accumulated up to the failure time of the first core. In particular, we have that:

$$F_{II|I}(t,T_H|t_I,T_L) = \left(1 \,-\, e^{-\left(\frac{t-t_I+t^*_{II}(t_I,T_L,T_H)}{\lambda(T_H)}\right)^{\beta}}\right)^2 \tag{5}$$

where $t^*_{II}(t_I,T_L,T_H)$ represents the time instant when a core working at temperature T_H would have reached the same reliability of a core working at temperature T_L at time instant t_I, that is:

$$R_{II}\left(t^*_{II}(t_I,T_L,T_H),T_H\right) = R_I(t_I,T_L) \tag{6}$$

After inserting the expression of the Weibull distribution we obtain:

$$t^*_{II}(t_I,T_L,T_H) = t_I\,\frac{\lambda(T_H)}{\lambda(T_L)} \tag{7}$$

The distribution of the second failure can then be computed by deconditioning Eq. (5):

$$F_{II}(t,T_L,T_H) \,=\, \int_0^t F_{II|I}(t,T_H|t_I,T_L)dF_I(t_I,T_L) \tag{8}$$

In this work we are interested in determining the system parameters, $\lambda_L = \lambda(T_L)$ and $\lambda_H = \lambda(T_H)$ from a set of samples that accounts for the first and second failure. Let us call u_i the samples from the distribution of the first failure, and v_i the ones from the second failure. As it can be seen in Fig. 2, neither u_i nor v_i belong to any of the two distributions we would like to fit. For what concerns the first failure (u_i), it matches the minimum of two Weibull distributions with parameter λ_L, i.e., Eq. (4). Matching the distribution of the second samples (Eq. (8) with v_i), would be quite complex due to deconditioning. Instead, we can rescale the samples by computing $v'_i = v_i - u_i + u_i\frac{\lambda_H}{\lambda_U}$: samples from v'_i correspond to the time the core would have failed if it had worked the whole time at temperature T_H instead of switching from T_L to T_H after the failure of the first core. This sample (addressed as "2nd F.S.R." in Fig. 2) matches the maximum of two Weibull distributions of scale parameter λ_H. The maximum of two Weibull distributions belongs to a family of distributions addressed as Exponentiated Weibull Distributions [15], characterised by many interesting properties and analytical results [13] that will be exploited in the following.

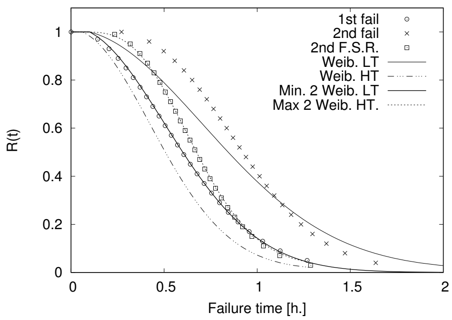

Fig. 2. Two failures model reliability.

III. Epistemic Uncertainty in a Weibull distribution

The input parameter values of a dependability model have uncertainty associated with them because they are inferred from subjective evaluation or are derived from a finite number of observations. The uncertainty in the input model parameters may be expressed with a probability distribution associated with them, so that the output measures of the model can be viewed as a random function of the given parameters.

In the present case, we assume that uncertainty is associated only with the scale parameter that is assumed to be a random variable Λ of CDF $G_\Lambda(\lambda)$ and density $g_\Lambda(\lambda)$. In the previous notation we have omitted the dependence of the scale parameter on the temperature T. The model output measures can be seen as a random function of the input parameters and the measures computed at a specific value of the input parameters can be considered to be conditional upon the used parameter values. To propagate the uncertainty the output measures must be unconditioned using the theorem of total probability. Thus, Eq. (2) is the conditional value $MTTF(\Lambda|\lambda)$, and the unconditional expected MTTF is:

