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Transverse–Spin Quark Distributions from Asymmetry Data and Symmetry Arguments

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Abstract: The transversity and the Sivers distribution functions of quarks incorporate important information about the transverse-spin and transverse-momentum structure of nucleons. We show how these distributions can be directly determined point by point from leptoproduction asymmetry data collected for various targets and produced hadrons by the COMPASS Collaboration. Only simple symmetry relations are used in the extraction.

Keywords: transverse spin; quark distribution functions; structure of the nucleon; semi-inclusive deep inelastic scattering

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1. Introduction

The investigation of the transverse-spin structure of nucleons has been an extremely active research area of hadronic physics in the past two decades [1,2]. Single-spin asymmetries in leptoproduction of hadrons from a transversely polarized target, $\ell N^{\uparrow} \to \ell' h X$, have been discovered and measured by various experiments (for a review, see, e.g., in [3]), and represent the primary way to access the transverse-spin distributions of quarks.

One of these asymmetries—the so-called Collins asymmetry—involves the transversity distribution h_1, a leading-twist and chirally odd distribution function which measures the transverse polarization of quarks inside a transversely polarized nucleon [4]. In the Collins asymmetry, transversity couples to a transverse-momentum-dependent chirally odd fragmentation function, $H_1^{\perp}$ (the "Collins function" [5]), which describes the fragmentation of a transversely polarized quark into a spinless hadron. Collins asymmetries have been measured by the HERMES [6,7] and the COMPASS experiments [8–10] on a proton target, and by COMPASS on a deuteron target [11–13].

Another single-spin asymmetry, associated with a different angular modulation of the cross section, originates from a correlation between the transverse spin of the nucleon and the transverse momentum of quarks, described by a leading-twist transverse-momentum dependent distribution (TMD), the "Sivers function" $f_{1T}^{\perp}$ [14,15]. A non-zero Sivers function causes the distributions of quark transverse momentum to be asymmetric with respect to the plane given by the directions of nucleon spin and momentum. This asymmetry, known as the Sivers effect, has been experimentally observed by the HERMES [6,16] and COMPASS collaborations [8,10–13,17] in the case of pion and kaon production.

Many phenomenological analyses of these asymmetries have been performed so far (see, for a review, the work in [18]). Most of them extract the quark distributions by fitting the data with a given functional form for their dependence on the Bjorken variable x. We adopt a more direct approach, taking advantage of the fact that the COMPASS experiment has provided data for different targets (proton and deuteron) and produced hadrons (positive and negative pions) with the same kinematics. By simple general arguments based on isospin symmetry and sea flavor symmetry, the asymmetries of the various leptoproduction processes can be related to each other and combined in such a way that the valence distributions u_v and d_v are separately extracted point by point in x (for details we refer the reader to the papers where this procedure was originally proposed and applied [19,20]). The approach is almost model-independent. In fact, although we adopt a Gaussian Ansatz for the transverse-momentum dependence of quark distributions (in order to factorize them from fragmentation functions), the Gaussian widths—representing the average transverse momenta of quarks—do not appear in the final results.

The transversity and Sivers quark sea distributions can be determined in the same way, but in this paper we will not consider them. As shown in [19,20], they turn out to be quite uncertain and compatible with zero within their errors.

The Sivers effect manifests itself also in the gluon sector [21]. The gluon Sivers function $f_{1T}^{\perp g}$ can be probed for instance in the inclusive leptoproduction of hadron pairs with large transverse momenta, as done by COMPASS [22]. There are three different elementary reactions contributing to this process: the leading-order scattering $\gamma^* q \to q$, the QCD Compton scattering $\gamma^* q \to qg$, and the photon–gluon fusion $\gamma^* g \to q\bar{q}$. As only the photon–gluon fusion involves the gluon Sivers distribution, the Sivers asymmetry data for dihadron production cannot be directly used to extract $f_{1T}^{\perp g}$. In [22], the photon–gluon fusion component of the Sivers asymmetry has been disentangled by means of a Monte Carlo method. The result, restricted to a single x value, shows that the gluonic contribution to the Sivers effect is definitely non-vanishing (and negative). Work is now in progress to determine the magnitude of $f_{1T}^{\perp g}$.

Extracting the transversity distributions from linear combinations of the Collins asymmetries requires some knowledge of the Collins fragmentation function $H_1^{\perp}$, which must be obtained independently from another class of processes, namely, inclusive dihadron production in e^+e^- annihilation, studied by various experiments [23–26]. An alternative way to determine the transversity from the Collins asymmetries alone is via the so-called "difference asymmetries", which allow extracting combinations of the u and d valence quark transversity without knowing the Collins fragmentation function. This method was proposed long time ago [27–29] to access the helicity distribution functions. Recently it has been revisited in the context of Sivers, Boer–Mulders and transversity distributions [30]. Here, we report the results of a recent paper of ours [31], where, using again the COMPASS measurements with proton and deuteron targets, we determined the transversity ratio $h_1^{u_v}/h_1^{d_v}$.

