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No Relation for Wigner's Friend

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Abstract
We argue that Wigner's friend thought experiment does not support observer dependence of quantum states. An analysis in terms of history vectors suggests that quantum collapse is to be understood as collapse of histories rather than collapse of states.

Keywords Quantum mechanics · Quantum measurement · History approach

The core of subjective or relational interpretations of QM (see for ex. [1–4]) is the alleged *observer dependence* of quantum states, meaning that different observers give different descriptions of the same quantum system. This is claimed by some authors to be a consequence of the usual quantum mechanical rules, when these are applied to "third person" situations, as in Wigner's friend thought experiment [5]. In the present note we argue that the usual textbook QM description of the experiment does not lead to an essential observer dependence.

Let us briefly recall the setting: an observer F and a system S are contained in a perfectly isolated laboratory. A second observer W stays outside of the laboratory. The initial state of S at $t = t_0$ is prepared in the superposition

$$\alpha|0\rangle + \beta|1\rangle \tag{1}$$

At $t = t_1$ the observer F effects a measurement on S in the $|0\rangle$, $|1\rangle$ basis [1]. According to the usual QM rules, F obtains the result 0 with probability $|\alpha|^2$ and the result 1 with probability $|\beta|^2$. Suppose for example that F obtains 1: "for him" the initial state $|\psi\rangle$ collapses into the state $|1\rangle$:

$$\alpha|0\rangle + \beta|1\rangle \longrightarrow |1\rangle \tag{2}$$

[1] i.e. a measurement of an observable having $|0\rangle$ and $|1\rangle$ as eigenvectors.

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How does the external observer W describe the evolution, from t_0 to t_1, of the composite system $S + F$? According to QM, in absence of any interaction with the environment, the system $S+F$ evolves unitarily. The initial state of the composite system is the tensor product $(\alpha|0\rangle + \beta|1\rangle) \otimes |init\rangle$, where $|init\rangle$ is the initial state of the observer F. A measurement by F on S is modeled by the $S + F$ (unitary) evolution

$$|0\rangle \otimes |init\rangle \longrightarrow |0\rangle \otimes |0_F\rangle \tag{3}$$

$$|1\rangle \otimes |init\rangle \longrightarrow |1\rangle \otimes |1_F\rangle \tag{4}$$

where $|0_F\rangle$ (resp. $|1_F\rangle$) is the state of F after he has obtained 0 (resp. 1), or equivalently the state of a notebook where F writes down the result of the measurement.

The crucial point is that the state of S becomes correlated to the state of F. Thus if the initial state of $S+F$ is $(\alpha|0\rangle+\beta|1\rangle)\otimes|init\rangle$, by linearity the system evolves from t_0 to t_1 as

$$(\alpha|0\rangle + \beta|1\rangle) \otimes |init\rangle \longrightarrow \alpha|0\rangle \otimes |0_F\rangle + \beta|1\rangle \otimes |1_F\rangle \tag{5}$$

i.e. in an *entangled* state of the $S + F$ system. All predictions on future measurements by W must be computed using this entangled state. In particular W could observe interference effects in $S + F$, since the state at t_1 is a superposition.

The observer F, on the contrary, "sees" no superposition. In fact he cannot observe $S + F$, since he is *part* of the composite system. Only S is accessible to him, and after his measurement at $t = t_1$, F describes the state of S as one of the two states $|0\rangle$, $|1\rangle$.

So far no difficulty arises. W and F are describing different physical systems: no wonder that their descriptions differ.

But suppose that W focuses his attention on the subsystem S. He can make measurements on S, since S is a subsystem of $S + F$ and therefore accessible to him. How does W describe the state of S after t_1, before making any measurement himself? Is this description different from the one made by F? Here the question acquires relevance, since both observers refer to the same system S. In fact it lies at the heart of the *measurement problem*[2] in QM, highlighting the clash between unitary evolution of isolated systems and nonunitary projection due to measurement.

The answer of textbook QM involves the reduced density operator $\rho^{(S)}$ for system S. The density operator $\rho^{(S+F)}$ for $S + F$ at $t = t_1$ is given by:

$$\rho^{(S+F)} = (\alpha|0\rangle \otimes |0_F\rangle + \beta|1\rangle \otimes |1_F\rangle)(\alpha^\star\langle 0| \otimes \langle 0_F| + \beta^\star\langle 1| \otimes \langle 1_F|) \tag{6}$$

The reduced density operator $\rho^{(S)}$ is defined by tracing on the subsystem F. Assuming $|0_F\rangle$ and $|1_F\rangle$ to be orthogonal[3] we find

$$\rho^{(S)} = Tr_F(\rho^{(S+F)}) = |\alpha|^2|0\rangle\langle 0| + |\beta|^2|1\rangle\langle 1| \tag{7}$$

Thus W describes S to be in a *mixed* state, and precisely in the state $|0\rangle$ with probability $|\alpha|^2$ and in the state $|1\rangle$ with probability $|\beta|^2$. Note that this mixed state is radically different from the superposed (pure) state $\alpha|0\rangle + \beta|1\rangle$. It is the description of system S given by W, after a measurement has been performed by F, but without acquisition of the result (since $S + F$ is isolated, F cannot communicate with W).

