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Instanton effects in $\mathcal{N} = 1$ brane models and the Kähler metric of twisted matter

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Abstract: We consider locally consistent systems of magnetized D9 branes on an orbifolded six-torus which support $\mathcal{N} = 1$ gauge theories. In such realizations, the matter multiplets arise from "twisted" strings connecting different stacks of branes. The introduction of Euclidean 5 branes (E5) wrapped on the six-dimensional compact space leads to instanton effects. For instance, if the system is engineered so as to yield SQCD, a single E5 brane may account for the ADS/TVY superpotential. We discuss the subtle interplay that exists between the annuli diagrams with an E5 boundary and the holomorphicity properties of the effective low-energy action of the $\mathcal{N} = 1$ theory. The consistency of this picture allows to obtain information on the Kähler metric of the chiral matter multiplets arising from twisted strings.

Keywords: D-branes, Intersecting branes models, Supersymmetric Effective Theories.

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Contents

1. Introduction

While non-perturbative instanton effects have been analyzed in great detail in field theory and can be evaluated by means of complete and clear algorithms (for reviews, see for instance refs. [1, 2]), the study of these effects in string theory is still at an early stage and, despite some remarkable progresses in the last few years, further work is still needed to reach a similar degree of accuracy in their computation. This would be very important not only for including string corrections to the effects that have been already computed with field theoretical methods, but especially to derive new non-perturbative effects of purely stringy origin that could play a relevant role in the applications of string theory to phenomenology. Recently this possibility has been intensively investigated from several different points of view and has received considerable attention [3]–[16].

However, in order to learn how to deal with non-perturbative effects in string theory and gain a good control on the results, it is very important also to reproduce, using string methods, the non-perturbative effects already known from field theory. To this aim, toroidal orbifolds of Type II string theory (for a review see ref. [17]) are very useful since they provide a concrete framework in which one can perform explicit calculations of instanton effects. For example, they can be used to engineer $\mathcal{N}=2$ super Yang-Mills (SYM) theories and study the instanton induced prepotential, as discussed in detail in ref. [18]. In a recent paper [19] we have extended this procedure by compactifying six dimensions on $(\mathcal{T}_2^{(1)} \times \mathcal{T}_2^{(2)})/\mathbb{Z}_2 \times \mathcal{T}_2^{(3)}$ and by including the contribution of the mixed annuli diagrams, as advocated in refs. [4, 7, 20]. In particular we have shown that the non-holomorphic terms in these annulus amplitudes precisely reconstruct the appropriate Kähler metric factors that are needed to write the instanton correlators in terms of purely holomorphic variables. In this way the correct holomorphic structure of the instanton induced low energy effective action in the Coulomb branch of the $\mathcal{N}=2$ SYM theory has been obtained.

In the present paper we apply this procedure to $\mathcal{N}=1$ SYM theories that we engineer by means of stacks of magnetized fractional D9 branes in a background given by the product of $\mathbb{R}^{1,3}$ times a six-dimensional orbifold $(\mathcal{T}_2^{(1)} \times \mathcal{T}_2^{(2)} \times \mathcal{T}_2^{(3)})/(\mathbb{Z}_2 \times \mathbb{Z}_2)$. A single stack of fractional D9 branes, that we call "color" branes, supports on its world-volume a pure $\mathcal{N}=1$ gauge theory. Matter chiral multiplets can be obtained by introducing a second stack of magnetized fractional D9 branes, called "flavor" branes, that belong in general to a different irreducible representation of the orbifold group, and by considering the massless open strings having one endpoint on the color branes and the other on the flavor branes. In this framework one can also engineer $\mathcal{N}=1$ super QCD by suitably introducing a third stack of magnetized fractional D9 branes, in such a way that the massless open strings connecting the color branes and the two types of flavor branes correspond respectively to the right and left-handed quarks and their super-partners, and hence give rise to a vector-like theory as described in section 2.

To study instanton effects in this set-up one has to add a stack of fractional Euclidean D5 branes (E5 branes for short) that completely wrap the internal manifold and hence describe point-like configurations from the four-dimensional point of view. If the wrapping numbers and magnetization of these E5 branes are the same as those of the color D9 branes, one has a stringy realization of ordinary gauge theory instantons.[1] If instead their wrapping numbers and magnetization are different from the color branes, one obtains "exhotic" instanton configurations of purely stringy nature. In this paper we will not explicitly consider this possibility, even if our methods could be used also in this case. On the contrary, following the procedure outlined in refs. [24, 18], we compute using string methods the superpotential in $\mathcal{N}=1$ SYM theories induced by gauge instantons. In doing so, the contribution of mixed annulus diagrams with a boundary attached to the E5 branes, which are of the same order in the string coupling constant as the disk diagrams which account for the moduli measure, has to be taken into account.

As noticed in the literature [4, 6, 7, 20], in supersymmetric situations these mixed annulus amplitudes are related in a precise way to the 1-loop corrections to the gauge coupling constant of the color gauge theory and the physical origin of this identification has been discussed in ref. [19]. This relation can then be used to compare the explicit

[1] In fact, these D9/E5 systems are essentially a T-dual version of the D3/D(–1) systems which, in uncompactified set-ups, are well-known to realize at the string theory level the gauge instantons and their moduli, described à la ADHM [21]–[24].

expression of the mixed annulus amplitudes to the general formula [25 − 27] that expresses the 1-loop corrections to the gauge coupling computed in string theory in terms of the fields and geometrical quantities that appear in the effective supergravity theory, such as the Kähler metrics for the various multiplets. Exploiting this fact, we explicitly compute the mixed annulus diagrams in our orbifold models and extract from them information on the Kähler metric for the matter multiplets. We then perform two checks on our results.

First, we consider the 1-instanton induced superpotential in the set-up corresponding to $\mathcal{N} = 1$ SQCD. In refs. [7, 10] it has already been shown that the stringy instanton calculus in this case reproduces the ADS/TVY superpotential [28] (see also ref. [29]). Here we discuss in detail the rôle of the mixed annuli contributions and show that they are crucial in making this superpotential holomorphic when expressed in terms of the variables appropriate to the low-energy supergravity description.

Second, we exploit the fact that the Kähler metrics of the matter multiplets enter crucially in the relation between the holomorphic superpotential couplings in the effective Lagrangian and the physical Yukawa couplings for the canonically normalized fields. We consider the expression of the latter provided in ref. [31] for the field-theory limit of magnetized brane models, and show that, after transforming it to the supergravity basis, it becomes purely holomorphic.

The paper is organized as follows. In section 2 we describe the set-up we utilize for realizing $\mathcal{N} = 1$ supersymmetric gauge theories. Section 3 is devoted to the description of the instanton calculus in this set-up. In section 4 we compute the mixed annulus diagrams while in section 5 we discuss the relation with the Kähler metric for the matter fields; furthermore we check the holomorphicity of the 1-instanton induced superpotential. In the last section we show that our expressions yield holomorphic cubic superpotential couplings of the matter multiplets if we start from the physical Yukawa couplings in magnetized brane models computed in ref. [31]. Finally, many technical details are given in the appendix.

2. Local $\mathcal{N} = 1$ brane models with chiral matter

A way to realize a $\mathcal{N} = 1$ SYM theory is to place a stack of fractional D9 branes in a background given by the product of $\mathbb{R}^{1,3}$ times a six-dimensional orbifold

$$\frac{\mathcal{T}_2^{(1)} \times \mathcal{T}_2^{(2)} \times \mathcal{T}_2^{(3)}}{\mathbb{Z}_2 \times \mathbb{Z}_2} \,. \tag{2.1}$$

For each torus $\mathcal{T}_2^{(i)}$, the string frame metric and the B-field[2] are parameterized by the Kähler and complex structure moduli, $T^{(i)} = T_1^{(i)} + \mathrm{i}\, T_2^{(i)}$ and $U^{(i)} = U_1^{(i)} + \mathrm{i}\, U_2^{(i)}$ respectively. For our precise conventions we refer to appendix A.1. The ten-dimensional string coordinates X^M and ψ^M are split as

$$X^M \to (X^\mu, Z^i) \quad \text{and} \quad \psi^M \to (\psi^\mu, \Psi^i)\,, \tag{2.2}$$

where $\mu = 0, 1, 2, 3$ and the complex coordinates Z^i and Ψ^i, defined in eq. (A.2), are orthonormal in the metric of the i-th torus. Also the (anti-chiral) spin-fields $S^{\dot{\mathcal{A}}}$ of the

[2]Without loss of generality, in the following we will actually set the B-field to zero.

RNS formalism in ten dimensions factorize in a product of four-dimensional and internal spin-fields, and the precise splitting is given in eq. (A.3). The $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold group in (2.1) contains three non-trivial elements h_i ($i = 1, 2, 3$). The element h_i leaves the i-th torus $\mathcal{T}_2^{(i)}$ invariant while acting as a reflection on the remaining two tori.

The above geometry can also be described in the so-called supergravity basis using the complex moduli s, $t^{(i)}$ and $u^{(i)}$, whose relation with the previously introduced quantities in the string basis is [32, 17]

$$\begin{aligned} \mathrm{Im}(s) \equiv s_2 &= \frac{1}{4\pi}\, \mathrm{e}^{-\phi_{10}}\, T_2^{(1)} T_2^{(2)} T_2^{(3)} , \\ \mathrm{Im}(t^{(i)}) \equiv t_2^{(i)} &= \mathrm{e}^{-\phi_{10}} T_2^{(i)} , \\ u^{(i)} = u_1^{(i)} + \mathrm{i}\, u_2^{(i)} &= U^{(i)} , \end{aligned} \tag{2.3}$$

where ϕ_{10} is the ten-dimensional dilaton. The real parts of s and $t^{(i)}$ are related to suitable RR potentials. In terms of these variables, the $\mathcal{N} = 1$ bulk Kähler potential is given by [33]

$$K = -\log(s_2) - \sum_{i=1}^{3} \log(t_2^{(i)}) - \sum_{i=1}^{3} \log(u_2^{(i)}) \ . \tag{2.4}$$

Colored and flavored branes. In this orbifold background we place a stack of N_a fractional D9 branes (hereinafter called colored branes and labeled by an index a) which for definiteness are taken to transform in the trivial irreducible representation R_0 of the orbifold group. The massless excitations of the open strings attached to these branes fill the $\mathcal{N} = 1$ vector multiplet in the adjoint representation of $\mathrm{U}(N_a)$. The disk interactions of the corresponding vertex operators reproduce, in the field theory limit $\alpha' \to 0$, the $\mathcal{N} = 1$ SYM action with $\mathrm{U}(N_a)$ gauge group, which in the Euclidean signature appropriate to discuss instanton effects, reads

$$S_{\mathrm{SYM}} = \frac{1}{g_a^2} \int d^4x \ \mathrm{Tr} \left\{ \frac{1}{2}\, F_{\mu\nu}^2 - 2\, \bar{\Lambda}_{\dot{\alpha}} \bar{\not{D}}^{\dot{\alpha}\beta} \Lambda_\beta \right\} , \tag{2.5}$$

where the tree-level Yang-Mills coupling constant g_a is given by

$$\frac{1}{g_a^2} = \frac{1}{4\pi}\, \mathrm{e}^{-\phi_{10}}\, T_2^{(1)} T_2^{(2)} T_2^{(3)} = s_2 \ . \tag{2.6}$$

Richer models can be found if we introduce additional stacks of fractional D9-branes, distinguished with a subscript b, that belong to various irreducible representations of the orbifold group and can be magnetized. In general, we will have N_b branes of type b, which we will call flavor branes, and $n_b^{(i)}$ will be their wrapping number around the i-th torus. These branes admit a constant magnetic field on the i-th torus

$$F_b^{(i)} = f_b^{(i)}\, dX^{2i+2} \wedge dX^{2i+3} = \mathrm{i}\, \frac{f_b^{(i)}}{T_2^{(i)}}\, dZ^i \wedge d\bar{Z}^i \ . \tag{2.7}$$

The generalized Dirac quantization condition requires that the first Chern class $c_1(F_b^{(i)})$ be an integer, which, in our conventions, implies that

$$2\pi\alpha' f_b^{(i)} = \frac{m_b^{(i)}}{n_b^{(i)}} \tag{2.8}$$

with $m_b^{(i)} \in \mathbb{Z}$. In terms of the angular parameters $\nu_b^{(i)}$, defined by

$$2\pi\alpha' \frac{f_b^{(i)}}{T_2^{(i)}} = \tan \pi\nu_b^{(i)} \quad \text{with} \quad 0 \leq \nu_b^{(i)} < 1\,, \tag{2.9}$$

it is possible to show that bulk $\mathcal{N} = 1$ supersymmetry is preserved if[3]

$$\nu_b^{(1)} - \nu_b^{(2)} - \nu_b^{(3)} = 0\ . \tag{2.10}$$

The presence of the magnetic fluxes implies that the open strings stretching between two different types of branes (e.g. the $D9_b/D9_a$ strings) are twisted. This means that the internal string coordinates Z^i and Ψ^i have the following twisted monodromy properties

$$Z^i\big(\mathrm{e}^{2\pi\mathrm{i}}z\big) = \mathrm{e}^{2\pi\mathrm{i}\nu_b^{(i)}}\, Z^i(z) \quad \text{and} \quad \Psi^i\big(\mathrm{e}^{2\pi\mathrm{i}}z\big) = \eta\, \mathrm{e}^{2\pi\mathrm{i}\nu_b^{(i)}}\, \Psi^i(z)\,, \tag{2.11}$$

where $\eta = +1$ for the NS sector and $\eta = -1$ for the R sector. If also the color branes are magnetized, we have to replace in (2.10) and (2.11) $\nu_b^{(i)}$ with $\nu_{ba}^{(i)} = \nu_b^{(i)} - \nu_a^{(i)}$, which describe the relative magnetization of the two stacks of branes. When no confusion is possible, we will denote the twist angles simply by $\nu^{(i)}$.

As is well-known, in a toroidal orbifold compactification with wrapped branes there are unphysical closed string tadpoles that must be canceled to have a globally consistent model. Usually this cancellation is achieved by introducing an orientifold projection and suitable orientifold planes. Like in other cases treated in the literature, in this paper we take a "local" point of view and assume that the brane systems we consider can be made fully consistent with an orientifold projection.

$\mathcal{N} = 1$ SQCD with magnetized branes. In the following, we will be mostly interested in studying instanton effects in $\mathcal{N} = 1$ SQCD with N_F flavors. In our orbifold background we can realize this model by taking two stacks of flavored fractional D9 branes, denoted by b and c respectively, both belonging to a different representation of the orbifold group with respect to the color branes; see figure 1 for a pictorial representation of the system we consider. For definiteness, we take the R_1 representation as defined in appendix A.1.