2. SIDIS with a Transversely Polarized Target

The process we are interested in is semi-inclusive DIS (SIDIS) with a transversely polarized target, $\ell N^{\uparrow} \to \ell' h X$. The produced hadrons h (of mass M_h and momentum $\boldsymbol{P}_h$) we will consider are positive and negative pions. Conventionally, all azimuthal angles are referred to the lepton scattering plane in a reference system in which the z axis is the virtual photon direction, while the x axis is directed along the transverse momentum of the outgoing lepton: ϕ_h is the azimuthal angle of $\boldsymbol{P}_h$, ϕ_S is the azimuthal angle of the nucleon spin vector. The transverse momenta are defined as follows. $\boldsymbol{k}_T$ is the transverse momentum of the quark inside the nucleon, $\boldsymbol{p}_T$ is the transverse momentum of the hadron with respect to the direction of the fragmenting quark, and $\boldsymbol{P}_{h\perp}$ is the measurable transverse momentum of the produced hadron with respect to the z axis.

The SIDIS cross section for a transversely polarized target can be synthetically written as

$$\sigma_t^{\pm} = \sigma_{0,t}^{\pm} + S_T \Big\{ \sigma_{S,t}^{\pm} \sin(\phi_h - \phi_S) + \sigma_{C,t}^{\pm} \sin(\phi_h + \phi_S) + \dots \Big\}, \tag{1}$$

where S_T is the transverse polarization of the target. The signs ± refer to the pion charge and $t = p, d$ is the target type. In Equation (1) we retained only two terms: the Sivers term, associated to the $\sin(\phi_h - \phi_S)$ modulation, and the Collins term, associated to the $\sin(\phi_h + \phi_S)$ modulation. The corresponding asymmetries are the Collins asymmetry

$$A^{\pm}_{C,t} = \frac{\sigma^{\pm}_{C,t}}{\sigma^{\pm}_{0,t}}, \tag{2}$$

and the Sivers asymmetry

$$A^{\pm}_{S,t} = \frac{\sigma^{\pm}_{S,t}}{\sigma^{\pm}_{0,t}}. \tag{3}$$

At leading twist and leading order in QCD, the Sivers component of the cross section couples the Sivers distribution $f_{1T}^{\perp}$ to the transverse-momentum-dependent unpolarized fragmentation function D_1, yielding the asymmetry [32–34]

$$A_S(x, z, Q^2) = \frac{\sum_{q,\bar{q}} e_q^2\, x \int \mathrm{d}^2 \boldsymbol{P}_{h\perp}\, \mathcal{C}\left[\frac{\boldsymbol{P}_{h\perp}\cdot \boldsymbol{k}_T}{M P_{h\perp}}\, f_{1T}^{\perp}\, D_1\right]}{\sum_{q,\bar{q}} e_q^2\, x \int \mathrm{d}^2 \boldsymbol{P}_{h\perp}\, \mathcal{C}\left[f_1 D_1\right]}, \tag{4}$$

where the convolution $\mathcal{C}$ is defined as (w is a function of transverse momenta)

$$\begin{aligned} \mathcal{C}\left[w f D\right] \;\; &= \int \mathrm{d}^2 \boldsymbol{k}_T \int \mathrm{d}^2 \boldsymbol{p}_T\, \delta^2(z\boldsymbol{k}_T + \boldsymbol{p}_T - \boldsymbol{P}_{h\perp}) \\ &\quad \times w(\boldsymbol{k}_T, \boldsymbol{p}_T)\, f(x, k_T^2, Q^2) D(z, p_T^2, Q^2)\,. \end{aligned} \tag{5}$$

The Collins term in the SIDIS cross section couples the transversity distribution h_1 to the Collins fragmentation function $H_1^{\perp}$, and the resulting asymmetry is [32,34]

$$A_C(x, z, Q^2) = \frac{\sum_{q,\bar{q}} e_q^2\, x \int \mathrm{d}^2 \boldsymbol{P}_{h\perp}\, \mathcal{C}\left[\frac{\boldsymbol{P}_{h\perp}\cdot \boldsymbol{p}_T}{z M_h P_{h\perp}}\, h_1\, H_1^{\perp}\right]}{\sum_{q,\bar{q}} e_q^2\, x \int \mathrm{d}^2 \boldsymbol{P}_{h\perp}\, \mathcal{C}\left[f_1 D_1\right]}. \tag{6}$$

In order to perform the convolutions, one should know the transverse-momentum-dependence of distribution and fragmentation functions. As there is no information on that, one must make some assumption. It is usual, and convenient for computational purpose, to use a Gaussian form, with its width as a free parameter. However, as we will see, Gaussian widths will not play any rôle in our analysis, so we do not need to know their values.

3. Sivers Distributions

We start from the extraction of the Sivers distributions, which is somehow easier, as it does not involve any unknown fragmentation function.