[2]For an introduction to this subject, see for example the book by A. Peres [6].

[3]since they are eigenvectors of a "position of the pointer" observable.

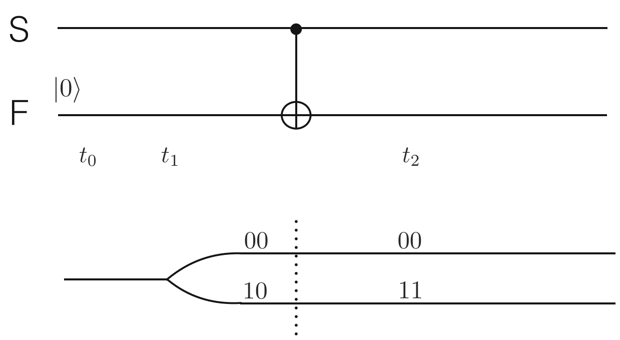

Fig. 1 Circuit describing the measurement of F on S, as seen by W, and the corresponding history diagram

In summary, the F and W descriptions of the state of S at t_0 and t_1 can be represented as follows:

$$\begin{array}{lll} & t_0 & \qquad\qquad t_1 \\ F: & \alpha|0\rangle+\beta|1\rangle \longrightarrow & |0\rangle \; or \; |1\rangle \;\; with\ probabilities\ |\alpha|^2 \; or \; |\beta|^2 \\ W: & \alpha|0\rangle+\beta|1\rangle \longrightarrow & mixed\ state\ (statistical\ ensemble\ of\ |0\rangle\ and\ |1\rangle) \\ & & with\ probabilities\ |\alpha|^2\ and\ |\beta|^2 \end{array}$$

The two descriptions do not coincide, because F knows the result of his measurement, whereas W does not. This is the only reason of the difference, due to ignorance of W, and reflected in the statistical ensemble he must use to describe S at t_1. But both F and W do agree that the initial state of S has collapsed in one of the two basis states $|0\rangle$ or $|1\rangle$, even if only F knows which one. This difference has no profound significance: F would describe S in exactly the same way as W, if he chose to measure S (producing collapse) without registering the result. Moreover the difference is not due to quantum mechanical effects, but only to incomplete information of one of the two observers. The same difference arises in a classical world, where S is for example a coin under a cup, and F "measures" it by removing the cup and registering whether S shows head or tails. If W cannot communicate with F ($S+F$ being isolated), the information on the result of F's measurement is not available, and he can only describe the state of S statistically, with a 50-50 probability of heads or tails. But this could hardly be a motivation to develop a relational interpretation of classical mechanics[4].

To acquire the missing information, W must interact with the $S+F$ system, for example with a measurement on S, or on F, or on the whole $S+F$. If he interacts with the whole of $S+F$, he will be able to detect that the composite system is in the superposition state (5). But if he limits his measurements to the subsystems S or F, these are not in superposed (pure) states, but in mixed states, and no interference effects can be detected.

Suppose that W measures S and obtains the result 0. The $S+F$ system collapses:

$$\alpha|0\rangle \otimes |0_F\rangle + \beta|1\rangle \otimes |1_F\rangle \longrightarrow |0\rangle \otimes |0_F\rangle \tag{8}$$

and by a subsequent measure on F, W can verify that 0 was indeed the result registered by F. Thus no contradiction can arise between the results of F and W.

[4]besides the obvious system of reference dependence of position, velocities etc.

But suppose that F has obtained 1 in his measurement on S. How is it possible that W has still a probability $|\alpha|^2$ of obtaining 0 on the same system, after the measurement of F? This "paradox" is resolved operationally, since Wigner's measurement will never reveal any disagreement with his friend's measurement.

In fact this situation illustrates the "history collapse" effect of measurements, discussed in [7, 8]. Consider the circuit version of Wigner's friend experiment in its simplest version, where S is a single qubit system and F is a single qubit "observer". The evolutions (3), (4) can be realized just with a CNOT gate[5] where S is the control and F is the target qubit, and choosing $|init\rangle = |0\rangle$:

As discussed in [8], a quantum system can be described by a *history vector* $|\Psi\rangle$ that encodes its whole time evolution:

$$|\Psi\rangle = \sum_{\gamma} A(\gamma)|\gamma\rangle \tag{9}$$

where γ are histories with nonvanishing amplitudes. A short summary follows, where we recall the definition of histories and corresponding amplitudes.