If the twist angles satisfy the $\mathcal{N} = 1$ supersymmetry condition $\nu_{ba}^{(1)} - \nu_{ba}^{(2)} - \nu_{ba}^{(3)} = 0$, the massless states of the $D9_b/D9_a$ strings fill up a chiral multiplet $q_{ba} \equiv q$, which transforms in the anti-fundamental representation $\bar{N}_a$ of the color group and appears with a flavor degeneracy

$$N_b|I_{ab}|\,, \tag{2.12}$$

where I_{ab} is the number of Landau levels for the (a, b) "intersection", namely

$$I_{ab} = \prod_{i=1}^{3} \big(m_a^{(i)} n_b^{(i)} - m_b^{(i)} n_a^{(i)}\big) = -I_{ba}\ . \tag{2.13}$$

[3]Other possibilities are $-\nu_b^{(1)} - \nu_b^{(2)} + \nu_b^{(3)} = 0$; $-\nu_b^{(1)} + \nu_b^{(2)} - \nu_b^{(3)} = 0$; $\nu_b^{(1)} + \nu_b^{(2)} + \nu_b^{(3)} = 2$. They are all related to the position (2.10) by obvious changes.

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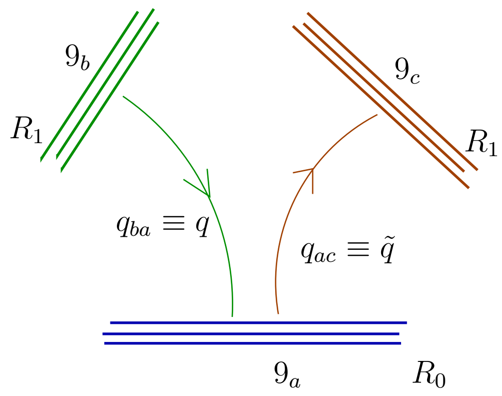

Figure 1: Schematization of the brane system we consider, and of its spectrum of chiral multiplets; see the text for more details.

The complex scalar, denoted with an abuse of notation by the same letter q used for the whole multiplet, arises from the NS sector and is described by the vertex operator (A.15). Its supersymmetric partner is a chiral fermion χ_α described by the vertex operator (A.16) of the R sector, which is connected to the scalar vertex by the $\mathcal{N} = 1$ supersymmetry generated by the open string supercharges preserved by the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold.

In an analogous way, we can analyze the open strings stretching between the color branes and the flavor branes of type c. If the twist angles are such that $\nu_{ac}^{(1)} - \nu_{ac}^{(2)} - \nu_{ac}^{(3)} = 0$, then the massless states of the D9$_a$/D9$_c$ strings (notice the orientation!) fill up a chiral multiplet $q_{ac} \equiv \tilde{q}$ that transforms in the fundamental representation N_a of the color group and appears with a flavor degeneracy

$$N_c |I_{ac}| \,, \tag{2.14}$$

where I_{ac} is the number of Landau levels for the (a, c) "intersection". The bosonic and fermionic components of the multiplet $\tilde{q}$ are described, respectively, by the vertex operators (A.26) and (A.27) which are also related to each other by the $\mathcal{N} = 1$ supersymmetry preserved by the orbifold.

This set-up provides a realization of $\mathcal{N} = 1$ SQCD if we arrange the branes in such a way that the flavor degeneracies (2.12) and (2.14) are equal:

$$N_b |I_{ab}| = N_c |I_{ac}| \equiv N_F \ . \tag{2.15}$$

In this way we engineer the same number N_F of fundamental and anti-fundamental chiral multiplets, which will be denoted by q_f and $\tilde{q}^f$ with $f = 1, \ldots, N_F$.

The field-theory limit of the disk amplitudes involving the fields of the chiral multiplets and those of the vector multiplet yields the $\mathcal{N} = 1$ SQCD action; for instance, the kinetic term of the scalars arises in the form

$$\int d^4x \, \sum_{f=1}^{N_F} \Big\{ D_\mu q^{\dagger f} \, D^\mu q_f + D_\mu \tilde{q}^f \, D^\mu \tilde{q}_f^\dagger \Big\} \,, \tag{2.16}$$

where we have explicitly indicated the sum over the flavor indices and suppressed the color indices. In the supergravity basis it is customary to use fields with a different normalization. The kinetic term for the scalars of the chiral multiplet is written as

$$\int d^4x \sum_{f=1}^{N_F} \Big\{ K_Q\, D_\mu Q^{\dagger f}\, D^\mu Q_f + K_{\tilde{Q}}\, D_\mu \tilde{Q}^f\, D^\mu \tilde{Q}_f^\dagger \Big\}\,, \tag{2.17}$$

where K_Q and $K_{\tilde{Q}}$ are the Kähler metrics. Upon comparison with (2.16), we see that relation between the fields q and $\tilde{q}$ appearing in the string vertex operators and the fields Q and $\tilde{Q}$ of the supergravity basis is

$$q = \sqrt{K_Q}\, Q \quad , \quad \tilde{q} = \sqrt{K_{\tilde{Q}}}\, \tilde{Q}\ . \tag{2.18}$$

Actually, the rescalings (2.18) apply not only to the scalar components, but to the entire chiral multiplets.

3. Instantonic brane effects

In this stringy set-up non-perturbative instantonic effects can be included by adding fractional Euclidean D5 branes (or E5 branes for short) that completely wrap the internal manifold. We choose these branes to be identical to the color $D9_a$ branes in the internal directions (i.e. they transform in the same representation of the orbifold group; they have, if any, the same magnetization etc.), while they are point-like in the space-time directions. Thus we call them $E5_a$, and they provide the stringy representation of ordinary instantons for the gauge theory on the $D9_a$ branes. Notice, however, that with respect to the gauge theory living on a different stack of D9 branes (like the branes $D9_b$ or $D9_c$), the $E5_a$ represent "exotic" instantons, whose properties are different from those of the ordinary gauge theory instantons. Recently, these "exotic" configurations have been investigated [3]–[16] from various points of view.

Our aim is to use the relation between the non-holomorphic corrections appearing in the string computation of instantonic effects and the Kähler metrics of the chiral multiplets in the supergravity basis to gain information on the latter. To elucidate the physical meaning of these corrections, we will examine in particular the one-instanton induced ADS/TVY superpotential [28] (see also ref. [29]), present in the case $N_F = N_a - 1$, whose stringy derivation has been recently reconsidered in [7, 10]. To proceed, let us first review how the instanton contributions to the superpotential arise in our specific set-up.

3.1 The instanton moduli

In presence of the $E5_a$ branes we have new types of open strings: the $E5_a/E5_a$ strings (neutral sector), the $D9_a/E5_a$ or $E5_a/D9_a$ strings (charged sector) and the $D9_b/E5_a$ or $E5_a/D9_c$ strings (flavored sectors). The states of such strings do not carry any space-time momentum and represent moduli rather than dynamical fields in space-time. The spectrum of moduli is summarized in table 1, and the corresponding vertex operators are listed in appendix A.2. Let us notice that the states of these strings can carry (discretized)

Sector		ADHM	Meaning	Chan-Paton	Dimension
$5_a/5_a$	NS	a_μ	centers	adj. $\mathrm{U}(k)$	(length)
		D_c	Lagrange mult.	$\vdots$	$(\text{length})^{-2}$
	R	M^α	partners	$\vdots$	$(\text{length})^{\frac{1}{2}}$
		$\lambda_{\dot\alpha}$	Lagrange mult.	$\vdots$	$(\text{length})^{-\frac{3}{2}}$
$9_a/5_a$	NS	$w_{\dot\alpha}$	sizes	$N_a \times \overline{k}$	(length)
$5_a/9_a$		$\bar{w}_{\dot\alpha}$	$\vdots$	$k \times \overline{N}_a$	$\vdots$
$9_a/5_a$	R	μ	partners	$N_a \times \overline{k}$	$(\text{length})^{\frac{1}{2}}$
$5_a/9_a$		$\bar{\mu}$	$\vdots$	$k \times \overline{N}_a$	$\vdots$
$9_b/5_a$	R	μ'	flavored	$N_F \times \overline{k}$	$(\text{length})^{\frac{1}{2}}$
$5_a/9_c$		$\tilde{\mu}'$	$\vdots$	$k \times \overline{N}_F$	$\vdots$

Table 1: The spectrum of moduli from the open strings with at least one boundary attached to the instantonic E5_a branes. See the text for more details and comments, and appendix A.2 for the expressions of the corresponding emission vertices.

momentum along the compact directions, when they are untwisted, i.e. when they belong to the neutral or charged sectors; such Kaluza-Klein copies of the moduli represent a genuine string feature.

Let us also recall that, in order to yield non-trivial interactions when $\alpha' \to 0$ [24], the emission vertices of some of the moduli, given in appendix A.2, have to be rescaled with factors of the dimensionful coupling constant on the E5_a, namely $g_{5_a} = g_a/(4\pi^2\alpha')$, with g_a given in (2.6). As a consequence, some of the moduli acquire unconventional scaling dimensions which, however, are the right ones for their interpretation as parameters of an instanton solution [1, 24].

The neutral moduli which survive the orbifold projection are the four physical bosonic excitations a_μ from the NS sector, related to the positions of the (multi-)centers of the instanton, and three auxiliary excitations D_c ($c = 1, 2, 3$). In the R sector, we find two chiral fermionic zero-modes M^α, and two anti-chiral ones $\lambda_{\dot\alpha}$. The M^α are the fermionic partners of the instanton centers. All of these moduli are $k \times k$ matrices and transform in the adjoint representation of $\mathrm{U}(k)$. If we write the $k \times k$ matrices a^μ and M^α as

$$a^\mu = x_0^\mu\, \mathbb{1}_{k\times k} + y_c^\mu\, T^c \quad , \quad M^\alpha = \theta^\alpha\, \mathbb{1}_{k\times k} + \zeta_c^\alpha\, T^c\,, \tag{3.1}$$

where T^c are the generators of $\mathrm{SU}(k)$, then the instanton center of mass, x_0^μ, and its fermionic partners, θ^α, can be identified respectively with the bosonic and fermionic coordinates of the $\mathcal{N} = 1$ superspace.

The charged instantonic sector contains, in the NS sector, two physical bosonic moduli $w_{\dot\alpha}$ with dimension of (length), related to the size and orientation in color space of the instanton, and a fermionic modulus μ. These moduli carry a fundamental $\mathrm{U}(k)$ index and a color one.

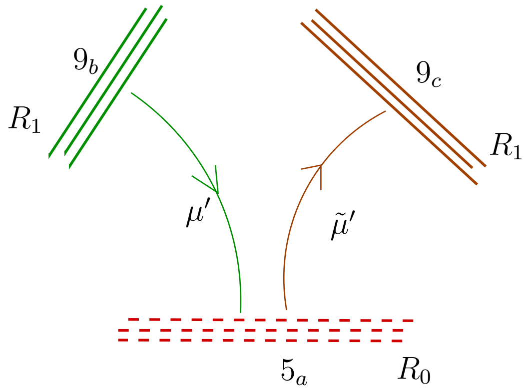

Figure 2: The flavored moduli of instantonic $E5_a$ branes in presence of $D9_b$ and $D9_c$ branes; see the text for more details.

In our realization of $\mathcal{N} = 1$ SQCD there are two flavored instantonic sectors corresponding to the open strings that stretch between the $E5_a$ branes and the flavor branes of type b or c, depicted in figure 2. In both cases the four non-compact directions have mixed Neumann-Dirichlet boundary conditions while all the internal complex coordinates are twisted. As a consequence, there are no bosonic physical zero-modes in the NS sector and the only physical excitations are fermionic ones from the R sector. A detailed analysis of the twisted conformal field theory shows that there are fermionic moduli μ'_f in the $D9_b/E5_a$ strings and fermionic moduli $\tilde{\mu}'^f$ in the $E5_a/D9_c$ strings. They are described respectively by the vertex operators (A.34) and (A.38). On the other hand no physical states survive the GSO projection in the $E5_a/D9_b$ and $D9_c/E5_a$ sectors. The fermionic moduli μ'_f and $\tilde{\mu}'^f$ are the counterparts of the chiral multiplets q_f and $\tilde{q}^f$ respectively, when the color $D9_a$ branes are replaced by the instantonic $E5_a$ branes.

The physical moduli we have listed above, collectively denoted by $\mathcal{M}_k$, are in one-to-one correspondence with the ADHM moduli of $\mathcal{N} = 1$ gauge instantons (for a more detailed discussion see, for instance, [1] and references therein). In all instantonic sectors we can construct many other open string states that carry a discretized momentum along the compact directions and/or have some bosonic or fermionic string oscillators. These "massive" states are not physical, *i.e.* they cannot be described by vertex operators of conformal dimension one; they can, however, circulate in open string loop diagrams.

3.2 The instanton induced superpotential

In the sector with instanton number k, the effective action for the gauge/matter fields is obtained by the "functional" integral over the instanton moduli of the exponential of all diagrams with at least part of their boundary on the $E5_a$ branes, possibly with insertions of moduli and gauge/matter fields [34, 35, 23, 24, 4, 19]. In the semi-classical approximation, only disk diagrams and annuli (the latter with no insertions) are retained. Focusing on the dependence from the scalar fields of the chiral multiplets in the Higgs branch, we have

$$S_k = \mathcal{C}_k \, \mathrm{e}^{-\frac{8\pi^2}{g_a^2}k} \, \mathrm{e}^{\mathcal{A}'_{5_a}} \int d\mathcal{M}_k \, \mathrm{e}^{-S_{\mathrm{mod}}(q,\tilde{q};\mathcal{M}_k)} \ . \tag{3.2}$$

Let us now analyze the various terms in this expression.

$\mathcal{C}_k$ is a normalization factor which compensates for the dimensions of the integration measure $d\mathcal{M}_k$, and may contain numerical constants and powers of the coupling g_a. Its dimensionality is determined by counting the dimensions (measured in units of α') of the various moduli $\mathcal{M}_k$ as given in the previous subsections, and the result is, up to overall numerical constants,

$$\mathcal{C}_k = \big(\sqrt{\alpha'}\big)^{-(3N_a-N_F)k} \, (g_a)^{-2N_a k} \ . \tag{3.3}$$

Notice the appearance of the one-loop coefficient $b_1 = (3N_a - N_F)$ of the β-function of the $\mathcal{N}=1$ SQCD with N_F flavors. The factor of $(g_a)^{-2N_a k}$ in (3.3) has been inserted following the discussion of ref. [1], but in principle it should also have a stringy interpretation, probably as a left-over of the cancellation between the bosonic and fermionic fluctuation determinants when the $\mathcal{N}=1$ gauge action is normalized as in (2.5).