Adopting a Gaussian Ansatz for the transverse-momentum dependence of functions, i.e.,

$$f_{1T}^{\perp}(x, k_T^2, Q^2) = f_{1T}^{\perp}(x, Q^2)\, \frac{\mathrm{e}^{-k_T^2/\langle k_T^2\rangle}}{\pi \langle k_T^2 \rangle}\,, \tag{7}$$

$$D_1(z, p_T^2, Q^2) = D_1(z, Q^2)\, \frac{\mathrm{e}^{-p_T^2/\langle p_T^2\rangle}}{\pi \langle p_T^2 \rangle}\,, \tag{8}$$

the Sivers asymmetry (4) takes the form [35,36]

$$A_S(x, z, Q^2) = G\, \frac{\sum_{q,\bar{q}} e_q^2 x f_{1T}^{\perp(1)q}(x, Q^2) z D_{1q}(z, Q^2)}{\sum_{q,\bar{q}} e_q^2 x f_1^q(x, Q^2) D_{1q}(z, Q^2)}. \tag{9}$$

Here, $f_{1T}^{\perp(1)}$ is the first k_T^2 moment of the Sivers function, defined as

$$f_{1T}^{\perp(1)}(x, Q^2) \equiv \int \mathrm{d}^2 \boldsymbol{k}_T\, \frac{k_T^2}{2M^2}\, f_{1T}^{\perp}(x, k_T^2, Q^2)\,, \tag{10}$$

and $D_1(z, Q^2)$ is the fragmentation function integrated over the transverse momentum. The factor G, resulting from the Gaussian integrations, is given by [35,36]

$$G = \frac{\sqrt{\pi} M}{\sqrt{\langle p_T^2 \rangle + z^2 \langle k_T^2 \rangle}}, \tag{11}$$

where $\langle k_T^2 \rangle$ and $\langle p_T^2 \rangle$ are the widths of the Sivers distribution and of the unpolarized fragmentation function, respectively. In the Gaussian model, G can be approximately related to the average transverse momentum of the produced hadrons, $\langle P_{h\perp} \rangle$, by

$$G \simeq \frac{\pi M}{2 \langle P_{h\perp} \rangle}. \tag{12}$$

As $\langle P_{h\perp} \rangle$ is an experimentally determined quantity, the values of the average transverse momenta of quarks are irrelevant.

Our purpose is to extract from the data the k_T^2 moment of the Sivers distribution, thus we integrate over z,

$$\widetilde{D}_1(Q^2) = \int \mathrm{d}z \, D_1(z, Q^2) \,, \quad \widetilde{D}_1^{(1)}(Q^2) = \int \mathrm{d}z \, z D_1(z, Q^2) \,, \tag{13}$$

and consider the integrated asymmetry

$$A_S(x, Q^2) = G \, \frac{\sum_{q,\bar{q}} e_q^2 x f_{1T}^{\perp(1)q}(x, Q^2) \widetilde{D}_{1q}^{(1)}(Q^2)}{\sum_{q,\bar{q}} e_q^2 x f_1^q(x, Q^2) \widetilde{D}_{1q}(Q^2)}. \tag{14}$$

Imposing isospin symmetry and SU(2) flavor symmetry of the pion sea, we can distinguish between favored and unfavored fragmentation functions as follows (superscripts ± refer to the pion charge),

$$D_{1,\mathrm{fav}} \equiv D_{1u}^{+} = D_{1d}^{-} = D_{1\bar{u}}^{-} = D_{1\bar{d}}^{+} \tag{15}$$

$$D_{1,\mathrm{unf}} \equiv D_{1u}^{-} = D_{1d}^{+} = D_{1\bar{u}}^{+} = D_{1\bar{d}}^{-}. \tag{16}$$

For the strange sector, following the work in [37] we set

$$D_{1s}^{\pm} = D_{1\bar{s}}^{\pm} = N_s \, D_{1,\mathrm{unf}} \,, \tag{17}$$

where N_s is a constant coefficient.

The denominators of the asymmetries $\sum_{q,\bar{q}} e_q^2 x f_1^q \widetilde{D}_{1q}$ for a proton and a deuteron target (p, d) and for charged pions, multiplied by 9, are given by (we use again isospin symmetry and ignore the charm components of the distribution functions, which are negligible in the kinematic region we will be considering)

$$p, \pi^+ : \quad x \left[4(f_1^u + \beta f_1^{\bar{u}}) + (\beta f_1^d + f_1^{\bar{d}}) + N_s \beta (f_1^s + f_1^{\bar{s}})\right] \widetilde{D}_{1,\mathrm{fav}} \equiv x f_p^{+} \, \widetilde{D}_{1,\mathrm{fav}}, \tag{18}$$

$$d, \pi^+ : \quad x \left[(4 + \beta)(f_1^u + f_1^d) + (1 + 4\beta)(f_1^{\bar{u}} + f_1^{\bar{d}}) + 2 N_s \beta (f_1^s + f_1^{\bar{s}})\right] \widetilde{D}_{1,\mathrm{fav}} \equiv x f_d^{\pi^+} \, \widetilde{D}_{1,\mathrm{fav}}, \tag{19}$$

$$p, \pi^- : \quad x \left[4(\beta f_1^u + f_1^{\bar{u}}) + (f_1^d + \beta f_1^{\bar{d}}) + N_s \beta (f_1^s + f_1^{\bar{s}})\right] \widetilde{D}_{1,\mathrm{fav}} \equiv x f_p^{-} \, \widetilde{D}_{1,\mathrm{fav}}, \tag{20}$$

$$d, \pi^- : \quad x \left[(1 + 4\beta)(f_1^u + f_1^d) + (4 + \beta)(f_1^{\bar{u}} + f_1^{\bar{d}}) + 2 N_s \beta (f_1^s + f_1^{\bar{s}})\right] \widetilde{D}_{1,\mathrm{fav}} \equiv x f_d^{-} \, \widetilde{D}_{1,\mathrm{fav}}, \tag{21}$$

with

$$\beta(Q^2) \equiv \frac{\widetilde{D}_{1,\mathrm{unf}}(Q^2)}{\widetilde{D}_{1,\mathrm{fav}}(Q^2)}. \tag{22}$$