By *history* we mean a sequence of possible measurement results, at discrete times $t_1, t_2, ...$ (time discretization is assumed for simplicity). The measured observables can be different at each t_i. With usual Born rules we can compute the probability $p(\psi, \gamma)$ of obtaining a particular sequence $\gamma = \gamma_1, \gamma_2, ...$ of results at times $t_1, t_2, ...$, if the system starts in state $|\psi\rangle$. We find

$$p(\psi, \gamma) = Tr(C_{\psi,\gamma} C^{\dagger}_{\psi,\gamma}) \tag{10}$$

where the chain operator $C_{\psi,\gamma}$ is defined as

$$C_{\psi,\gamma} = P_{\gamma_n} U(t_n, t_{n-1})\ P_{\gamma_{n-1}}\ U(t_{n-1}, t_{n-2}) \cdots P_{\gamma_1}\ U(t_1, t_0) P_{\psi} \tag{11}$$

with $P_{\psi} = |\psi\rangle\langle\psi|$; P_{γ_i} are projectors on eigensubspaces of observables corresponding to the eigenvalues γ_i and $U(t_{i+1}, t_i)$ is the unitary evolution operator between times t_i and t_{i+1}. If γ_n is nondegenerate, the chain operator can be written as:

$$C_{\psi,\gamma} = |\gamma_n\rangle A(\psi, \gamma) \langle\psi| \tag{12}$$

the *amplitude* $A(\psi, \gamma)$ being defined by

$$A(\psi, \gamma) = \langle\gamma_n| P_{\gamma_{n-1}}\ U(t_{n-1}, t_{n-2}) \cdots P_{\gamma_1}\ U(t_1, t_0) |\psi\rangle \tag{13}$$

and the probability (10) is just the square modulus of the amplitude (13). The generalization to a degenerate eigenvalue γ_n is straightforward (see [8]). The vectors $|\gamma\rangle$ in (9) are in 1-1 correspondence with all possible histories $\gamma_1, ...\gamma_n$, and assumed to form an orthonormal basis. Probabilities for (any sequences of) measurements can be easily computed using appropriate projections of the history vector (9) [8].

We apply now this formalism to the circuit that models Wigner's friend experiment, given in Fig. 1. Note that it represents the system $S + F$ accessible only to W: the measurement of F on the subsystem S is represented by the unitary gate $CNOT$, and not by a projection.

[5] The CNOT is a two-qubit gate acting on the computational basis as as $|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$, $|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle$, $|1\rangle|0\rangle \rightarrow |1\rangle|1\rangle$, $|1\rangle|1\rangle \rightarrow |1\rangle|0\rangle$, the first qubit being the *control* and the second being the *target*.

Starting from an initial state $|\psi\rangle|0\rangle$ the history vector contains only two histories, since the only nonvanishing amplitudes are

$$A(00, 00) = \langle 00|CNOT|00\rangle\langle 00|\psi, 0\rangle = \alpha \tag{14}$$
$$A(10, 11) = \langle 11|CNOT|10\rangle\langle 10|\psi, 0\rangle = \beta \tag{15}$$

Suppose now that W at time t_2 measures F and obtains 1. This result is compatible only with the history (10, 11). Its probability is given by the square modulus of the corresponding amplitude, i.e. $|\beta|^2$. The history vector collapses to

$$|\Psi\rangle = |10, 11\rangle \tag{16}$$

The measurement is performed at time t_2, but the history involves also values at time $t_1 < t_2$: in this sense the collapse "modifies the past". Even if at t_1 F had obtained 0 in his measurement of S, the measurement of W "undoes" this event and realigns it to agree with his outcome at $t = t_2$. This "undoing" of past events has been discussed by various authors (see for ex. [9]), and is really a matter of *interpretation*, since there is no possibility of "checking" the undoing. However note that history collapse is *not* a matter of interpretation, if we require simultaneity of collapse in entangled spacelike separated systems to be valid in every reference frame [7].

In conclusion, at first sight it may appear that Wigner (W) and his friend (F) give different descriptions of F's measurement of S. In particular F's description involves a collapse, whereas W describes a unitary evolution. W could detect interference effects in measurements on $S + F$, whereas F sees no such effects after his measurement on S. But this difference is only due to W and F considering *different* physical systems: W describes $S + F$, whereas F can only describe S. If we compare instead their descriptions *of the same system* S, the difference essentially vanishes as discussed above. The only residual difference is due to lack of information, *not to quantum mechanical effects*, and in our opinion does not motivate any "relational" or subjective interpretation of quantum states.

Note in recent times Gedanken experiments [10, 11] extending Wigner's friend setup have been proposed to probe the issue of observer independence. Real experiments have been carried out [12, 13] in 2019, showing violation of Bell-type inequalities derived by assuming free-choice, locality and observer independence in [12], and absoluteness of observed events (AOE), no-superdeterminism and locality in [13]. However care is necessary to uncover all assumptions made in real or Gedanken experiments. For example the analysis in ref. Frauchiger and Renner [10] has been criticized by a number of authors [14–16], and inconsistences have been pointed out. In [13] the assumption AOE includes marginal probability relations that are in general violated in QM. It is therefore not clear to us that these experiments could directly support observer independence of quantum states or events.

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