$$E[MTTF] = \int MTTF(\Lambda|\lambda)\, g_\Lambda(\lambda)\, d\lambda$$

In order to clarify such approach, let us consider a single-core system whose failure process is represented by a random variable X of CDF $F_X(t|\lambda)$ and density $f_X(t|\lambda)$ following a Weibull distribution with normalized scale $\lambda(T_L) = 1$ and shape $\beta = 2$. To estimate the scale parameter, we observe a sample of n independent and identical distributed (iid) random realizations of X, i.e. $x_1, x_2, \ldots, x_n$. Given the set of observations, the maximum likelihood estimate for the parameter λ can be computed as in [15]:

$$\hat{\lambda}_{\Lambda|X} = \left(\frac{\sum_i x_i^\beta}{n}\right)^{\frac{1}{\beta}} \tag{9}$$

where $\hat{\lambda}_{\Lambda|X}$ represents the estimated value of λ conditioned on the observations $x_1, x_2, \ldots, x_n$; to simplify the notation in the following we will omit the subscript. By iterating such experiment $k \to \infty$ times, we can estimate the CDF $G_\Lambda(\lambda)$ and the density $g_\Lambda(\lambda)$ of the random variable Λ.

The CDFs computed with an increasing sizes n of the sample set are shown in Fig. 3. The accuracy of the estimated scale parameter increases with the size of the sample set, thus the probability mass of the CDFs tends to be concentrated around the real scale value $\lambda = 1$.

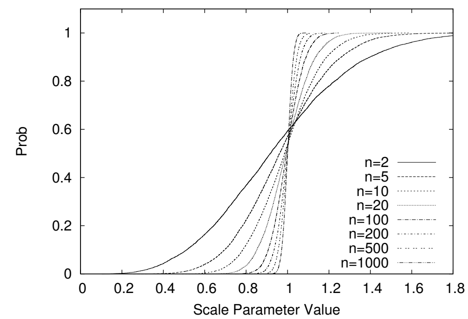

Fig. 3. Epistemic uncertainty on the scale parameter.

Using the indicator function $\mathbf{1}(\phi)$ that returns 1 if predicates ϕ is true and zero otherwise, we can compute the distribution of the expected MTTF due to the epistemic uncertainty as:

$$\begin{aligned} Pr\{MTTF \le x\} \;=\; & \int \mathbf{1}\!\left(\int (1 - F_X(t|\lambda))\ dt \le x\right) \cdot \\ \cdot\;\; & g_\Lambda(\lambda)\ d\lambda \end{aligned} \tag{10}$$

As shown in Fig. 4, also the CDFs of the conditional MTTF given the epistemic uncertainty focus around the real value of $MTTF = 1 \cdot \Gamma(1 + \frac{1}{2}) = 0.88623$.

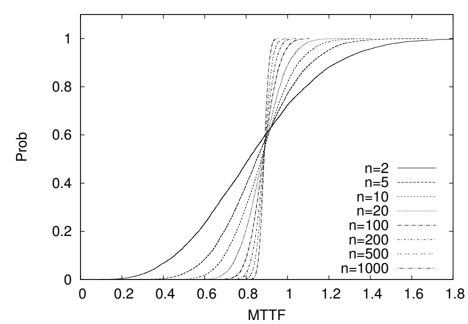

Fig. 4. CDF of the conditional MTTF of a Weibull distribution given the epistemic uncertainty on the scale parameter.

IV. Epistemic distribution of the system

In order to compute the Epistemic distribution of the system, first we must define how the parameters of the model, λ_L and λ_H can be determined from a set of n measurements. For what concerns the first failure, since the minimum of two Weibull distributions is still a Weibull distribution, we could use the same techniques introduced in Section III, and estimate it as:

$$\hat{\lambda}_L = \left(\frac{2 \sum_i x_i^\beta}{k} \right)^{\frac{1}{\beta}} \tag{11}$$