Similar expressions can be written for the numerator of Equation (14), $\sum_{q,\bar{q}} e_q^2 x f_{1T}^{\perp(1)q} \widetilde{D}_{1q}^{(1)}$, with the replacements $\widetilde{D}_1 \to \widetilde{D}_1^{(1)}$, $f_1 \to f_{1T}^{\perp(1)}$, and $\beta \to \beta'$, where

$$\beta'(Q^2) = \frac{\widetilde{D}^{(1)}_{1,\mathrm{unf}}(Q^2)}{\widetilde{D}^{(1)}_{1,\mathrm{fav}}(Q^2)}. \tag{23}$$

Introducing the ratio of the first to the zeroth moment of the fragmentation functions,

$$\rho(Q^2) = \frac{\widetilde{D}^{(1)}_{1,\mathrm{fav}}(Q^2)}{\widetilde{D}_{1,\mathrm{fav}}(Q^2)}, \tag{24}$$

we find for the pion asymmetries with a proton target (for simplicity we drop the S of Sivers)

$$A_p^+ = G\,\rho\,\frac{4(f_{1T}^{\perp(1)u} + \beta' f_{1T}^{\perp(1)\bar{u}}) + (\beta' f_{1T}^{\perp(1)d} + f_{1T}^{\perp(1)\bar{d}}) + N_s\beta'(f_{1T}^{\perp(1)s} + f_{1T}^{\perp(1)\bar{s}})}{f_p^+}, \tag{25}$$

$$A_p^- = G\,\rho\,\frac{4(\beta' f_{1T}^{\perp(1)u} + f_{1T}^{\perp(1)\bar{u}}) + (f_{1T}^{\perp(1)d} + \beta' f_{1T}^{\perp(1)\bar{d}}) + N_s\beta'(f_{1T}^{\perp(1)s} + f_{1T}^{\perp(1)\bar{s}})}{f_p^-}, \tag{26}$$

and for the deuteron target

$$A_d^+ = G\,\rho\,\frac{(4+\beta')(f_{1T}^{\perp(1)u} + f_{1T}^{\perp(1)d}) + (1+4\beta')(f_{1T}^{\perp(1)\bar{u}} + f_{1T}^{\perp(1)\bar{d}}) + 2N_s\beta'(f_{1T}^{\perp(1)s} + f_{1T}^{\perp(1)\bar{s}})}{f_d^+}, \tag{27}$$

$$A_d^- = G\,\rho\,\frac{(1+4\beta')(f_{1T}^{\perp(1)u} + f_{1T}^{\perp(1)d}) + (4+\beta')(f_{1T}^{\perp(1)\bar{u}} + f_{1T}^{\perp(1)\bar{d}}) + 2N_s\beta'(f_{1T}^{\perp(1)s} + f_{1T}^{\perp(1)\bar{s}})}{f_d^-}. \tag{28}$$

The combinations

$$f_p^+ A_p^+ - f_p^- A_p^- = G\,\rho(1-\beta')(4 f_{1T}^{\perp(1)u_v} - f_{1T}^{\perp(1)d_v}), \tag{29}$$

$$f_d^+ A_d^+ - f_d^- A_d^- = 3G\,\rho(1-\beta')(f_{1T}^{\perp(1)u_v} + f_{1T}^{\perp(1)d_v}), \tag{30}$$

select the valence Sivers distributions. From Equations (29) and (30), we get the valence distributions for u and d quarks, separately:

$$x f_{1T}^{\perp(1)u_v} = \frac{1}{5\,G\rho(1-\beta')}\left[(x f_p^+ A_p^+ - x f_p^- A_p^-) + \frac{1}{3}(x f_d^+ A_d^+ - x f_d^- A_d^-)\right], \tag{31}$$

$$x f_{1T}^{\perp(1)d_v} = \frac{1}{5\,G\rho(1-\beta')}\left[\frac{4}{3}(x f_d^+ A_d^+ - x f_d^- A_d^-) - (x f_p^+ A_p^+ - x f_p^- A_p^-)\right]. \tag{32}$$

The asymmetry data we use to extract the Sivers distributions come from COMPASS measurements on proton [10] and deuteron targets [13] (we treat deuteron as the incoherent sum of a proton and a neutron). The unpolarized distribution functions f_1^q are taken from the CTEQ5D global fit [38]. The unpolarized fragmentation functions are taken from the DSS parametrization [37]. Notice that in the DSS fit D_{1u}^+ is not assumed to be equal to $D_{1\bar{d}}^+$, but their difference is rather small. Thus, we identify $D_{1,\mathrm{fav}}$ with $(D_{1u}^+ + D_{1\bar{d}}^+)/2$ as given by DSS. In the DSS parametrization the factor N_s is found to be 0.83.

The normalization of the Sivers distributions is determined by the quantity $G = \pi M / 2\langle P_{h\perp}\rangle$. The values of $\langle P_{h\perp}\rangle$, measured by COMPASS, slightly depend on x, so that G ranges from 2.8 to 3.1.