As explained in refs. [34, 24], the disk diagrams with no insertions account for the exponential of (minus) the classical instanton action $8\pi^2 k/g_a^2$, where g_a is interpreted as the Yang-Mills coupling constant at the string scale. This explains the second factor in (3.2). The third factor contains $\mathcal{A}'_{5_a}$ which accounts for the open string annuli diagrams with at least one boundary on the E5_a branes and no insertions [4, 6, 7, 19]. Since the functional integration over the ADHM moduli $\mathcal{M}_k$ is explicitly performed in (3.2), to avoid double counting only the contribution of the "massive" string excitations has to be taken into account in these annuli: this is the reason of the $'$ notation which reminds that only the "massive" instantonic string excitations must circulate in the loop.

Finally, in the integrand of (3.2) we find the moduli action $S_{\mathrm{mod}}(q,\tilde{q};\mathcal{M}_k)$. This can be computed following the procedure explained in ref. [24] from all disk scattering amplitudes involving the ADHM moduli and the scalar fields q and $\tilde{q}$ in the limit $\alpha' \to 0$ (with g_a fixed). The result is

$$\begin{aligned} S_{\mathrm{mod}}(q,\tilde{q};\mathcal{M}_k) = \mathrm{tr}_k\Big\{ & \mathrm{i}D_c\Big(\bar{w}_{\dot\alpha}(\tau^c)^{\dot\alpha}_{\ \dot\beta} w^{\dot\beta} + \mathrm{i}\bar\eta^c_{\mu\nu}\big[a^\mu, a^\nu\big]\Big) \\ & - \mathrm{i}\lambda^{\dot\alpha}\Big(\bar\mu w_{\dot\alpha} + \bar{w}_{\dot\alpha}\mu + \big[a_\mu, M^\alpha\big]\sigma^\mu_{\alpha\dot\alpha}\Big)\Big\} \\ & + \mathrm{tr}_{\,k} \sum_{f=1}^{N_F} \Big\{ \bar{w}_{\dot\alpha}\big[q^{\dagger f} q_f + \tilde{q}^f \tilde{q}^\dagger_f\big] w^{\dot\alpha} - \frac{\mathrm{i}}{2}\,\bar\mu\, q^{\dagger f}\mu'_f + \frac{\mathrm{i}}{2}\,\tilde{\mu}'^f \tilde{q}^\dagger_f\,\mu \Big\} \ . \end{aligned} \tag{3.4}$$

The first two lines above have the only effect of implementing the (super) ADHM constraints, for which D_c and $\lambda^{\dot\alpha}$ act as Lagrange multipliers. The last line arises from the disk diagrams which contain insertions of the chiral scalars and survive in the field theory limit, depicted in figure 3.

There are other non-zero disk diagrams with moduli and matter fields that survive in the field theory limit. However, these other diagrams can be related to the ones in figure 3 by means of supersymmetry Ward identities [23, 24]. This implies that the complete

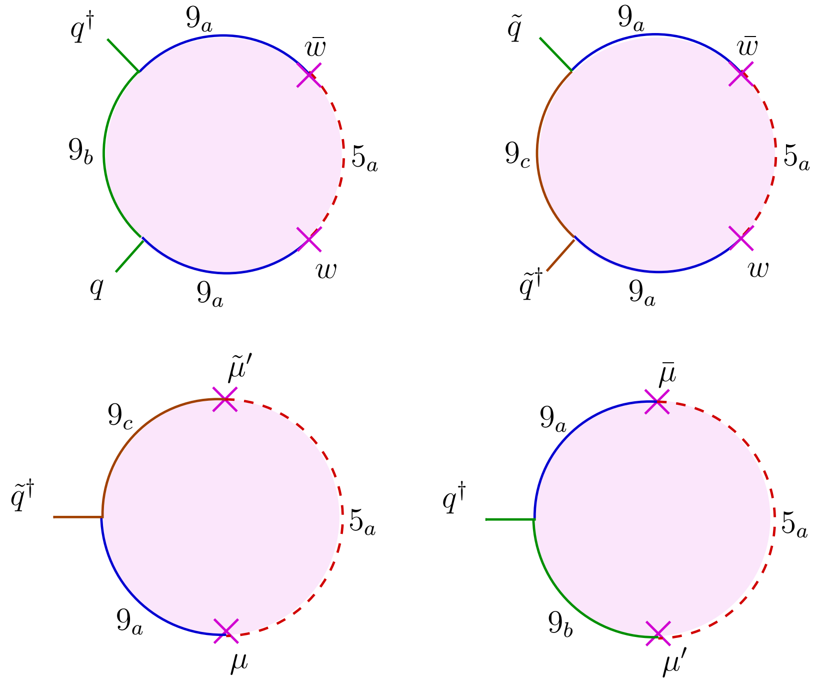

Figure 3: Disk interactions between the moduli and the chiral scalars which survive in the field theory limit

result is obtained simply by replacing in (3.4) the scalars q and $\tilde{q}$ and their conjugate with the corresponding chiral and anti-chiral superfields. From now on, we will assume this replacement. Notice that the multiplets q and $\tilde{q}$ appear in (3.4) differently from their conjugates $q^\dagger$ and $\tilde{q}^\dagger$; this fact has important consequences on the holomorphicity properties of the instanton-induced correlators, as we will see later.

In the moduli action (3.4), the superspace coordinates x_0^μ and θ^α, defined in (3.1), appear only through superfields $q(x_0,\theta), \tilde{q}(x_0,\theta), \ldots$. It is therefore convenient to separate these coordinates from the remaining centered moduli, denoted by $\widehat{\mathcal{M}}_k$, and rewrite the effective action (3.2) in terms of a k-instanton induced superpotential W_k, namely

$$S_k = \int d^4x_0\, d^2\theta\; W_k(q,\tilde{q})\,, \tag{3.5}$$

where

$$W_k(q,\tilde{q}) = \mathcal{C}_k\; \mathrm{e}^{-\frac{8\pi^2}{g_a^2}k}\; \mathrm{e}^{\mathcal{A}'_{5_a}} \int d\widehat{\mathcal{M}}_k\; \mathrm{e}^{-S_{\mathrm{mod}}(q,\tilde{q};\widehat{\mathcal{M}}_k)}\;. \tag{3.6}$$

Even if $S_{\mathrm{mod}}(q,\tilde{q};\widehat{\mathcal{M}}_k)$ has an explicit dependence on $q^\dagger$ and $\tilde{q}^\dagger$, this dependence disappears upon integrating over $\widehat{\mathcal{M}}_k$ as a consequence of the cohomology properties of the integration measure on the instanton moduli space [36, 1, 18]. Thus, $W_k(q,\tilde{q})$ depends holomorphically on the chiral superfields q and $\tilde{q}$. However, the annulus amplitude $\mathcal{A}'_{5_a}$ that appears in the prefactor of eq. (3.6) could introduce a non-holomorphic dependence on the complex and Kähler structure moduli of the compactification space. On the other hand, the multiplets

q and $\tilde{q}$ have to be rescaled according to eq. (2.18) to express the result in the supergravity variables, and the holomorphic Wilsonian renormalization group invariant scale $\Lambda_{\rm hol}$ has to be introduced.

We will consider the interplay of all these observations in section 5, after explicitly evaluating the instantonic annulus amplitude $\mathcal{A}'_{5_a}$ in section 4. Before this, however, we briefly comment on the non-perturbative superpotential for $k = 1$.

3.3 The ADS/TVY superpotential

The measure $d\widehat{\mathcal{M}}_k$ in eq. (3.6) contains many fermionic zero modes. Among them, the $\lambda^{\dot\alpha}$ are Lagrange multipliers for the fermionic ADHM constraints but, after enforcing these constraints, the μ's, $\bar\mu$'s, μ''s and $\tilde\mu'$'s must be exactly compensated otherwise the entire integral vanishes. The single instanton case, $k = 1$, is already very interesting. First of all, in this case it is easy to see that the balancing of the fermionic zero-modes requires that $N_F = N_a - 1$. After having integrated over the fermions, we are left with a (constrained) Gaussian integration over the bosonic moduli $w_{\dot\alpha}$ and $\bar w_{\dot\alpha}$, which can be explicitly performed e.g. by going to a region of the moduli space where the chiral fields are diagonal, up to rows/columns of zeroes. Furthermore, the D-terms in the gauge sector constrain the superfields to obey $q^{\dagger f} q_f = \tilde q^f \tilde q_f^\dagger$, so that the bosonic integration brings the square of a simple determinant in the denominator, which cancels the anti-holomorphic contributions produced by the fermionic integrals. In the end, one finds [1] (see also refs. [7, 10])

$$W_{k=1}(q,\tilde q) = \mathcal{C}_k \; \mathrm{e}^{-\frac{8\pi^2}{g_a^2}\,k} \; \mathrm{e}^{\mathcal{A}'_{5_a}} \, \frac{1}{\det\left(\tilde q q\right)} \,, \tag{3.7}$$

which has the same form of the ADS/TVY superpotential [28, 29]. As we will explicitly see in the following, the prefactor $\mathrm{e}^{\mathcal{A}'_{5_a}}$ is crucial to establish the correct holomorphic properties of this superpotential when everything is expressed in terms of the supergravity variables (2.3), the chiral superfields are normalized with their Kähler metrics and the Wilsonian scale $\Lambda_{\rm hol}$ is introduced.

4. The mixed annuli

To describe explicitly the instanton induced effects on the low energy effective action, the only ingredient yet to be specified is the annulus amplitude $\mathcal{A}_{5_a}$, whose "primed" part appears in the equations from eq. (3.2) on. This amplitude represents the 1-loop vacuum energy of open strings with at least one end point on the wrapped instantonic branes E5$_a$. Because of supersymmetry, the annulus amplitude associated to the E5$_a$/E5$_a$ strings identically vanishes, so $\mathcal{A}_{5_a}$ receives contributions only from mixed annuli with one boundary on the E5$_a$'s and the other on the D9 branes. In particular, the 1-loop contribution of the charged instantonic open strings is denoted as

$$\mathcal{A}_{5_a;9_a} = \mathcal{A}(9a/5a) + \mathcal{A}(5a/9a)\,, \tag{4.1}$$

where on the r.h.s. we distinguish the contributions of the D9$_a$/E5$_a$ and E5$_a$/D9$_a$. Similarly, for the flavored instantonic open strings

$$\mathcal{A}_{5_a;9_b} = \mathcal{A}(9b/5a) + \mathcal{A}(5a/9b) \quad \text{and} \quad \mathcal{A}_{5_a;9_c} = \mathcal{A}(9c/5a) + \mathcal{A}(5a/9c) \tag{4.2}$$

for the two different stacks of flavor branes used to engineer $\mathcal{N}=1$ SQCD.

It has been noticed in the literature [6, 7] that the computation of mixed annuli is related to the stringy computation of the 1-loop threshold corrections to the coupling of the color gauge group living on the D9$_a$. In ref. [19] we showed that this relation is explained by the fact that, in a supersymmetric theory, the mixed annuli compute just the running coupling by expanding around the classical instanton background, namely

$$\mathcal{A}_{5_a} = -\frac{8\pi^2 k}{g_a^2(\mu)}\Bigg|_{\text{at 1-loop}} . \tag{4.3}$$

Applying this argument to our system and keeping distinct the charged and flavored sectors, we expect therefore to find

$$\mathcal{A}_{5_a;9_a} = -8\pi^2 k \left(\frac{3N_a}{16\pi^2}\log(\alpha'\mu^2) + \Delta_{\text{color}}\right) , \tag{4.4a}$$

$$\mathcal{A}_{5_a;9_b} + \mathcal{A}_{5_a;9_c} = -8\pi^2 k \left(-\frac{N_F}{16\pi^2}\log(\alpha'\mu^2) + \Delta_{\text{flavor}}\right) . \tag{4.4b}$$

In these expressions, μ is the scale that regularizes the IR divergences of the annuli amplitudes[4] due to the massless states circulating in the loop, and the coefficients of the logarithms arise by counting (with appropriate sign and weight) the bosonic and fermionic ground states of mixed open strings with one end point on the E5$_a$s branes, i.e. the charged and flavored instanton moduli that we listed in section 3. This counting agrees, as it should, with the 1-loop β-function coefficients that are appropriate, respectively, for the gauge and the flavor multiplets. Let us now describe the explicit form of the various annuli amplitudes.

Charged sector. For a given open string orientation, we have

$$\mathcal{A}(9_a/5_a) = \int_0^\infty \frac{d\tau}{2\tau}\left[\text{Tr}_{\text{NS}}\left(P_{\text{GSO}}^{(9_a/5_a)}\, P_{\text{orb.}}\, q^{L_0}\right) - \text{Tr}_{\text{R}}\left(P_{\text{GSO}}^{(9_a/5_a)}\, P_{\text{orb.}}\, q^{L_0}\right)\right] , \tag{4.5}$$

where $q=\exp(-2\pi\tau)$, $P_{\text{GSO}}^{(9_a,5_a)}$ is the appropriate GSO projector, and

$$P_{\text{orb.}} = \frac{1}{4}\left(1+\sum_{i=1}^{3} h_i\right) \tag{4.6}$$

is the orbifold projector, with h_i being the three non-trivial elements of the $\mathbb{Z}_2\times\mathbb{Z}_2$ orbifold action of our background. Each element h_i is in fact the generator of a $\mathbb{Z}_2$ subgroup which leaves invariant the i-th torus (see appendix A.1). The corresponding term in the amplitude is therefore identical in form to the one encountered in the computation of the $9a/5a$ amplitude in a $\mathcal{N}=2$ background $\mathcal{T}_2^{(j)}\times\mathcal{T}_2^{(k)}$ (with $j,k\neq i$). This computation is described, for instance, in section 4 of ref. [19] to which we refer for notations and details.

[4]These open string annulus amplitudes exhibit both UV and IR divergences. The UV divergences, corresponding to IR divergences in the dual closed string channel, cancel in consistent tadpole-free models; even if in this paper we take only a local point of view, we assume that globally the closed string tadpoles are absent so that we can ignore the UV divergences.