As introduced in Section II, the maximum of two Weibull distributions is distributed according to an Exponentiated Weibull Distribution, which is characterised by the following CDF $F(t)$ and PDF $f(t)$:

$$F(t) = \left(1 - e^{-\left(\frac{t}{\lambda}\right)^\beta}\right)^a \tag{12}$$

$$f(t) = \frac{a\beta}{\lambda} \left(\frac{t}{\lambda}\right)^{\beta-1} \left(1 - e^{-\left(\frac{t}{\lambda}\right)^\beta}\right)^{a-1} e^{-\left(\frac{t}{\lambda}\right)^\beta} \tag{13}$$

where a is the second shape parameter of the Exponentiated Weibull. In [15], a maximum likelihood estimator for parameter λ of this distribution is given. In particular, it is shown that the log-likelihood function can be expressed as:

$$\begin{aligned} L(a, \beta, \lambda) &= k \ln a + k \ln \beta - k\beta \log \lambda + (\beta - 1) \sum x_i + \\ &+ (a-1) \sum \ln \left[1 - e^{-\left(\frac{x_i}{\lambda}\right)^\beta}\right] - \sum \left(\frac{x_i}{\lambda}\right)^\beta \end{aligned}$$

Since parameters a and β in this case are both known ($a = \beta = 2$), only the partial derivative about λ is needed to determine the scale parameter λ_H of the distribution:

$$\hat{\lambda}_H = \left(\frac{1}{k} \sum_i \left[x_i^2 \left(1 - \frac{e^{-\left(\frac{x_i}{\hat{\lambda}_H}\right)^2}}{1 - e^{-\left(\frac{x_i}{\hat{\lambda}_H}\right)^2}} \right) \right] \right)^{\frac{1}{2}} \tag{14}$$

Equation (14) is an implicit expression since $\hat{\lambda}_H$ appears on both sides. However in [15] it has been proven that it has very nice properties: in particular it can be solved with a fixed point algorithm in very few iterations (in our experience, usually 10 iterations have been enough to determine $\hat{\lambda}_H$ up to the machine double floating point precision). In particular, we can estimate parameters $\hat{\lambda}_L$ and $\hat{\lambda}_H$ from a set of n samples u_i and v_i of the failure times of the first and second cores using the following procedure $\mathcal{P}$:

1) Determine $\hat{\lambda}_L$ from u_i using Eq. (11).
2) Initially set $\hat{\lambda}_H = \hat{\lambda}_L$.
3) Until $\hat{\lambda}_H$ convereges to a fixed point solution:
   - Compute $v'_i = v_i - u_i \left(1 - \frac{\hat{\lambda}_H}{\hat{\lambda}_L}\right)$.
   - Determine an improvement of $\hat{\lambda}_H$ from v'_i using Eq. (14).

In the previous procedure, v'_i computes the rescaled failure time of the second core, which has been shown to correspond to the maximum of two Weibull distributions of parameter λ_H in Section II. Fig. 5 shows the marginal epistemic uncertainty distributions of the two parameters that we can obtain by using the proposed estimation method. As expected, especially for a reduced number of samples, the uncertainty on the high temperature parameter λ_H is much higher than the one on λ_L. Moreover, since the estimation of λ_H depends on the value computed for $\hat{\lambda}_L$, the two parameters are not independent. Fig.

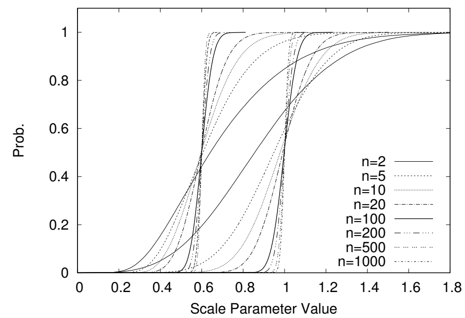

Fig. 5. Epistemic uncertainty on the two scale parameters.

6 shows the correlation of the two values. It is interesting to see that $\hat{\lambda}_H$ is almost never estimated at a value that is less than about $\hat{\lambda}_L/4$. As expected, as the number of samples increases, the epistemic distribution of the two parameters approaches a bi-variate normal distribution centred in their exact value. Note

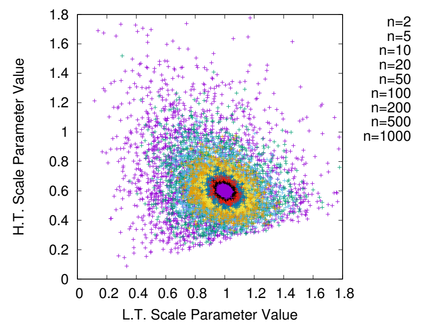

Fig. 6. Correlation of the two scale parameters.