We can now use Equations (31) and (32) to extract point-by-point the valence Sivers distributions from asymmetry data. The results are displayed in Figure 1. The error bars are the statistical uncertainties of the measured asymmetries. The x points correspond to different Q^2 values, ranging from 1.2 GeV^2 to 20 GeV^2, with an average value $\langle Q^2\rangle \approx$ 4 GeV^2.

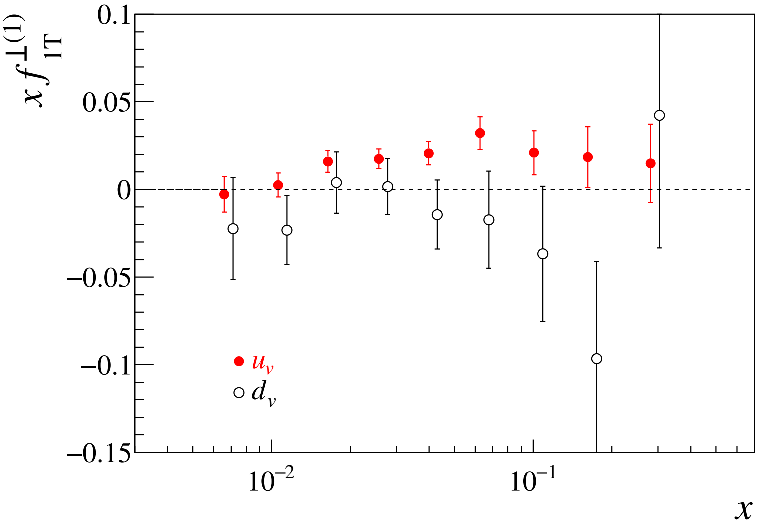

Figure 1. The first k_T^2 moments of the Sivers valence distributions, $xf_{1T}^{\perp(1)u_v}$ (red solid circles) and $xf_{1T}^{\perp(1)d_v}$ (black open circles).

The u_v Sivers distribution is determined more precisely than the d_v distribution, as the asymmetry measurements on the proton are considerably more accurate than the corresponding ones on the deuteron, in particular in the valence region (the COMPASS Collaboration has taken much less data on deuterons than on protons). Although affected by larger uncertainties, the d_v distribution appears to be negative. The COMPASS experiment has also provided data on kaon leptoproduction. These measurements have been analyzed in [20], where the resulting Sivers distributions were shown to be well compatible with those extracted from pion data and presented here.

We recall that the Sivers functions can be disentangled from the transverse momentum convolution and extracted with no need for the Gaussian model by considering the asymmetries weighted with $P_{h\perp}$ [33]. The COMPASS Collaboration has recently performed this analysis obtaining a set of Sivers distributions in agreement with those presented here [39].

4. Transversity Distributions from Collins Asymmetries

Let us now move to the point-by-point determination of transversity from the Collins asymmetry data.

Using again a Gaussian Ansatz for the transversity distribution and the Collins fragmentation function,

$$h_1(x,k_T^2,Q^2) = h_1(x,Q^2)\,\frac{e^{-k_T^2/\langle k_T^2\rangle}}{\pi\langle k_T^2\rangle_S}\,, \tag{33}$$

$$H_1^{\perp}(z,p_T^2,Q^2) = H_1^{\perp}(z,Q^2)\,\frac{e^{-p_T^2/\langle p_T^2\rangle}}{\pi\langle p_T^2\rangle}\,, \tag{34}$$

the Collins asymmetry (6) becomes [40]

$$A_C(x,z,Q^2) = G\,\frac{\sum_{q,\bar q} e_q^2 x h_1^q(x,Q^2) H_{1q}^{\perp(1/2)}(z,Q^2)}{\sum_{q,\bar q} e_q^2 x f_1^q(x,Q^2) D_{1q}(z,Q^2)}\,. \tag{35}$$

The "half-moment" of $H_1^{\perp}$ is defined as

$$H_1^{\perp(1/2)}(z, Q^2) \equiv \int \mathrm{d}^2\boldsymbol{p}_T \, \frac{p_T}{zM_h} \, H_1^{\perp}(z, p_T^2, Q^2)\,, \tag{36}$$

and in the Gaussian model is proportional to $H_1^{\perp}(z, Q^2)$, as defined in Equation (34):

$$H_1^{\perp(1/2)}(z, Q^2) = \frac{\sqrt{\pi \langle p_T^2 \rangle}}{2zM_h} \, H_1^{\perp}(z, Q^2)\,. \tag{37}$$

The factor G in Equation (35) is

$$G = \frac{1}{\sqrt{1 + z^2 \langle k_T^2 \rangle / \langle p_T^2 \rangle}}. \tag{38}$$

With the reasonable assumption $z^2 \langle k_T^2 \rangle / \langle p_T^2 \rangle \ll 1$, we can approximately set $G \simeq 1$.