It turns out that the GSO projection in the R sector has to be defined differently for the two string orientations (see appendix A.2) so that the amplitude $\mathcal{A}(9a/5a)$ vanishes, and one remains with

$$\mathcal{A}_{5_a;9_a} = \mathcal{A}(5a/9a) = N_a k \sum_{i=1}^{3} \int_0^\infty \frac{d\tau}{2\tau} \mathcal{Y}^{(i)} . \tag{4.7}$$

In the end all string excitations cancel and only the zero-modes contribute: they correspond to the charged instanton moduli listed in table 1, and their Kaluza-Klein partners on the torus $\mathcal{T}_2^{(i)}$ fixed by the element h_i of the orbifold group; these states reconstruct the sum

$$\mathcal{Y}^{(i)} \equiv \sum_{(r_1,r_2)\in\mathbb{Z}^2} q^{\frac{|r_1 U^{(i)} - r_2|^2}{U_2^{(i)} T_2^{(i)}}} . \tag{4.8}$$

The integration over the modular parameter can be done [37, 19] with the assumption that the UV divergence for $\tau \to 0$, which corresponds to an IR divergences in the closed string channel, cancel in a globally consistent, i.e. tadpole-free, model (of which here we are considering just the "local" aspects on some given stacks of branes far from the orientifold planes). The IR divergence for $\tau \to \infty$ requires the introduction of cut-offs $m_{(i)}$ which are conveniently taken to be complex, as advocated in [38, 39, 19]. The resulting amplitude is then

$$\mathcal{A}_{5_a;9_a} = -\frac{N_a k}{2} \sum_{i=1}^{3} \Big(\log(\alpha' m_{(i)}^2) + \log\big(U_2^{(i)} T_2^{(i)} |\eta(U^{(i)}|^4)\big) \Big) . \tag{4.9}$$

Choosing [19]

$$m_{(i)} \;=\; \mu\, \mathrm{e}^{\mathrm{i}\varphi_{(i)}} \;=\; \mu\, \mathrm{e}^{2\mathrm{i}\arg(\eta(U^{(i)}))} , \tag{4.10}$$

the final result is

$$\mathcal{A}_{5_a;9_a} = -8\pi^2 k \left[\frac{3N_a}{16\pi^2} \log(\alpha'\mu^2) + \frac{N_a}{16\pi^2} \sum_{i=1}^{3} \log\Big(U_2^{(i)} T_2^{(i)} (\eta(U^{(i)})^4\Big) \right] , \tag{4.11}$$

which is of the expected form (4.4a).

Flavored sectors. The amplitude $\mathcal{A}_{5_a;9_b}$ receives contributions from the two possible orientations of the open strings, as described in eq. (4.2). These contributions, and therefore also the total amplitude $\mathcal{A}_{5_a;9_b}$, are defined in perfect analogy with eq. (4.5), in particular they contain the orbifold projector $P_{\rm orb}$ given in (4.6). Taking into account that the D5$_a$ branes transform according to the trivial representation of the orbifold group, while the D9$_b$ and D9$_c$ transform according to the representation R_1 defined in table A.6, we can make explicit the orbifold action on the Chan-Paton factors. We can then write the total amplitude as the sum of four sectors corresponding to the insertions of the various group elements as follows:[5]

$$\mathcal{A}_{5_a;9_b} \equiv \frac{1}{4} \sum_{I=0}^{3} R_1(h_I)\, \mathcal{A}_{5_a;9_b}^{h_I} = \frac{1}{4} \Big\{ \mathcal{A}_{5_a;9_b}^{e} + \mathcal{A}_{5_a;9_b}^{h_1} - \mathcal{A}_{5_a;9_b}^{h_2} - \mathcal{A}_{5_a;9_b}^{h_3} \Big\} . \tag{4.12}$$

[5]In this notation the identity element e corresponds to h_0.

The annulus amplitudes $\mathcal{A}^{h_I}_{5_a;9_b}$ take into account the action of the orbifold elements h_I on the string fields Z^i and Ψ^i, in the various sectors, as described in appendix A.1. Such amplitudes are computed in detail in appendix A.3; here we simply write the final results. In the untwisted sector we find

$$\mathcal{A}^{e}_{5_a;9_b} = \frac{\mathrm{i}\,k\,N_F}{2\pi}\int_0^\infty \frac{d\tau}{2\tau}\left[\sum_{i=1}^{3} R_1(h_i)\,\partial_z \log\theta_1(z|\mathrm{i}\tau)\big|_{z=\mathrm{i}\tau\nu^{(i)}}\right], \tag{4.13}$$

while in the three twisted sectors we have

$$\begin{aligned}\mathcal{A}^{h_i}_{5_a;9_b} =& \frac{\mathrm{i}\,k\,N_F}{2\pi}\int_0^\infty \frac{d\tau}{2\tau} \\ &\times\left[\partial_z \log\theta_1(z|\mathrm{i}\tau)\big|_{z=\mathrm{i}\tau\nu^{(i)}} + R_1(h_i)\sum_{j\neq i=1}^{3} R_1(h_j)\,\partial_z\log\theta_2(z|\mathrm{i}\tau)\big|_{z=\mathrm{i}\tau\nu^{(j)}}\right].\end{aligned} \tag{4.14}$$

It is worth pointing out that the contribution of the odd spin-structure, which is divergent due to the superghost zero-modes, actually cancels out when in each sector we sum over the two open string orientations, leading to finite and well-defined expressions.

Inserting the amplitudes (4.13) and (4.14) in (4.12), we find

$$\mathcal{A}_{5_a;9_b} = \frac{\mathrm{i}\,k\,N_F}{4\pi}\int_0^\infty \frac{d\tau}{2\tau}\left\{\sum_{i=1}^{3} R_1(h_i)\,\partial_z \log\left[\theta_1(z|\mathrm{i}\tau)\,\theta_2(z|\mathrm{i}\tau)\right]\big|_{z=\mathrm{i}\tau\nu^{(i)}}\right\}. \tag{4.15}$$

Then, if we use the identity

$$\theta_1(z|i\tau)\,\theta_2(z|i\tau) = \theta_1(2z|2i\tau)\prod_{n=1}^{\infty}\left(\frac{1-q^{2n}}{1+q^{2n}}\right), \tag{4.16}$$

where $q = \exp(-2\pi\tau)$, it is easy to see that the total flavored amplitude (4.15) becomes

$$\mathcal{A}_{5_a;9_b} = \frac{\mathrm{i}\,k\,N_F}{2\pi}\int_0^\infty \frac{d\tau}{2\tau}\left[\sum_{i=1}^{3} R_1(h_i)\,\partial_z \log\theta_1(z|\mathrm{i}\tau)\big|_{z=\mathrm{i}\tau\nu^{(i)}}\right]. \tag{4.17}$$

Notice that this is identical to the contribution (4.13) of the untwisted sector. This means that the flavored amplitude of the orbifold theory is the same as the one without the orbifold, so that the $\mathcal{N}=1$ structure realized with the magnetic fluxes is fully preserved by the orbifold projection. The mixed amplitude (4.17) agrees with the quadratic term in the gauge field f of the annulus amplitude $\mathcal{A}_{9a;9b}(f)$ computed in ref. [37] to evaluate the gauge threshold corrections in intersecting brane models (see also refs. [6, 7, 40]).

We now need to evaluate the integral over τ that appears in eq. (4.17). It is not difficult to realize that this integral is divergent both in the ultraviolet ($\tau \to 0$) and in the infrared ($\tau \to \infty$). The ultraviolet divergence can be eliminated by considering tadpole free models as mentioned above, while the infrared divergence can be cured by introducing, for example, a regulator $R(\tau) = \left(1 - \mathrm{e}^{-1/(\alpha' m^2\tau)}\right)$ with the cut-off $m \to 0$. The original evaluation [37] of the τ integral appearing in (4.17) has been recently revisited in ref. [40]. Using this revised result in our case,[6] we obtain

$$\mathcal{A}_{5_a;9_b} = 8\pi^2 k \left(\frac{N_F}{32\pi^2} \log(\alpha' m^2) + \frac{N_F}{32\pi^2} \log \mathbf{\Gamma}_{ba} \right) , \tag{4.18}$$

where

$$\mathbf{\Gamma}_{ba} = \frac{\Gamma(1-\nu_{ba}^{(1)})}{\Gamma(\nu_{ba}^{(1)})} \frac{\Gamma(\nu_{ba}^{(2)})}{\Gamma(1-\nu_{ba}^{(2)})} \frac{\Gamma(\nu_{ba}^{(3)})}{\Gamma(1-\nu_{ba}^{(3)})} . \tag{4.19}$$

Considering also the contribution of the flavor branes of type c that are characterized by twist angles $\nu_{ac}^{(i)}$, and writing $m = \mu\, \mathrm{e}^{\mathrm{i}\varphi}$, for our realization of $\mathcal{N} = 1$ SQCD the total flavored annulus amplitude is

$$\mathcal{A}_{5_a;9_b} + \mathcal{A}_{5_a;9_c} = 8\pi^2 k \left(\frac{N_F}{16\pi^2} \big[\log(\alpha' \mu^2) + 2\mathrm{i}\varphi \big] + \frac{N_F}{32\pi^2} \log \left(\mathbf{\Gamma}_{ba}\, \mathbf{\Gamma}_{ac} \right) \right) , \tag{4.20}$$

which is indeed of the expected form (4.4b).

5. Relation to the matter Kähler metric

In this section we elaborate on the previous results. In particular we will rewrite the annulus amplitudes (4.11) and (4.20) in terms of the variables (2.3) of the supergravity basis in order to obtain information on the Kähler metrics for the fundamental $\mathcal{N} = 1$ chiral multiplets, and then check that the instanton induced superpotential W_k acquires the correct holomorphy properties required by $\mathcal{N} = 1$ supersymmetry.

5.1 Holomorphic coupling redefinition

As remarked already in refs. [26, 27], the UV cutoff that has to be used in the field theory analysis of a string model is the four-dimensional Planck mass M_P, which is related to α' as follows:

$$M_P^2 = \frac{1}{\alpha'}\, \mathrm{e}^{-\phi_{10}}\, s_2 , \tag{5.1}$$

where ϕ_{10} is the ten-dimensional dilaton. In terms of this cut-off, eqs. (4.11) and (4.20) become, respectively,

$$\mathcal{A}_{5_a;9_a} = -8\pi^2 k \left(\frac{3N_a}{16\pi^2} \log \frac{\mu^2}{M_P^2} + \widetilde{\Delta}_{\text{color}} \right) \tag{5.2}$$

with

$$\widetilde{\Delta}_{\text{color}} = \frac{N_a}{16\pi^2} \left(3 \log(\mathrm{e}^{-\phi_{10}}\, s_2) + \sum_{i=1}^{3} \log \left(U_2^{(i)} T_2^{(i)} (\eta(U^{(i)})^4) \right) \right) , \tag{5.3}$$

and

$$\mathcal{A}_{5_a;9_b} + \mathcal{A}_{5_a;9_c} = -8\pi^2 k \left(-\frac{N_F}{16\pi^2} \log \frac{\mu^2}{M_P^2} + \widetilde{\Delta}_{\text{flavor}} \right) \tag{5.4}$$

[6] See in particular eq. (3.16) of ref. [40] with all numerical additive constants absorbed in a redefinition of the cut-off m.

with

$$\widetilde{\Delta}_{\text{flavor}} \;=\; -\,\frac{N_F}{16\pi^2}\left(\log(\mathrm{e}^{-\phi_{10}}\,s_2) \,+\, 2\mathrm{i}\varphi \,+\, \frac{1}{2}\log\left(\boldsymbol{\Gamma}_{ba}\,\boldsymbol{\Gamma}_{ac}\right)\right)\,. \tag{5.5}$$

Since in $\widetilde{\Delta}_{\text{flavor}}$ there are no analytic terms, we can consistently set $\varphi = 0$ in the following.

We now rewrite the above expressions in terms of the geometrical variables of the supergravity basis. For the charged amplitude $\mathcal{A}_{5_a;9_a}$ the procedure is very similar to the one we have applied in the $\mathcal{N}=2$ case [19]. In fact, using the tree-level relation between the string and the supergravity moduli given in (2.3) and the bulk Kähler potential (2.4), eq. (5.2) can be recast in the following form:

$$\mathcal{A}_{5_a;9_a} = k\left[-\frac{3N_a}{2}\log\frac{\mu^2}{M_P^2} - N_a\sum_{i=1}^{3}\log\left(\eta(u^{(i)})^2\right) + \frac{N_a}{2}K + N_a\log g_a^2\right]\,. \tag{5.6}$$

Turning to the flavored amplitude (5.4), we easily see that it can be rewritten as follows:

$$\mathcal{A}_{5_a;9_b} + \mathcal{A}_{5_a;9_c} = k\left[\frac{N_F}{2}\log\frac{\mu^2}{M_P^2} - \frac{N_F}{2}K + \frac{N_F}{2}\log(\mathcal{Z}_Q\mathcal{Z}_{\widetilde{Q}})\right]\,, \tag{5.7}$$

where the quantities $\mathcal{Z}_Q$ and $\mathcal{Z}_{\widetilde{Q}}$, defined through the equation

$$\log(\mathrm{e}^{-\phi_{10}}\,s_2) \,+\, \frac{1}{2}\log\left(\boldsymbol{\Gamma}_{ba}\boldsymbol{\Gamma}_{ca}\right) \;=\; -K + \log(\mathcal{Z}_Q\mathcal{Z}_{\widetilde{Q}})\,, \tag{5.8}$$

are explicitly given by

$$\mathcal{Z}_Q = \left(4\pi\,s_2\right)^{-\frac{1}{4}}\left(t_2^{(1)}t_2^{(2)}t_2^{(3)}\right)^{-\frac{1}{4}}\left(u_2^{(1)}u_2^{(2)}u_2^{(3)}\right)^{-\frac{1}{2}}\left(\boldsymbol{\Gamma}_{ba}\right)^{\frac{1}{2}}\,, \tag{5.9}$$

and

$$\mathcal{Z}_{\widetilde{Q}} = \left(4\pi\,s_2\right)^{-\frac{1}{4}}\left(t_2^{(1)}t_2^{(2)}t_2^{(3)}\right)^{-\frac{1}{4}}\left(u_2^{(1)}u_2^{(2)}u_2^{(3)}\right)^{-\frac{1}{2}}\left(\boldsymbol{\Gamma}_{ac}\right)^{\frac{1}{2}}\,. \tag{5.10}$$

Eqs. (5.6) and (5.7) can be combined in the general formula at 1-loop

$$\mathcal{A} = k\left[-\frac{b}{2}\log\frac{\mu^2}{M_P^2} \,+\, f^{(1)} \,+\, \frac{c}{2}K \,-\, T(G_a)\log\left(\frac{1}{g_a^2}\right) + \sum_r n_r\,T(r)\,\log\mathcal{Z}_r\right] \tag{5.11}$$

where $f^{(1)}$ is a holomorphic function and

$$\begin{aligned} T(r)\,\delta_{AB} &= \mathrm{Tr}_{\,r}\big(T_AT_B\big)\,, & T(G_a) &= T(\mathrm{adj})\,,\\ b &= 3\,T(G_a) - \sum_r n_r\,T(r)\,, & c &= T(G_a) - \sum_r n_r\,T(r) \end{aligned} \tag{5.12}$$

with T_A being the generators of the gauge group G_a and n_r the number of $\mathcal{N}=1$ chiral multiplets transforming in the representation r. Indeed, eq. (5.6) is obtained from eq. (5.11) when we consider the $\mathcal{N}=1$ vector multiplet (i.e. $b = 3N_a$ and $c = N_a$), and take

$$f^{(1)} = -\,N_a\sum_{i=1}^{3}\log\left(\eta(u^{(i)})^2\right)\,. \tag{5.13}$$

On the contrary eq. (5.7) is obtained from eq. (5.11) by considering the fundamental matter fields of $\mathcal{N}=1$ SQCD with N_F flavors (i.e. $b=c=-N_F$), taking $f^{(1)}=0$ and identifying $\mathcal{Z}_r$ with $\mathcal{Z}_Q$ and $\mathcal{Z}_{\widetilde{Q}}$ of eqs. (5.9) and (5.10).