that when the number of samples is very low, the previous procedure might not work, since the inner-most summation might become negative, and thus produce a complex number when powered to $1/2$. Table I reports the percentage of times the procedure did not find a solution as function of the number of samples. As it can be seen, with a reasonable number of samples (i.e., $k = 50$) we could find only one case out of $500K$ experiments where the procedure did not converge. By applying the proposed procedure we can compute the

TABLE I
PERCENTAGE OF TIMES PROCEDURE $\mathcal{P}$ FAILS TO FIND A SOLUTION

k	2	5	10	20	50	100	500
%err	17.5%	7.3%	2.1%	0.2%	$2{\cdot}10^{-04}$%	0	0

epistemic density $f_\Lambda(\lambda_L, \lambda_H)$ of the two parameters. The epistemic distribution can be used to propagate the uncertainty to output measures such as the MTTF of the system. In particular, we can compute the average value of the MTTF (shown in Fig. 7) as:

$$MTTF = \iiint (1 - F_{II}(t|\lambda_L, \lambda_H))\, f_\Lambda(\lambda_L, \lambda_H)\, dt\, d\lambda_L\, d\lambda_H$$

Notice that the distribution $F_{II}(t|\lambda_L, \lambda_H)$ can be estimated by simulation of the state-based model, the results shown in Fig. 7 and 8 were computed in such a way.

With a simple extension of the approach used to derive Eq. (10), we can compute the distribution of the expected MTTF due to the epistemic uncertainty (shown in Fig. 8) as:

$$Pr\{MTTF \leq x\} = \iint \mathbf{1}\left(\int (1 - F_{II}(t|\lambda_L, \lambda_H))\, dt \leq x \right) \cdot \\ \cdot \quad f_\Lambda(\lambda_L, \lambda_H)\ d\lambda_L\ d\lambda_H$$

The previous expression can also be used to compute more accurate confidence intervals: for example, the 98% confidence interval can be computed by finding the values t_L and t_U such that:

$$Pr\{MTTF \leq t_L\} = 0.01, \qquad Pr\{MTTF \leq t_U\} = 0.99$$

The evolution of the 90%, 95%, 98% and 99% confidence intervals for the MTTF are also reported in Fig. 7. Such plots can help determine the optimal number of samples required to have a given accuracy, with a given confidence level, on the output measures.

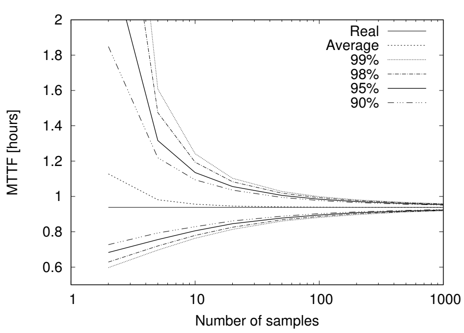

Fig. 7. Uncertainty propagation.

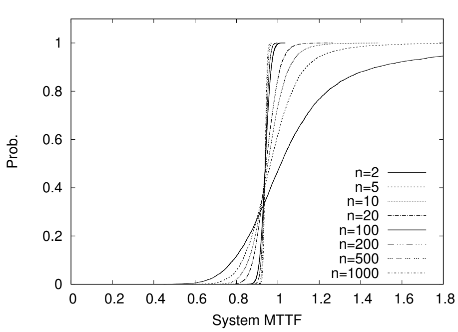

Fig. 8. CDF of the conditional MTTF of a Weibull distribution given the epistemic uncertainty on the scale parameter.

V. Conclusions

In this paper we considered uncertainty propagation applied to a two-core system-on-chip embedded system. In particular, we considered the epistemic distribution caused by parameters estimated from a reduced number of samples, and propagated its effect on the evaluation of the MTTF of the system. Future works will consider more complex systems, composed of m cores, and n-out-of-m redundancy, and the estimation of the shape parameter. We will also consider how uncertainty propagates to more sophisticated measures, such as percentiles of the time to failure distribution.

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