Being interested in the extraction of the transversity distributions, we can integrate over z,

$$\widetilde{H}_1^{\perp(1/2)}(Q^2) = \int \mathrm{d}z \, H_1^{\perp(1/2)}(z, Q^2)\,, \quad \widetilde{D}_1(Q^2) = \int \mathrm{d}z \, D_1(z, Q^2)\,, \tag{39}$$

and write the integrated asymmetry as

$$A_C(x, Q^2) = \frac{\sum_{q,\bar{q}} e_q^2 x h_1^q(x, Q^2) \widetilde{H}_{1q}^{\perp(1/2)}(Q^2)}{\sum_{q,\bar{q}} e_q^2 x f_1^q(x, Q^2) \widetilde{D}_{1q}(Q^2)}. \tag{40}$$

The favored and unfavored fragmentation functions D_1 are the same as in Equations (15) and (16). The corresponding relations for $H_1^{\perp}$, based on isospin and flavor symmetries, are

$$H_{1,\mathrm{fav}}^{\perp} = H_{1u}^{\perp +} = H_{1d}^{\perp -} = H_{1\bar{u}}^{\perp -} = H_{1\bar{d}}^{\perp +} \tag{41}$$

$$H_{1,\mathrm{unf}}^{\perp} = H_{1u}^{\perp -} = H_{1d}^{\perp +} = H_{1\bar{u}}^{\perp +} = H_{1\bar{d}}^{\perp -}. \tag{42}$$

We assume $H_{1s}^{\perp} = H_{1\bar{s}}^{\perp} = 0$, as suggested by some models, and we ignore the c components of the distribution functions, which are negligible at the x, Q^2 values of interest here. The denominators of the asymmetries $\sum_{q,\bar{q}} e_q^2 x f_1^q \widetilde{D}_{1q}$ for a proton and a deuteron target (p, d) and for charged pions, multiplied by 9, are

$$p, \pi^+ : \quad x\,[4(f_1^u + \beta f_1^{\bar{u}}) + (\beta f_1^d + f_1^{\bar{d}}) + N\beta(f_1^s + f_1^{\bar{s}})]\,\widetilde{D}_{1,\mathrm{fav}} \equiv x f_p^+ \,\widetilde{D}_{1,\mathrm{fav}}, \tag{43}$$

$$d, \pi^+ : \quad x\,[(4+\beta)(f_1^u + f_1^d) + (1+4\beta)(f_1^{\bar{u}} + f_1^{\bar{d}}) + 2N\beta(f_1^s + f_1^{\bar{s}})]\,\widetilde{D}_{1,\mathrm{fav}} \equiv x f_d^+ \,\widetilde{D}_{1,\mathrm{fav}}, \tag{44}$$

$$p, \pi^- : \quad x\,[4(\beta f_1^u + f_1^{\bar{u}}) + (f_1^d + \beta f_1^{\bar{d}}) + N\beta(f_1^s + f_1^{\bar{s}})]\,\widetilde{D}_{1,\mathrm{fav}} \equiv x f_p^- \,\widetilde{D}_{1,\mathrm{fav}}, \tag{45}$$

$$d, \pi^- : \quad x\,[(1+4\beta)(f_1^u + f_1^d) + (4+\beta)(f_1^{\bar{u}} + f_1^{\bar{d}}) + 2N\beta(f_1^s + f_1^{\bar{s}})]\,\widetilde{D}_{1,\mathrm{fav}} \equiv x f_d^- \,\widetilde{D}_{1,\mathrm{fav}}, \tag{46}$$

where β is defined in Equation (22) and can be taken from standard parametrizations of fragmentation functions.

Similar expressions are obtained for the numerator of Equation (40), $\sum_{q,\bar{q}} e_q^2 x h_1^q \widetilde{H}_{1q}^{\perp(1/2)}$, with the replacements $\widetilde{D}_1 \to \widetilde{H}_1^{\perp}$, $f_1 \to h_1$, and $\beta \to \alpha$, where

$$\alpha(Q^2) \equiv \frac{\widetilde{H}_{1,\mathrm{unf}}^{\perp(1/2)}(Q^2)}{\widetilde{H}_{1,\mathrm{fav}}^{\perp(1/2)}(Q^2)}. \tag{47}$$

Introducing the analyzing power

$$a_P(Q^2) = \frac{\widetilde{H}_{1,\mathrm{fav}}^{\perp(1/2)}(Q^2)}{\widetilde{D}_{1,\mathrm{fav}}(Q^2)}\,, \tag{48}$$

we find for the proton target (for simplicity we drop the C of Collins)

$$A_p^+ = a_P \frac{4(h_1^u + \alpha h_1^{\bar{u}}) + (\alpha h_1^d + h_1^{\bar{d}})}{f_p^+}, \tag{49}$$

$$A_p^- = a_P \frac{4(\alpha h_1^u + h_1^{\bar{u}}) + (h_1^d + \alpha h_1^{\bar{d}})}{f_p^-}, \tag{50}$$

and for the deuteron target

$$A_d^+ = a_P \frac{(4+\alpha)(h_1^u + h_1^d) + (1+4\alpha)(h_1^{\bar{u}} + h_1^{\bar{d}})}{f_d^+}, \tag{51}$$

$$A_d^- = a_P \frac{(1+4\alpha)(h_1^u + h_1^d) + (4+\alpha)(h_1^{\bar{u}} + h_1^{\bar{d}})}{f_d^-}. \tag{52}$$