In view of the relation (4.3), we see that by adding the disk contribution to the above annulus amplitude one obtains the running coupling constant of the effective theory in the 1-loop approximation, namely

$$\mathcal{A}_{1-\mathrm{loop}} = -\frac{8\pi^2 k}{g_a^2} + \mathcal{A} \tag{5.14}$$

with $\mathcal{A}$ given by (5.11), i.e. by the sum of eqs. (5.6) and (5.7). On the other hand, according to refs. [25 − 27, 30] this has to be expressed in terms of the Wilsonian gauge coupling $\tilde{g}_a$, the (tree-level) bulk Kähler potential K and the (tree-level) Kähler metrics K_r of the chiral multiplets in the representation r of the gauge group G_a as follows

$$\mathcal{A}_{1-\mathrm{loop}} = -\frac{8\pi^2 k}{\tilde{g}_a^2} + k\left[-\frac{b}{2}\log\frac{\mu^2}{M_P^2} + f^{(1)} + \frac{c}{2}K - T(G_a)\log\left(\frac{1}{\tilde{g}_a^2}\right) + \sum_r n_r\, T(r)\log K_r\right] \tag{5.15}$$

with $f^{(1)}$ being a holomorphic function, and b and c defined as in (5.12).

Notice that the gauge coupling constant g_a obtained from the disk amplitude may not coincide with the Wilsonian coupling $\tilde{g}_a$ appearing in (5.15): in general there may be loop effects, related to sigma-model anomalies in the low energy supergravity theory [41], so that

$$\frac{1}{g_a^2} = \frac{1}{\tilde{g}_a^2} + \frac{\delta}{8\pi^2}\ . \tag{5.16}$$

Since $\tilde{g}_a$ is the Wilsonian coupling, it has to be (the imaginary part of) a chiral field: at tree-level this is indeed what happens (see eq. (2.6)), but such a relation may be spoiled by loop corrections leading to $\delta \neq 0$. Furthermore, $\tilde{g}_a$ runs only at 1-loop and its β-function is given by

$$\beta_{\mathrm{W}}(\tilde{g}_a) = -\frac{b_1\,\tilde{g}_a^3}{16\pi^2}\ . \tag{5.17}$$

Comparing eq. (5.15) with the string expression (5.14) for $\mathcal{A}_{1-\mathrm{loop}}$, we see:

- that in the right hand side of eq. (5.11) the $\log(1/g_a^2)$ term can be replaced by $\log(1/\tilde{g}_a^2)$ since the difference yields a higher order correction;
- that $f^{(1)}$ is given by (5.13) and
- that if δ contains a term $\delta^{(0)}$ of order $\tilde{g}_a^0$, the tree-level Kähler metrics of the chiral fields are given by

$$K_r = \mathcal{Z}_r \mathcal{X}_r \tag{5.18}$$

  where the non-holomorphic factors $\mathcal{X}_r$ are such that

$$\delta^{(0)} + \sum_r n_r\, T(r)\log \mathcal{X}_r = 0\ . \tag{5.19}$$

Note that if δ is of higher order in $\tilde{g}_a$, i.e. if $\delta^{(0)} = 0$, then $\mathcal{X}_r = 1$ and the tree-level Kähler metric of the chiral multiplets reduces to $\mathcal{Z}_r$ (see eqs. (5.9) and (5.10)), that is to what can be directly read from the string annulus amplitude (5.11).

In the following subsection we will check the consistency of this result by showing that the instanton induced superpotential has the correct holomorphy properties required by $\mathcal{N} = 1$ supersymmetry when everything is expressed in the appropriate variables of the low energy effective action. Moreover in section 6 we will compare our findings against the holomorphy properties of the Yukawa superpotential computed in ref. [31] for systems of magnetized D9 branes in the field theory limit.

5.2 Field redefinitions and the instanton induced superpotential

The threshold corrections $\widetilde{\Delta}_{\text{color}}$ and $\widetilde{\Delta}_{\text{flavor}}$, and especially their non-holomorphic parts, play an important rôle since they are related to the "primed" part of the annulus amplitude that appears in the prefactor of the instantonic correlators; in fact

$$\mathcal{A}'_{5_a} = -8\pi^2 k \left(\widetilde{\Delta}_{\text{color}} + \widetilde{\Delta}_{\text{flavor}}\right) . \tag{5.20}$$

In ref. [20] it has been suggested that some of the terms of $\mathcal{A}'_{5_a}$ are related to the rescalings of the fields appearing in the instanton induced correlator that are necessary in order to have a pure holomorphic expression. In ref. [19] we have showed in detail that for $\mathcal{N} = 2$ models, where the instantons determine corrections to the gauge prepotential, this is indeed what happens. Here we show that the same is true also for the instanton-induced superpotential for $\mathcal{N} = 1$ theories, thus clarifying the general procedure.

We concentrate in the one-instanton case ($k = 1$), where one finds, for $N_F = N_a - 1$, the instanton-induced ADS/TVY-like superpotential of eq. (3.7). For $k = 1$, the "primed" amplitude (5.20) explicitly reads

$$\mathcal{A}'_{5_a} = -N_a \sum_{i=1}^{3} \log\left(\eta(u^{(i)})^2\right) + N_a \log g_a^2 + \frac{N_a - N_F}{2} K + \frac{N_F}{2} \log(\mathcal{Z}_Q \mathcal{Z}_{\widetilde{Q}}) . \tag{5.21}$$

Using this expression in eq. (3.7) and introducing the Kähler metrics $K_Q = \mathcal{Z}_Q \mathcal{X}_Q$ and $K_{\widetilde{Q}} = \mathcal{Z}_{\widetilde{Q}} \mathcal{X}_{\widetilde{Q}}$, with $\mathcal{Z}_Q$ and $\mathcal{Z}_{\widetilde{Q}}$ given in (5.9) and (5.10), and $\mathcal{X}_Q$ and $\mathcal{X}_{\widetilde{Q}}$ such that

$$\delta^{(0)} + \frac{N_F}{2} \log(\mathcal{X}_Q \mathcal{X}_{\widetilde{Q}}) = 0 , \tag{5.22}$$

we obtain

$$W_{k=1} = \mathrm{e}^{K/2} \prod_{i=1}^{3} \left(\eta(u^{(i)})^{-2N_a}\right) \left((\sqrt{\alpha'})^{-(2N_a+1)}\, \mathrm{e}^{-\frac{8\pi^2}{\tilde{g}_a^2}}\right) \left(K_Q K_{\widetilde{Q}}\right)^{\frac{N_a-1}{2}} \frac{1}{\det(\tilde{q}q)} . \tag{5.23}$$

where we have used eq. (5.16) and the fact that $N_F = N_a - 1$. If we now introduce the chiral multiplets Q and $\widetilde{Q}$ in the supergravity basis through the rescalings (2.18), and the holomorphic renormalization group invariant scale through the β-function (5.17), namely

$$\Lambda^b_{\text{hol}} = (\sqrt{\alpha'})^{-b}\, \mathrm{e}^{-\frac{8\pi^2}{\tilde{g}_a^2}} , \tag{5.24}$$

we find

$$W_{k=1}=\mathrm{e}^{K/2}\prod_{i=1}^{3}\left(\eta(u^{(i)})^{-2N_a}\right)\,\Lambda_{\rm hol}^{2N_a+1}\,\frac{1}{\det(\widetilde{Q}\,Q)}\equiv\mathrm{e}^{K/2}\,\widehat{\Lambda}_{\rm hol}^{2N_a+1}\,\frac{1}{\det(\widetilde{Q}Q)}\;. \tag{5.25}$$

In the last step we have absorbed the moduli dependent factors of $\eta(u^{(i)})$ with a holomorphic redefinition of the Wilsonian scale $\Lambda_{\rm hol}$ into $\widehat{\Lambda}_{\rm hol}$.

The final form of eq. (5.25) is the correct one for a holomorphic ADS/TVY superpotential term in a non-trivial background. The factor of $\mathrm{e}^{K/2}$ is the contribution of the bulk Kähler potential, while the remaining part

$$\widehat{W}_{k=1}=\widehat{\Lambda}_{\rm hol}^{2N_a+1}\,\frac{1}{\det(\widetilde{Q}Q)} \tag{5.26}$$

is a holomorphic expression in the appropriate variables of the Wilsonian scheme. Thus, the various pieces of the “primed” instantonic annulus amplitude $\mathcal{A}'_{5_a}$ have conspired to reproduce the required factors to obtain a fully holomorphic ADS/TVY superpotential $\widehat{W}_{k=1}$.

6. Comparison with the Yukawa couplings

It is well known that the Kähler metrics of the chiral multiplets play a key rôle in relating the holomorphic superpotential couplings in the effective supergravity Lagrangian to the physical Yukawa couplings of the canonically normalized matter fields. This relation represents therefore a possible test on the structure of the Kähler metrics K_Q and $K_{\tilde{Q}}$.

Let us recall some basic points, and set up appropriate notations. When various stacks of branes, of types $a, b, c, \ldots$, are present, there are chiral multiplets arising from the massless open strings stretching between them. We will denote as q^{ba} the chiral multiplet (as well as the scalar therein) coming from the $\mathrm{D9}_b/\mathrm{D9}_a$ strings, which we formerly indicated as q, and as q^{ac} the chiral multiplet corresponding to $\mathrm{D9}_a/\mathrm{D9}_c$ strings, which was previously indicated as $\tilde{q}$. We will then similarly have the multiplets q^{cb}, ... The corresponding multiplets in the “supergravity” basis will be denoted as Q^{ba}, Q^{ac}, Q^{cb}, ..., and their Kähler metrics will be K_{ba} (formerly K_Q), K_{ac} (formerly $K_{\tilde{Q}}$) and so on. These metrics will contain the appropriate factors $\boldsymbol{\Gamma}_{ba}$, $\boldsymbol{\Gamma}_{ac}$, $\boldsymbol{\Gamma}_{cb}$, ... , which in turn are given by the analogue of eq. (4.19) in terms of the twist angles $\nu_{ba}^{(i)}$, $\nu_{ac}^{(i)}$, $\nu_{cb}^{(i)}$, ...

In this situation, there are non-trivial interactions supported on disks whose boundary is partly attached to three different branes, say of types a, c and b, provided the twist angles $\nu_{ba}^{(i)}$, $\nu_{ac}^{(i)}$, $\nu_{cb}^{(i)}$ for each i are either the internal or the external angles of a triangle. These interactions in the field-theory limit correspond to Yukawa couplings between the fields of the chiral multiplets q^{ac}, q^{cb} and q^{ba}, like for instance the one associated to figure 4:

$$\int d^4x\; Y_{acb}\,\mathrm{Tr}\left(\chi^{ac}q^{cb}\chi^{ba}\right), \tag{6.1}$$

plus its supersymmetric completion terms. Altogether such interactions can be encoded in the cubic superpotential

$$W_{\rm Y}=Y_{acb}\,\mathrm{Tr}\left(q^{ac}q^{cb}q^{ba}\right)\;. \tag{6.2}$$

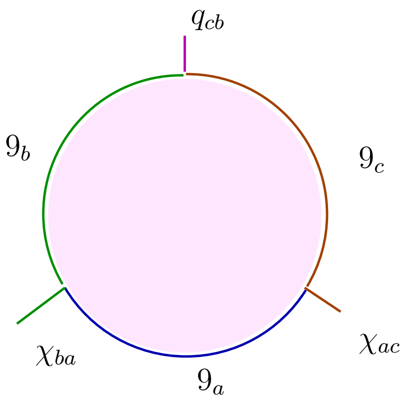

Figure 4: A disk diagram leading to a Yukawa coupling.

If we rewrite the above superpotential in terms of the multiplets in the supergravity basis via the rescalings (2.18), we obtain

$$W_{\rm Y} = Y_{acb} \left(K_{ac}\, K_{bc}\, K_{ba}\right)^{\frac{1}{2}}\ \mathrm{Tr}\left(Q^{ac}Q^{cb}Q^{ba}\right)\ . \tag{6.3}$$

On the other hand, in the effective supergravity action this superpotential must take the form

$$W_{\rm Y} = \mathrm{e}^{K/2}\, \widehat{W}_{acb}\, \mathrm{Tr}\left(Q^{ac}Q^{cb}Q^{ba}\right), \tag{6.4}$$

where $\mathrm{e}^{K/2}$ is the standard contribution of the bulk Kähler potential and $\widehat{W}_{acb}$ are purely holomorphic functions of the geometric moduli. Comparing these last two equations, we deduce that

$$Y_{acb} = \mathrm{e}^{K/2}\left(K_{ac}\, K_{cb}\, K_{ba}\right)^{-\frac{1}{2}} \widehat{W}_{acb}\ . \tag{6.5}$$

If we now use the bulk Kähler potential (2.4) and the Kähler metric $K_{ba} = \mathcal{Z}_{ba}\mathcal{X}_{ba}$, with $\mathcal{Z}_{ba}$ given in (5.9) and similarly for K_{ac} and K_{cb}, we easily obtain

$$\begin{aligned} Y_{acb} &= (4\pi)^{\frac{3}{8}}\, s_2^{-\frac{1}{8}}\left(t_2^{(1)}t_2^{(2)}t_2^{(3)}\right)^{-\frac{1}{8}}\left(u_2^{(1)}u_2^{(2)}u_2^{(3)}\right)^{\frac{1}{4}}\left(\mathbf{\Gamma}_{ac}\mathbf{\Gamma}_{cb}\mathbf{\Gamma}_{ba}\right)^{-\frac{1}{4}}\left(\mathcal{X}_{ac}\mathcal{X}_{cb}\mathcal{X}_{ba}\right)^{-\frac{1}{2}}\widehat{W}_{acb} \\ &= \sqrt{4\pi}\ \mathrm{e}^{\frac{\phi_4}{2}}\left(u_2^{(1)}u_2^{(2)}u_2^{(3)}\right)^{\frac{1}{4}}\left(\mathbf{\Gamma}_{ac}\mathbf{\Gamma}_{cb}\mathbf{\Gamma}_{ba}\right)^{-\frac{1}{4}}\left(\mathcal{X}_{ac}\mathcal{X}_{cb}\mathcal{X}_{ba}\right)^{-\frac{1}{2}}\widehat{W}_{acb}\,, \end{aligned} \tag{6.6}$$

where $\phi_4 = \phi_{10} - \frac{1}{2}\sum_i \log(T_2^{(i)})$ is the four-dimensional dilaton.

We now compare this finding with the results of ref. [31] for the physical Yukawa couplings of toroidal models with magnetized D9 branes.[7] The expression for Y_{acb} is given in their eq. (7.13). Setting to zero the value of the Wilson lines, and rewriting it in terms of the supergravity moduli through eqs (5.47)–(5.49) of the same reference, in our notation it reads

$$Y_{acb} = \mathrm{e}^{\frac{\phi_4}{2}}\left(u_2^{(1)}u_2^{(2)}u_2^{(3)}\right)^{\frac{1}{4}}\prod_{i=1}^{3}\left|\frac{\vartheta_1^{(i)}\vartheta_2^{(i)}}{\vartheta_1^{(i)}+\vartheta_2^{(i)}}\right|^{\frac{1}{4}} W'_{acb}\ . \tag{6.7}$$

[7] In the T-dual intersecting brane version, these couplings have been studied in refs. [42–44, 32].

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The detailed expression of the quantities $\vartheta_1^{(i)}$, $\vartheta_2^{(i)}$ and W'_{acb} is not relevant here; the only important points that we want to emphasize are that W'_{acb} is a holomorphic function of the complex structure moduli $u^{(i)}$ and that the expression (6.7) has been obtained starting from the non-abelian Yang-Mills theory on the D9 branes, rather than from the full fledged DBI action. Therefore one expects that it only represents the field theory limit of the string result.[8] However, as already argued in ref. [31], one can extend eq. (6.7) by observing that in the field theory limit $\alpha' \to 0$ (i.e. in the small twist limit) one has

$$\left(\mathbf{\Gamma}_{ac}\mathbf{\Gamma}_{cb}\mathbf{\Gamma}_{ba}\right)^{-\frac{1}{4}} \sim \prod_{i=1}^{3}\left|\frac{\vartheta_1^{(i)}\vartheta_2^{(i)}}{\vartheta_1^{(i)}+\vartheta_2^{(i)}}\right|^{\frac{1}{4}} . \tag{6.8}$$

With this understanding, eq. (6.7) can then be generalized as

$$Y_{acb} = \mathrm{e}^{\frac{\phi_4}{2}}\left(u_2^{(1)}u_2^{(2)}u_2^{(3)}\right)^{\frac{1}{4}}\left(\mathbf{\Gamma}_{ac}\mathbf{\Gamma}_{cb}\mathbf{\Gamma}_{ba}\right)^{-\frac{1}{4}} W'_{acb}\,, \tag{6.9}$$

which agrees with eq. (6.6) by taking $W'_{acb} = \sqrt{4\pi}\,\widehat{W}_{acb}$, provided the non-holomorphic factors obey

$$\mathcal{X}_{ac}\mathcal{X}_{cb}\mathcal{X}_{ba} = 1 . \tag{6.10}$$

Indeed, stripping off the various factors of the Kähler potential and of the Kähler metrics from the physical Yukawa couplings Y_{acb} according to eq. (6.6), we can obtain the expected holomorphic structure of the superpotential only if (6.10) is satisfied.

The simplest solution to this constraint is clearly $\mathcal{X}_{ba} = \mathcal{X}_{cb} = \mathcal{X}_{ac} = 1$. This would imply that the Kähler metrics for the twisted chiral matter fields are given by the expressions (5.9) and (5.10). Notice that in this case the only dependence on the twist parameters would be through the Γ-functions contained in the factors $\left(\mathbf{\Gamma}_{ba}\right)^{\frac{1}{2}}$ and $\left(\mathbf{\Gamma}_{ac}\right)^{\frac{1}{2}}$ (see eq. (4.19)). Such factors are the same as the ones that can be obtained directly from a 3-point string scattering amplitude involving one (closed string) geometric modulus and two twisted scalar fields, as explained in refs. [32, 46]. On the other hand, the possibility of non-trivial $\mathcal{X}$ factors has been considered in refs. [20, 47]. Besides satisfying the constraint (6.10), such non-holomorphic factors should also be related to a non-vanishing sigma-model anomaly term $\delta^{(0)}$, as explained in section 5.1. It would be very interesting to do an independent calculation to check this point.

We close with a few concluding remarks. Loop corrections to the bulk Kähler potential or to the Einstein term in the bulk action, which for Type II theories have been computed in refs. [33, 48], in general induce shifts of the supergravity variables and in particular are responsible for a non-vanishing δ-term in eq. (5.16). However, such a δ-term appears to be of order $g_s \sim \tilde{g}_a^2$ and thus it does not affect the form of the Kähler metric for the $\mathcal{N}=1$ twisted matter at tree-level. Furthermore, using the known expressions for the Kähler potential and Kähler metrics, in ref. [19] we have explicitly checked that in $\mathcal{N}=2$ SQCD no shift $\delta^{(0)}$ is produced. It would be very interesting to explore further this issue and see whether and how an anomalous term $\delta^{(0)}$ in $\mathcal{N}=1$ theories with magnetized branes is possible.

[8]See ref. [45] for a string theory calculation of the Yukawa couplings and a direct derivation of the $\mathbf{\Gamma}$ factors.

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A. Technicalities

In this appendix we provide some technical details. In particular in appendix A.1 we describe the background geometry and the $\mathbb{Z}_2 \times \mathrm{Z}_2$ orbifold action. In appendix A.2 we write the vertex operators for the physical states in the various open string sectors and their GSO properties, and finally in appendix A.3 we present the calculation of the mixed annulus amplitudes for the flavored sectors.

A.1 The background geometry

The metric and the B-field on the torus $\mathcal{T}_2^{(i)}$ are expressed in terms of the complex moduli $T^{(i)}$ and of the Kähler moduli $T^{(i)}$ as follows:

$$G^{(i)} = \frac{T_2^{(i)}}{U_2^{(i)}} \begin{pmatrix} 1 & U_1^{(i)} \\ U_1^{(i)} & |U^{(i)}|^2 \end{pmatrix} \quad \text{and} \quad B^{(i)} = \begin{pmatrix} 0 & -T_1^{(i)} \\ T_1^{(i)} & 0 \end{pmatrix} . \tag{A.1}$$

With respect to the above metric, the orthonormal complex coordinates and the corresponding string fields on the torus $\mathcal{T}_2^{(i)}$ are given by:

$$Z^i = \sqrt{\frac{T_2^{(i)}}{2U_2^{(i)}}} \left(X^{2i+2} + U^{(i)} X^{2i+3} \right) , \quad \Psi^i = \sqrt{\frac{T_2^{(i)}}{2U_2^{(i)}}} \left(\psi^{2i+2} + U^{(i)} \psi^{2i+3} \right) \tag{A.2}$$

for $i = 1, 2, 3$.

The (anti-chiral) spin-fields $S^{\dot{\mathcal{A}}}$ of the RNS formalism in ten dimensions factorize in a product of four-dimensional and internal spin-fields, according to

$$S^{\dot{\mathcal{A}}} \to (S_\alpha S_{---}, S_\alpha S_{-++}, S_\alpha S_{+-+}, S_\alpha S_{++-}, S^{\dot\alpha} S^{+++}, S^{\dot\alpha} S^{+--}, S^{\dot\alpha} S^{-+-}, S^{\dot\alpha} S^{--+}) , \tag{A.3}$$

where the index α ($\dot\alpha$) denotes positive (negative) chirality in $\mathbb{R}^{1,3}$ and the labels $(\pm, \pm, \pm)$ on the internal spin-fields denote charges $(\pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2})$ under the three internal U(1)'s.

The orbifold group $\mathbb{Z}_2 \times \mathbb{Z}_2$ contains three non-trivial elements h_i (subject to the relation $h_1 h_2 = h_3$) acting on the internal coordinates as follows

$$\begin{aligned} h_1 :\ & (Z^1, Z^2, Z^3) \to (Z^1, -Z^2, -Z^3) , \\ h_2 :\ & (Z^1, Z^2, Z^3) \to (-Z^1, Z^2, -Z^3) , \\ h_3 :\ & (Z^1, Z^2, Z^3) \to (-Z^1, -Z^2, Z^3) , \end{aligned} \tag{A.4}$$

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and similarly for the $\Psi^{1,2,3}$ fields. We may summarize the transformation properties (A.4) for the conformal fields ∂Z^i and Ψ^i $(i = 1, 2, 3)$ in the Neveu-Schwarz sector by means of the following table:

conf. field	irrep
∂Z^i, Ψ^i	R_i

, (A.5)

where $\{R_I\} = \{R_0, R_i\}$ are the irreducible representations of $\mathbb{Z}_2 \times \mathbb{Z}_2$, identified by writing the character table of the group

	e	h_1	h_2	h_3
R_0	1	1	1	1
R_1	1	1	-1	-1
R_2	1	-1	1	-1
R_3	1	-1	-1	1

. (A.6)

The Clebsh-Gordan series for these representations is simply given by

$$R_0 \otimes R_I = R_I \quad , \qquad R_i \otimes R_j = \delta_{ij} R_0 + |\epsilon_{ijk}| R_k \ . \tag{A.7}$$

The action of the orbifold group on spin fields and spinor states is given by a spinor representation of the geometrical rotations of π in the various tori defined in eq. (A.4). In particular, we choose it to be given by

$$\begin{aligned} h_1 &= \quad \mathbb{1} \otimes \sigma_3 \otimes \sigma_3 \, , \\ h_2 &= -\sigma_3 \otimes \mathbb{1} \ \otimes \sigma_3 \, , \\ h_3 &= -\sigma_3 \otimes \sigma_3 \otimes \mathbb{1} \, , \end{aligned} \tag{A.8}$$

which corresponds to the following table:

anti-chiral	chiral	h_1	h_2	h_3	irrep
S_{-++}	S^{+--}	1	1	1	R_0
S_{---}	S^{+++}	1	-1	-1	R_1
S_{++-}	S^{--+}	-1	1	-1	R_2
S_{-+-}	S^{+-+}	-1	-1	1	R_3

. (A.9)

In particular, the only invariant spin fields are S_{-++} and S^{+--}.

A.2 The string vertices

We describe now in some more detail the string realization of the massless $\mathcal{N} = 1$ multiplets corresponding to the system of magnetized branes we consider in the paper. This system is pictorially represented in figures 1 and 2, where however only the flavored multiplets and moduli respectively are indicated.

$D9_a/D9_a$ strings. These strings are untwisted. The NS massless vertices in the (-1) superghost picture:

$$V_A(z) = (\pi\alpha')^{\frac{1}{2}}\, A_\mu\, \psi^\mu(z)\, \mathrm{e}^{-\varphi(z)}\, \mathrm{e}^{\mathrm{i}p_\mu X^\mu(z)}\,, \tag{A.10}$$

and those of the R sector, which we write in the $(-1/2)$ picture:

$$V_\Lambda(z) = (2\pi\alpha')^{\frac{3}{4}}\, \Lambda^\alpha\, S_\alpha(z) S_{-++}(z)\, \mathrm{e}^{-\frac{1}{2}\varphi(z)}\, \mathrm{e}^{\mathrm{i}p_\mu X^\mu(z)} \tag{A.11}$$

contain the d.o.f. of the $\mathcal{N}=1$ gauge multiplet. In these vertices, the polarizations have canonical dimensions (this explains the dimensional prefactors[9]) and are $N_a \times N_a$ matrices transforming in the adjoint representation of $\mathrm{SU}(N_a)$; here we neglect the $\mathrm{U}(1)$ factor associated to the center of mass of the N_a D9 branes. With respect to the orbifold group, the $\mathrm{D9}_a/\mathrm{D9}_a$ strings carry Chan-Paton factors in the representation $R_0 \times R_0 = R_0$. Also the operator part of the vertices (A.10) and (A.11) must therefore be invariant under the orbifold. This is clearly the case for V_A, and the gaugino vertex (A.11) contains the spin field S_{-++} which, according to (A.9) is indeed invariant.

$D9_b/D9_a$ and $D9_a/D9_b$ strings. Next we consider the strings stretching between two different stacks of branes. For definiteness we focus on those connected on one end to the color $\mathrm{D9}_a$ branes, and the other end to the flavor $\mathrm{D9}_b$ branes. These strings have Chan-Paton factors which, with respect to the orbifold group, belong to the representation

$$R_1 \otimes R_0 = R_1\ . \tag{A.12}$$

To write the vertex operators it is convenient to introduce the following notation

$$\sigma(z) \equiv \prod_{i=1}^{3} \sigma_{\nu_{ba}^{(i)}}(z) \quad , \quad s(z) \equiv \prod_{i=1}^{3} S_{\nu_{ba}^{(i)}}(z)\,, \tag{A.13}$$

where $\sigma_{\nu_{ba}^{(i)}}$ and $S_{\nu_{ba}^{(i)}}$ are respectively the bosonic and fermionic twist fields in the i-th torus whose conformal dimensions are

$$h_\sigma^{(i)} = \frac{1}{2}\nu_{ba}^{(i)}\big(1-\nu_{ba}^{(i)}\big) \quad \text{and} \quad h_S^{(i)} = \frac{1}{2}\big(\nu_{ba}^{(i)}\big)^2\ . \tag{A.14}$$

Then, the physical massless state in the NS sector of the $\mathrm{D9}_b/\mathrm{D9}_a$ is a complex scalar described by the following vertex operator:

$$V_q(z) = (2\pi\alpha')^{\frac{1}{2}}\, q\ \sigma(z) :\bar{\Psi}^1(z)s(z):\, \mathrm{e}^{-\varphi(z)}\, \mathrm{e}^{\mathrm{i}p_\mu X^\mu(z)}\,, \tag{A.15}$$

which can be easily checked[10] to have conformal dimension 1 for $p^2=0$ when the twists satisfy the supersymmetry condition $\nu_{ba}^{(1)} = \nu_{ba}^{(2)} + \nu_{ba}^{(3)}$. The operator $s(z)$ is invariant under the orbifold, so that $:\bar{\Psi}^1(z)s(z):$ belongs to the representation R_1 and compensates the non-trivial transformation of the Chan-Paton factors (see eq. (A.12)).

[9] See for example ref. [24] for details on the normalizations of vertex operators and scattering amplitudes.

[10] Remember that the conformal dimension of $\mathrm{e}^{-\varphi(z)}$ and $\mathrm{e}^{-\frac{1}{2}\varphi(z)}$ is respectively equal to $\frac{1}{2}$ and $\frac{3}{8}$.