The combinations

$$f_p^+ A_p^+ - f_p^- A_p^- = a_P(1-\alpha)(4h_1^{u_v} - h_1^{d_v}) \tag{53}$$

$$f_d^+ A_d^+ - f_d^- A_d^- = a_P 3(1-\alpha)(h_1^{u_v} + h_1^{d_v}) \tag{54}$$

select the valence transversity distributions. From Equations (53) and (54), we get the valence distributions for u and d quarks, separately:

$$x h_1^{u_v} = \frac{1}{5}\frac{1}{a_P(1-\alpha)}\left[(x f_p^+ A_p^+ - x f_p^- A_p^-) + \frac{1}{3}(x f_d^+ A_d^+ - x f_d^- A_d^-)\right], \tag{55}$$

$$x h_1^{d_v} = \frac{1}{5}\frac{1}{a_P(1-\alpha)}\left[\frac{4}{3}(x f_d^+ A_d^+ - x f_d^- A_d^-) - (x f_p^+ A_p^+ - x f_p^- A_p^-)\right]. \tag{56}$$

The analyzing power $\tilde{a}_P^h$ is obtained from inclusive two-hadron production in electron–positron annihilation, $e^+ e^- \to h_1 h_2 X$, with the two hadrons in different hemispheres. In this process, the Collins effect is observed in the combination of the fragmenting processes of a quark and an antiquark, resulting in the product of two Collins functions.

The ratio of the unfavored to favored Collins function, Equation (47), is not constrained by the data, so we have to make some hypothesis. We assume the unfavored Collins function to be equal and opposite to the favored one,

$$H_{1,\mathrm{fav}}^{\perp(1/2)}(z, Q^2) = -H_{1,\mathrm{unf}}^{\perp(1/2)}(z, Q^2)\,, \tag{57}$$

that is, we set $\alpha(Q^2) = -1$. This assumption is suggested by the fact that the asymmetries for positive and negative pions are found to have approximately the same size but an opposite sign.

Using Equation (57), we find that the favored Collins function extracted from the Belle data [23] can be fitted as

$$H_{1,\mathrm{fav}}^{\perp(1/2)}(z, Q_B^2) = N z(1-z)^{\gamma}\, D_{1,\mathrm{fav}}(z, Q_B^2), \quad Q_B^2 = 110\ \mathrm{GeV}^2/c^2\,, \tag{58}$$

with $C = 0.46 \pm 0.03$ and $\gamma = 0.49 \pm 0.07$. The fragmentation functions from the Belle value of the momentum transfer $Q_B^2 = 110\ \mathrm{GeV}^2/c^2$ to the Q^2 values of COMPASS data. The evolution of $H_1^{\perp(1/2)}(z, Q^2)$ involves unknown twist-3 fragmentation functions and cannot be implemented. Therefore, we simply assume that the analyzing power is constant in Q^2. The value we obtain is $a_P = 0.122$.

Using the CTEQ5D unpolarized distribution functions [38] and the DSS unpolarized fragmentation functions [37], and the asymmetries measured by COMPASS into Equations (55) and (56), we find the valence transversity distributions plotted in Figure 2.

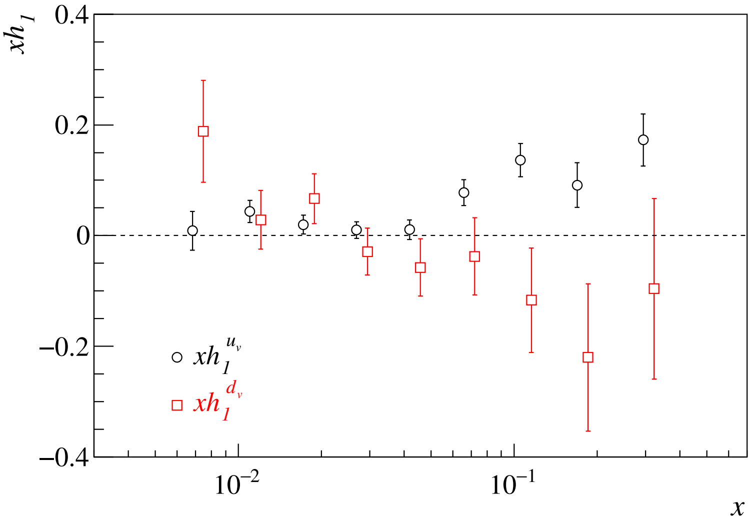

Figure 2. Valence transversity distributions. Black circles represent $xh_1^{u_v}$ and red squares represent $xh_1^{d_v}$.

The valence u quark transversity distribution is positive and well determined, while the d quark has about the same size but an opposite sign and considerably larger uncertainties.

We have checked the robustness of our results against different assumptions about the relation between the favored and the unfavored Collins function, and different hypotheses on the evolution of the fragmentation functions. The effects of all these changes are very small and negligible within the present uncertainties.