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In the R sector the massless states are described by the following vertex operator:

$$V_\chi(z) = (2\pi\alpha')^{\frac{3}{4}}\,\chi^\alpha\,S_\alpha(z)\,\sigma(z)\Sigma(z)\,\mathrm{e}^{-\frac{1}{2}\varphi(z)}\,\mathrm{e}^{\mathrm{i}p_\mu X^\mu(z)}\,, \tag{A.16}$$

where

$$\Sigma(z) = \prod_{i=1}^{3} S_{(\nu_{ba}^{(i)}-\frac{1}{2})}(z)\ . \tag{A.17}$$

Again, one can easily check that this vertex operator has conformal dimension 1 for $p^2 = 0$ when the twists satisfy the supersymmetry condition. With respect to the orbifold group, the operator $\Sigma(z)$ transforms in the representation R_1, just as the spin field S_{---} to which it reduces in the limit of vanishing twists (see eq. (A.9)). The chirality of the spinor χ is fixed by the GSO projection, with

$$P_{\mathrm{GSO}}^{9b/9a} = \frac{1+(-)^F}{2}\,, \tag{A.18}$$

and the fact that the vertex (A.16) survives the projection means that on the corresponding state we have

$$(-)^F|S_\alpha\sigma\Sigma\rangle = +|S_\alpha\sigma\Sigma\rangle\ . \tag{A.19}$$

The vertices (A.15) and (A.16) belong to the $\bar{N}_a$ representation of the color group, and have a flavor degeneracy described in the main text (see eqs. (2.12) and (2.13)).

If we consider the D9_a/D9_b strings we find the conjugate scalar $q^\dagger$:

$$V_{q^\dagger}(z) = (2\pi\alpha')^{\frac{1}{2}}\,q^\dagger\;\bar{\sigma}(z):\Psi^1(z)\bar{s}(z):\,\mathrm{e}^{-\varphi(z)}\,\mathrm{e}^{\mathrm{i}p_\mu X^\mu(z)} \tag{A.20}$$

and the conjugate fermion $\chi^\dagger$:

$$V_{\chi^\dagger}(z) = (2\pi\alpha')^{\frac{3}{4}}\,\chi_{\dot\alpha}\,S^{\dot\alpha}(z)\,\bar{\sigma}(z)\bar{\Sigma}(z)\,\mathrm{e}^{-\frac{1}{2}\varphi(z)}\,\mathrm{e}^{\mathrm{i}p_\mu X^\mu(z)}\,, \tag{A.21}$$

where the anti-twist fields are defined as follows:

$$\bar{\sigma}(z) \equiv \prod_{i=1}^{3}\sigma_{1-\nu_{ba}^{(i)}}(z)\,,\quad \bar{s}(z) \equiv \prod_{i=1}^{3} S_{-\nu_{ba}^{(i)}}(z)\,,\quad \bar{\Sigma}(z) \equiv \prod_{i=1}^{3} S_{\frac{1}{2}-\nu_{ba}^{(i)}}\ . \tag{A.22}$$

These vertices transform in the fundamental representation N_a of the color group and are degenerate in flavor as the q and χ vertices.

Let us notice that the operator appearing in eq. (A.21) is not the CFT conjugate of the one appearing in eq. (A.16), which would be given[11] by $S_\beta\bar{\sigma}\bar{\Sigma}$. This latter has the same F-parity as in eq. (A.19), so that the state corresponding to the vertex (A.21) must have the opposite one:

$$(-)^F|S_{\dot\alpha}\bar{\sigma}\bar{\Sigma}\rangle = -|S_{\dot\alpha}\bar{\sigma}\bar{\Sigma}\rangle\ . \tag{A.23}$$

The anti-chiral vertex (A.21) is selected because in the R sector of the D9_a/D9_b strings we take

$$P_{\mathrm{GSO}}^{9a/9b} = \frac{1-(-)^F}{2}\,, \tag{A.24}$$

as opposed to eq. (A.18). This has important consequences for the annulus partition function of the flavored strings; see the discussion of this point after eq. (4.34) of ref. [19], in an $\mathcal{N}=2$ context.

[11]In fact, the nontrivial overlap between the 4d spin fields is $\langle S_\beta|S_\alpha\rangle \propto \epsilon_{\alpha\beta}$.

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D9$_a$/D9$_c$ and D9$_c$/D9$_a$ strings. The open strings stretching between the color D9$_a$ branes and the flavor branes of type D9$_c$ have essentially the same characteristics of the D9$_b$/D9$_a$ and D9$_a$/D9$_b$ ones, provided one replaces the twist angles $\nu_{ba}^{(i)}$ with

$$\nu_{ac}^{(i)} = \nu_a^{(i)} - \nu_c^{(i)} \ . \tag{A.25}$$

In the NS sector of the D9$_a$/D9$_c$ strings we find a complex scalar $\tilde{q}$ associated to the massless vertex

$$V_{\tilde{q}}(z) = (2\pi\alpha')^{\frac{1}{2}}\, \tilde{q}\ \tilde{\sigma}(z) : \bar{\Psi}^1(z)\tilde{s}(z) : \mathrm{e}^{-\varphi(z)}\, \mathrm{e}^{\mathrm{i}p_\mu X^\mu(z)} \,, \tag{A.26}$$

while in the R sector we have a chiral spinor:

$$V_{\tilde{\chi}}(z) = (2\pi\alpha')^{\frac{3}{4}}\, \tilde{\chi}^\alpha\, S_\alpha(z)\, \tilde{\sigma}(z)\tilde{\Sigma}(z)\, \mathrm{e}^{-\frac{1}{2}\varphi(z)}\, \mathrm{e}^{\mathrm{i}p_\mu X^\mu(z)} \ . \tag{A.27}$$

Here the twist/spin fields $\tilde{\sigma}$, $\tilde{s}$ and $\tilde{\Sigma}$ are defined analogously to eqs (A.13) and (A.17) in terms of the angles $\nu_{ac}^{(i)}$ of eq. (A.25). These vertices transform in the fundamental representation N_a of the color group and are degenerate in flavor as indicated in eq. (2.14) in the main text. Notice that for the (a, c) "intersection" the rôle of the color and flavor branes is switched, with respect to the (b, a) intersection, for what concerns the GSO/chirality projection.

E5$_a$/E5$_a$ strings. The polarizations in all vertices arising from these strings are $k \times k$ matrices transforming in the adjoint of U(k), and belong to the trivial representation of the orbifold group. The operator part of the vertices must therefore also be invariant under the orbifold.

In the NS sector we have the vertices

$$V_a(z) = g_{5_a}\, (2\pi\alpha')^{\frac{1}{2}}\, a_\mu\, \psi^\mu(z)\, \mathrm{e}^{-\varphi(z)} \,, \tag{A.28a}$$

$$V_D(z) = D_c\, (\pi\alpha')^{\frac{1}{2}}\, \bar{\eta}^c_{\mu\nu}\, \psi^\nu(z)\psi^\mu(z) \,, \tag{A.28b}$$

where $\bar{\eta}^c_{\mu\nu}$ are the three anti-self-dual 't Hooft symbols. In the R sector, we have

$$V_M(z) = \frac{g_{5_a}}{\sqrt{2}}\, (2\pi\alpha')^{\frac{3}{4}}\, M^\alpha\, S_\alpha(z) S_{-++}(z)\, \mathrm{e}^{-\frac{1}{2}\varphi(z)} \,, \tag{A.29}$$

$$V_\lambda(z) = \lambda_{\dot\alpha}\, (2\pi\alpha')^{\frac{3}{4}}\, S^{\dot\alpha}(z) S^{+--}(z)\, \mathrm{e}^{-\frac{1}{2}\varphi(z)} \ . \tag{A.30}$$

D9$_a$/E5$_a$ and E5$_a$/D9$_a$ strings. In the NS sector of D9$_a$/E5$_a$ strings we have the vertices

$$V_w(z) = \frac{g_{5_a}}{\sqrt{2}}\, (2\pi\alpha')^{\frac{1}{2}}\, w_{\dot\alpha}\, \Delta(z) S^{\dot\alpha}(z)\, \mathrm{e}^{-\varphi(z)} \ . \tag{A.31}$$

Here Δ is the twist operator with conformal weight 1/4 which changes the boundary conditions of the non-compact coordinates X^μ from Neumann to Dirichlet.

In the R sector there is the vertex

$$V_\mu(z) = \frac{g_{5_a}}{\sqrt{2}}\, (2\pi\alpha')^{\frac{3}{4}}\, \mu\, \Delta(z) S_{-++}(z)\, \mathrm{e}^{-\frac{1}{2}\varphi(z)} \ . \tag{A.32}$$

Both in (A.31) and (A.32) the polarizations are $N_a \times k$ matrices which transform in the bi-fundamental representation $N_a \times \bar{k}$ of $\mathrm{U}(N_a) \times \mathrm{U}(k)$. The Chan-Paton factors and the operator part of these vertices are invariant under the orbifold group.

The charged moduli associated to the $\mathrm{E}5_a/\mathrm{D}9_a$ strings, denoted by $\bar{w}_{\dot{\alpha}}$ and $\bar{\mu}$, transform in the $\bar{N}_a \times k$ representation and are described by vertex operators of the same form as (A.31) and (A.32) with $\Delta(z)$ replaced by the anti-twist $\bar{\Delta}(z)$, corresponding to DN (instead of ND) boundary conditions along the space-time directions. In particular, we have

$$V_{\bar{\mu}}(z) = \frac{g_{5_a}}{\sqrt{2}} (2\pi\alpha')^{\frac{3}{4}} \, \bar{\mu} \, \bar{\Delta}(z) S_{-++}(z) \, \mathrm{e}^{-\frac{1}{2}\varphi(z)} \ . \tag{A.33}$$

Notice that this vertex is not the CFT conjugate of eq. (A.32), which would contain the operator $\bar{\Delta} S^{+--}$.

$\mathbf{D9_b/E5_a}$ and $\mathbf{E5_a/D9_b}$ strings. The Chan-Paton factors for these strings transform, analogously to eq. (A.12), in the representation R_1 of the orbifold group.

Since there is no momentum available for these strings, it is not possible to construct physical vertices in the NS sector. The only physical vertex for $\mathrm{D}9_b/\mathrm{E}5_a$ strings is in the R sector:

$$V_{\mu'}(z) = \frac{g_{5_a}}{\sqrt{2}} (2\pi\alpha')^{\frac{3}{4}} \, \mu' \, \Delta(z) \, \sigma(z) \, \Sigma(z) \, \mathrm{e}^{-\frac{1}{2}\varphi(z)} \ . \tag{A.34}$$

Notice that the operator part of this vertex transforms in the R_1 representation (see the discussion after eq. (A.17) above), and this appropriately compensates the transformation of the Chan-Paton factors. The polarization transforms in the anti-fundamental of $\mathrm{U}(k)$ and carries a flavor degeneracy as in eq. (2.12). The vertex (A.34) survives the GSO projection

$$P_{\mathrm{GSO}}^{9b/5a} = \frac{1 + (-)^F}{2} \tag{A.35}$$

hence, on the corresponding state, we have

$$(-)^F |\Delta\sigma\Sigma\rangle = +|\Delta\sigma\Sigma\rangle \ . \tag{A.36}$$

If we consider now the mixed strings with the opposite orientation, the $\mathrm{E}5_a/\mathrm{D}9_b$ ones, we can only construct a massless vertex using the operator $\bar{\Delta}\bar{\sigma}\bar{\Sigma}$. This operator is the CFT conjugate of the operator appearing in eq. (A.34), and has therefore the same F-parity. However, in the R sector, as usual we have to take the opposite projection with respect to eq. (A.35), namely

$$P_{\mathrm{GSO}}^{5a/9b} = \frac{1 - (-)^F}{2} \ . \tag{A.37}$$

Therefore, no physical state with this orientation survives the projection.

$\mathbf{E5_a/D9_c}$ and $\mathbf{D9_c/E5_a}$ strings. In this sector, we find a physical vertex in the R sector of the $\mathrm{E}5_a/\mathrm{D}9_c$ given by

$$V_{\tilde{\mu}'}(z) = \frac{g_{5_a}}{\sqrt{2}} (2\pi\alpha')^{\frac{3}{4}} \, \tilde{\mu}' \, \Delta(z) \, \tilde{\sigma}(z) \, \tilde{\Sigma}(z) \, \mathrm{e}^{-\frac{1}{2}\varphi(z)} \ . \tag{A.38}$$

The polarization transforms in the fundamental of U(k) and carries a flavor degeneracy as in eq. (2.14).

In the mixed strings D9_c/E5_a with the opposite orientation, no physical state survives the GSO projection.

A.3 The flavored annuli

In this section we compute in more detail the contribution of E5_a/D9_b and D9_b/E5_a strings to the annulus amplitude, deriving the expressions given in eqs (4.13) and (4.14) for the one-loop traces $\mathcal{A}^{h_I}_{5_a;9_b}$ containing the insertions of the various elements of the orbifold group.

The untwisted sector corresponds to the insertion the identity element $h_0 \equiv e$. For the even spin structures, one finds that contributions corresponding to the two possible orientations are equal. Adding them, one finds

$$\begin{aligned} \mathcal{A}^{e}_{5_a;9_b} = \frac{\mathrm{i}\,k\,N_F}{2\pi}\int_0^\infty \frac{d\tau}{2\tau} \sum_{\substack{\alpha,\beta=0\\(\alpha,\beta)\neq(1,1)}}^{1} (-1)^{\alpha+\beta+\alpha\beta}\ \frac{\theta[^\alpha_\beta](0|\mathrm{i}\tau)}{\theta_1'(0|\mathrm{i}\tau)} \\ \times\left(\frac{\theta[^\alpha_\beta](\frac{\mathrm{i}\tau}{2}|\mathrm{i}\tau)\,\theta_1'(0|\mathrm{i}\tau)}{\theta[^\alpha_\beta]\,(0|\mathrm{i}\tau)\,\theta_1(\frac{\mathrm{i}\tau}{2}|\mathrm{i}\tau)}\right)^2 \prod_{i=1}^{3}\frac{\theta[^\alpha_\beta](\mathrm{i}\nu^{(i)}\tau|\mathrm{i}\tau)}{\theta_1(\mathrm{i}\nu^{(i)}\tau|\mathrm{i}\tau)} \end{aligned} \tag{A.39}$$

where N_F is given in eq. (2.15) and we have used the following notations[12]

$$\theta[^0_0] \equiv \theta_3 \quad ; \quad \theta[^0_1] \equiv \theta_4 \quad ; \quad \theta[^1_0] \equiv \theta_2 \quad ; \quad \theta[^1_1] \equiv \theta_1\ . \tag{A.40}$$

Moreover, we have denoted with primes the derivatives of the θ-functions with respect to their first argument, i.e.

$$\theta_1'(\nu|\mathrm{i}\tau) \equiv \partial_z\theta_1(z|\mathrm{i}\tau)\big|_{z=\nu}\ . \tag{A.41}$$

The contribution of the odd spin structure R$(-1)^F$ for a given orientation is actually infinite and cannot be regularized using the procedure described in ref. [49]. However, because of the different GSO projection to be employed in the two oriented sectors (see appendix A.2), one gets a complete cancellation between the two orientations.