5. Transversity Distributions from Difference Asymmetries

Some information on the transversity distributions, with no need for an independent measurement of the Collins function, can be obtained from SIDIS by considering the so-called difference asymmetries, namely,

$$\mathcal{A}_{C,t} \equiv \frac{\sigma^+_{C,t} - \sigma^-_{C,t}}{\sigma^+_{0,t} + \sigma^-_{0,t}}. \tag{59}$$

When taking the ratios of the asymmetries on deuteron and proton, the Collins fragmentation functions cancel out:

$$\frac{\mathcal{A}_{C,d}}{\mathcal{A}_{C,p}} = 3\left[\frac{(4f_1^u + 4f_1^{\bar{u}} + f_1^d + f_1^{\bar{d}})(D_{1,\mathrm{fav}} + D_{1,\mathrm{unf}}) + 2(f_1^s + f_1^{\bar{s}})D_{1,s}}{5(f_1^u + f_1^d + f_1^{\bar{u}} + f_1^{\bar{d}})(D_{1,\mathrm{fav}} + D_{1,\mathrm{unf}}) + 4(f_1^s + f_1^{\bar{s}})D_{1,s}}\right]\frac{h_1^{u_v} + h_1^{d_v}}{4h_1^{u_v} - h_1^{d_v}}, \tag{60}$$

and the only unknowns are the transversity distributions. Thus, by measuring $\mathcal{A}_C$ on p and d, one obtains the ratio $h_1^{d_v}/h_1^{u_v}$ in terms of known quantities.

The procedure for calculating the difference asymmetries from COMPASS data is described in [31]. The quantities $(h_1^{u_v} + h_1^{d_v})/(4h_1^{u_v} - h_1^{d_v})$ have been determined by using Equation (60) and standard parametrizations for the unpolarized parton distributions [38] and fragmentation functions [37]. Finally, from the quantities $(h_1^{u_v} + h_1^{d_v})/(4h_1^{u_v} - h_1^{d_v})$, the valence ratio $h_1^{d_v}/h_1^{u_v}$ is obtained. This ratio is plotted in Figure 3 (solid circles) for the higher x bins (centered at 0.062, 0.100, 0.161, 0.280). The points at smaller x have much too large uncertainties, as the proton asymmetries in that region are compatible with zero, and have not been plotted. As expected, the uncertainties are large, but the results agree with those obtained by the Collins asymmetry analysis presented in the previous Section, as discussed in [31]. Averaging over the four points, one finds the ratio $h_1^{d_v}/h_1^{u_v}$ to be -0.82 ± 0.43 in the x range spanned by the measurement. The large uncertainty is mainly due to the large uncertainty of the deuteron data.

It is interesting however to apply this method using the Collins asymmetry data of the 2010 proton run [9] and the projections for the new measurements which COMPASS plans to perform in 2021 and 2022 on a transversely polarized deuteron target [41]. The new run will balance the world data on proton and deuteron, making isospin separation much easier and more precise. The projections for the ratio $h_1^{d_v}/h_1^{u_v}$ are also plotted in Figure 3 (open circles). As one can see, the gain of accuracy is impressive: the uncertainty of the weighted mean of the four points goes from ±0.43 to ±0.11.

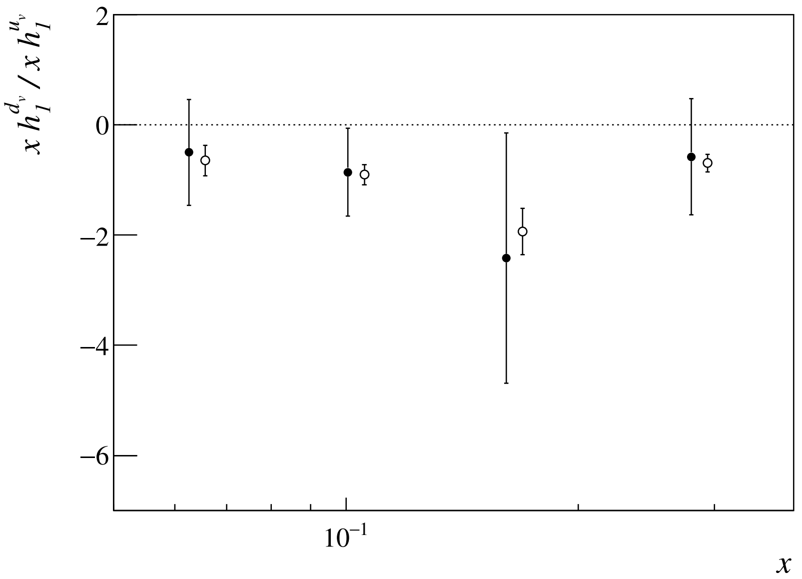

Figure 3. Ratio $h_1^{d_v}/h_1^{u_v}$ from the difference asymmetries. Solid circles: determination from existing measurements. Open circles: projection for future COMPASS run.

6. Conclusions

We determined in a simple and direct way the Sivers distributions and transversity distributions of valence quarks from the COMPASS measurements of charged pion leptoproduction on proton and deuteron targets. Taking advantage of the variety of processes investigated by the COMPASS experiment with the same kinematics, we extracted the quark distributions point by point by combining only observable quantities on the basis of isospin symmetry. In order to factorize the distribution functions from the fragmentation functions we used a Gaussian model for the transverse momentum dependence, but the final results do not depend on the Gaussian widths. Thus, our approach does not involve any free parameter.

Both the transversity and the Sivers u_v and d_v distributions obtained in our analysis are in good agreement with the results of previous phenomenological analyses, which fitted the data with a given functional form for the distributions in x.

In general, while the u_v distributions are determined quite accurately, the d_v distributions are more uncertain. A better knowledge of the d_v sector would require more data with a deuteron target. This is one of the goals of the next COMPASS run.

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Abbreviations

The following abbreviations are used in this paper:

DSS	de Florian, Sassot, Stratmann
QCD	Quantum chromodynamics
SIDIS	Semi-inclusive deep inelastic scattering

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