Eq. (A.39), which coincides with eq. (2.11) of ref. [6], can be recast in a simpler form by first using the following relation

$$\left(\frac{\theta[^\alpha_\beta](\frac{\mathrm{i}\tau}{2}|\mathrm{i}\tau)\ \theta_1'(0|\mathrm{i}\tau)}{\theta[^\alpha_\beta](0|\mathrm{i}\tau)\ \theta_1(\frac{\mathrm{i}\tau}{2}|\mathrm{i}\tau)}\right)^2 = \frac{\theta''[^\alpha_\beta](0|\mathrm{i}\tau)}{\theta[^\alpha_\beta](0|\mathrm{i}\tau)} - \partial_z^2\log\theta_1(z|\mathrm{i}\tau)\big|_{z=\frac{\mathrm{i}\tau}{2}}\ , \tag{A.42}$$

and then the Riemann identity

$$\begin{aligned} &\sum_{\alpha,\beta=0}^{1}(-1)^{\alpha+\beta+\alpha\beta}\ \ \theta[^\alpha_\beta](z|\mathrm{i}\tau)\prod_{i=1}^{3}\theta[^\alpha_\beta](\mathrm{i}\tau\nu^{(i)}|\mathrm{i}\tau) \\ &= -\,2\,\theta_1\big(\frac{z}{2}|\mathrm{i}\tau\big)\,\theta_1\big(-\frac{z}{2}+\mathrm{i}\tau\nu^{(1)}|\mathrm{i}\tau\big)\,\theta_1\big(\frac{z}{2}+\mathrm{i}\tau\nu^{(2)}|\mathrm{i}\tau\big)\,\theta_1\big(\frac{z}{2}+\mathrm{i}\tau\nu^{(3)}|\mathrm{i}\tau\big)\ . \end{aligned} \tag{A.43}$$

[12]For the θ-functions we use the notations listed in appendix A of ref. [39] where also the Riemann identity, that will be used several times in the following, is given.

The last term of eq. (A.42) does not contribute when this is inserted into eq. (A.39) since the right hand side of eq. (A.43) is zero for $\nu = 0$. Therefore we get

$$\mathcal{A}^{e}_{5_a;9_b} = \frac{\mathrm{i}\,k\,N_F}{2\pi}\int_0^\infty \frac{d\tau}{2\tau} \sum_{\substack{\alpha,\beta=0\\(\alpha,\beta)\neq(1,1)}}^{1} (-1)^{\alpha+\beta+\alpha\beta}\, \frac{\theta''[^\alpha_\beta](0|\mathrm{i}\tau)}{\theta'_1(0|\mathrm{i}\tau)} \prod_{i=1}^{3} \frac{\theta[^\alpha_\beta](\mathrm{i}\nu^{(i)}\tau|\mathrm{i}\tau)}{\theta_1(\mathrm{i}\nu^{(i)}\tau|\mathrm{i}\tau)}\ . \tag{A.44}$$

Taking the second derivative with respect to z of the Riemann identity (A.43) and evaluating it at $z = 0$, we obtain another identity which allows us to rewrite eq. (A.44) as follows:

$$\begin{aligned}\mathcal{A}^{e}_{5_a;9_b} &= \frac{\mathrm{i}\,k\,N_F}{2\pi}\int_0^\infty \frac{d\tau}{2\tau} \\ &\times \Big[\partial_z \log\theta_1(z|\mathrm{i}\tau)\big|_{z=\mathrm{i}\tau\nu^{(1)}} - \partial_z \log\theta_1(z|\mathrm{i}\tau)\big|_{z=\mathrm{i}\tau\nu^{(2)}} - \partial_z \log\theta_1(z|\mathrm{i}\tau)\big|_{z=\mathrm{i}\tau\nu^{(3)}}\Big]\ .\end{aligned} \tag{A.45}$$

This is precisely eq. (4.13) of the main text.

Let us now turn to the three amplitudes $\mathcal{A}^{h_i}_{5_a;9_b}$ containing the insertion on the non-trivial elements h_i of the orbifold group. Again the (divergent) contribution of the odd spin structure $\mathrm{R}(-1)^F$ cancels when we sum over the two orientations of the flavored strings, while the even spin structures give the following contribution

$$\begin{aligned}\mathcal{A}^{h_i}_{5_a;9_b} &= \frac{\mathrm{i}\,k\,N_F}{2\pi}\int_0^\infty \frac{d\tau}{2\tau}\,\frac{1}{\theta'_1(0|\mathrm{i}\tau)} \sum_{\substack{\alpha,\beta=0\\(\alpha,\beta)\neq(1,1)}}^{1} (-1)^{\beta+\alpha\beta}\ [R_1(h_i)]^\alpha \\ &\Big(\theta''[^\alpha_\beta](0|\mathrm{i}\tau) - \theta[^\alpha_\beta](0|i\tau)\,\partial_z^2 \log\theta_1(z|\mathrm{i}\tau)\big|_{z=\frac{\mathrm{i}\tau}{2}}\Big) \frac{\theta[^\alpha_\beta](\mathrm{i}\tau\nu^{(i)}|\mathrm{i}\tau)}{\theta_1(\mathrm{i}\tau\nu^{(i)}|\mathrm{i}\tau)} \prod_{j\neq i=1}^{3} \frac{\theta[^{\ \alpha}_{\beta+1}](\mathrm{i}\tau\nu^{(j)}|\mathrm{i}\tau)}{\theta_2(\mathrm{i}\tau\nu^{(j)}|\mathrm{i}\tau)}\ ,\end{aligned} \tag{A.46}$$

where in the second line we have already used the relation (A.42). To proceed we have to distinguish the two cases corresponding to h_1, for which $R_1(h_1) = 1$, and to h_2, h_3, for which $R_1(h_2) = R_1(h_3) = -1$. In the first case eq. (A.46) becomes

$$\begin{aligned}\mathcal{A}^{h_1}_{5_a;9_b} = \frac{\mathrm{i}\,k\,N_F}{2\pi}&\int_0^\infty \frac{d\tau}{2\tau}\,\frac{1}{\theta'_1(0|\mathrm{i}\tau)} \sum_{\substack{\alpha,\beta=0\\(\alpha,\beta)\neq(1,1)}}^{1} (-1)^{\alpha+\beta+\alpha\beta} \\ &\times \Big(\theta''[^\alpha_\beta](0|\mathrm{i}\tau) - \theta[^\alpha_\beta](0|i\tau)\,\partial_z^2 \log\theta_1(z|\mathrm{i}\tau)\big|_{z=\frac{\mathrm{i}\tau}{2}}\Big) \\ &\times \frac{\theta[^\alpha_\beta](\mathrm{i}\tau\nu^{(1)}|\mathrm{i}\tau)}{\theta_1(\mathrm{i}\tau\nu^{(1)}|\mathrm{i}\tau)}\ \frac{\theta[^{\ \alpha}_{\beta+1}](\mathrm{i}\tau\nu^{(2)}|\mathrm{i}\tau)}{\theta_2(\mathrm{i}\tau\nu^{(2)}|\mathrm{i}\tau)}\ \frac{\theta[^{\ \alpha}_{\beta-1}](\mathrm{i}\tau\nu^{(3)}|\mathrm{i}\tau)}{\theta_2(\mathrm{i}\tau\nu^{(3)}|\mathrm{i}\tau)}\ ,\end{aligned} \tag{A.47}$$

where we have used the fact that

$$\theta[^{\ \alpha}_{\beta-1}](z|\mathrm{i}\tau) = (-1)^\alpha \theta[^{\ \alpha}_{\beta+1}](z|\mathrm{i}\tau)\ . \tag{A.48}$$

We can then exploit the identity

$$\begin{aligned}&\sum_{\alpha,\beta=0}^{1} (-1)^{\alpha+\beta+\alpha\beta}\,\theta[^\alpha_\beta](z|\mathrm{i}\tau)\,\theta[^\alpha_\beta](\mathrm{i}\tau\nu^{(1)}|\mathrm{i}\tau)\,\theta[^{\ \alpha}_{\beta+1}](\mathrm{i}\tau\nu^{(2)}|\mathrm{i}\tau)\,\theta[^{\ \alpha}_{\beta-1}](\mathrm{i}\tau\nu^{(3)}|\mathrm{i}\tau) \\ &\quad = 2\,\theta_1\big(\tfrac{z}{2}|\mathrm{i}\tau\big)\,\theta_1\big(-\tfrac{z}{2}+\mathrm{i}\tau\nu^{(1)}|\mathrm{i}\tau\big)\,\theta_2\big(\tfrac{z}{2}+\mathrm{i}\tau\nu^{(2)}|\mathrm{i}\tau\big)\,\theta_2\big(\tfrac{z}{2}+\mathrm{i}\tau\nu^{(3)}|\mathrm{i}\tau\big)\end{aligned} \tag{A.49}$$

to check that the term containing the second derivative of the logarithm of θ_1 in eq. (A.47) does not give any contribution. The remaining terms can be computed by differentiating twice with respect to z both sides of eq. (A.49) and putting $z = 0$. In this way we get

$$\begin{aligned}\mathcal{A}^{h_1}_{5_a;9_b} &= \frac{\mathrm{i}\, k\, N_F}{2\pi}\int_0^\infty \frac{d\tau}{2\tau} \\ &\times \left[\partial_z \log\theta_1(z|\mathrm{i}\tau)\big|_{z=\mathrm{i}\tau\nu^{(1)}} - \partial_z \log\theta_2(z|\mathrm{i}\tau)\big|_{z=\mathrm{i}\tau\nu^{(2)}} - \partial_z \log\theta_2(z|\mathrm{i}\tau)\big|_{z=\mathrm{i}\tau\nu^{(3)}}\right] .\end{aligned} \tag{A.50}$$

Let us now consider the amplitude with h_2 inserted. In this case eq. (A.46) becomes

$$\begin{aligned}\mathcal{A}^{h_2}_{5_a;9_b} = \frac{\mathrm{i}\, k\, N_F}{2\pi}\int_0^\infty \frac{d\tau}{2\tau}\frac{1}{\theta_1'(0|\mathrm{i}\tau)} &\sum_{\substack{\alpha,\beta=0\\(\alpha,\beta)\neq(1,1)}}^{1} (-1)^{\alpha+\beta+\alpha\beta} \\ &\times \left(\theta''\left[\begin{smallmatrix}\alpha\\ \beta-1\end{smallmatrix}\right](0|\mathrm{i}\tau) - \theta\left[\begin{smallmatrix}\alpha\\ \beta-1\end{smallmatrix}\right](0|\mathrm{i}\tau)\,\partial_z^2\log\theta_1(z|\mathrm{i}\tau)\big|_{z=\frac{\mathrm{i}\tau}{2}}\right) \\ &\times \frac{\theta\left[\begin{smallmatrix}\alpha\\ \beta+1\end{smallmatrix}\right](i\tau\nu^{(2)}|\mathrm{i}\tau)}{\theta_1(i\tau\nu^2|i\tau)}\,\frac{\theta\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right](\mathrm{i}\tau\nu^{(3)}|\mathrm{i}\tau)}{\theta_2(\mathrm{i}\tau\nu^{(3)}|\mathrm{i}\tau)}\,\frac{\theta\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right](\mathrm{i}\tau\nu^{(1)}|\mathrm{i}\tau)}{\theta_2(\mathrm{i}\tau\nu^{(1)}|\mathrm{i}\tau)}\end{aligned} \tag{A.51}$$

after making the substitution $\beta + 1 \to \beta$ and using eq. (A.48). Then, by exploiting the following relation:

$$\begin{aligned}\sum_{\alpha,\beta=0}^{1} (-1)^{\alpha+\beta+\alpha\beta}\quad &\theta\left[\begin{smallmatrix}\alpha\\ \beta-1\end{smallmatrix}\right](z|\mathrm{i}\tau)\,\theta\left[\begin{smallmatrix}\alpha\\ \beta+1\end{smallmatrix}\right](\mathrm{i}\tau\nu^{(2)}|\mathrm{i}\tau)\,\theta\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right](\mathrm{i}\tau\nu^{(3)}|\mathrm{i}\tau)\,\theta\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right](\mathrm{i}\tau\nu^{(1)}|\mathrm{i}\tau) \\ &= 2\,\theta_1\big(\frac{z}{2}|\mathrm{i}\tau\big)\,\theta_1\big(\frac{z}{2}+\mathrm{i}\tau\nu^{(2)}|\mathrm{i}\tau\big)\,\theta_2\big(\frac{z}{2}+\mathrm{i}\tau\nu^{(3)}|\mathrm{i}\tau\big)\,\theta_2\big(-\frac{z}{2}+\mathrm{i}\tau\nu^{(1)}|\mathrm{i}\tau\big)\,,\end{aligned} \tag{A.52}$$

we can show that the second term in the second line in eq. (A.51) is zero. The first term can instead be computed by differentiating twice both sides of eq. (A.52), and we get

$$\begin{aligned}\mathcal{A}^{h_2}_{5_a;9_b} &= \frac{\mathrm{i}\, k\, N_F}{2\pi}\int_0^\infty \frac{d\tau}{2\tau} \\ &\times \left[\partial_z \log\theta_1(z|\mathrm{i}\tau)\big|_{z=\mathrm{i}\tau\nu^{(2)}} - \partial_z \log\theta_2(z|\mathrm{i}\tau)\big|_{z=\mathrm{i}\tau\nu^{(3)}} - \partial_z \log\theta_2(z|\mathrm{i}\tau)\big|_{z=\mathrm{i}\tau\nu^{(1)}}\right] .\end{aligned} \tag{A.53}$$

Finally the amplitude with the insertion of h_3 is obtained from the previous expression by the exchange $(i = 2) \leftrightarrow (i = 3)$, namely

$$\begin{aligned}\mathcal{A}^{h_3}_{5_a;9_b} &= \frac{\mathrm{i}\, k\, N_F}{2\pi}\int_0^\infty \frac{d\tau}{2\tau} \\ &\times \left[\partial_z \log\theta_1(z|\mathrm{i}\tau)\big|_{z=\mathrm{i}\tau\nu^{(3)}} - \partial_z \log\theta_2(z|\mathrm{i}\tau)\big|_{z=\mathrm{i}\tau\nu^{(2)}} - \partial_z \log\theta_2(z|\mathrm{i}\tau)\big|_{z=\mathrm{i}\tau\nu^{(1)}}\right] .\end{aligned} \tag{A.54}$$

Eqs. (A.50), (A.53) and (A.54) can be written in the compact form reported in eq. (4.14) of the main text.

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