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ORIGINAL RESEARCH

# Inequality of Opportunity and Variance Decomposition: A Methodological Insight

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## Abstract

This paper introduces a new variance-based measure of inequality of opportunity (EOp) grounded in a decomposition of the Coefficient of Variation (CV). Our approach disentangles overall inequality into a *between-group* component driven by circumstances beyond individual control (e.g., gender, family background, or region of birth) and a *within-group* component which captures effort and idiosyncratic heterogeneity. Within this framework, we characterize three increasingly stringent EOp criteria, capturing equality in group means, equality in within-group dispersion, and their joint validity. Using EU-SILC income data, we illustrate the conceptual and empirical relevance of the method data and highlight substantial cross-country heterogeneity in both average outcomes and within-type variability across European countries. The results show that mean-based evaluations alone may fail to capture important differences in outcome dispersion across circumstance groups, underscoring the value of a variance-based approach for the analysis of equality of opportunity.


**Keywords** Equality of opportunity · Coefficient of variation · Decomposition by population subgroups · Mean · Variance

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## 1 Introduction

As a fundamental tension between fairness and inequality, Equality of Opportunity (EOp) represents one of the cornerstones of modern economic and social research. The EOp concept captures the duality of real-world policy which requires, on one hand, rewarding individual responsibility and effort, and on the other hand compensating for disadvantages beyond one's control. Extending the ideas of Roemer (1998), EOp theory explicitly distinguishes between inequalities ascribed to *circumstances*, such as family background, gender, and country of birth, and inequalities from personal responsibility, *effort*. Essentially, societies will accept and may even endorse some inequality of effort type as 'fair' reward, but any inequality attributable to circumstance is 'unfair' and should be handled in policy. This distinction reflects the two fundamental principles of EOp: *compensation* – neutralizing outcome differences caused by circumstances – and *reward* – preserving outcome differences due to effort (Fleurbaey & Peragine, 2013). The imperative to equalize opportunities has motivated a large and growing literature on measuring and decomposing inequality of opportunity across countries and over time. Many methodological approaches have been proposed to quantify inequality of opportunity (e.g., Lefranc et al., 2009; Checchi and Peragine, 2010; Ferreira and Gignoux, 2011), typically finding substantial cross-country heterogeneity in the share of total inequality attributable to circumstances. For example, Marrero and Rodriguez (2013) report that very high inequality of opportunity can impede economic growth. Taken together, these contributions underscore both the empirical importance of addressing inequalities rooted in circumstances beyond individual control and the responsibility factor, emphasizing the critical need for policy-actionable measurement tools.

Building upon previous methodological contributions in inequality and opportunity measurement – such as Dagum (1997); Deutsch and Silber (1997), and recent analyses of the interplay between effort and circumstances (e.g., Abatemarco, 2010) – this paper develops a unified and robust methodological framework to measure and interpret inequality of opportunity. We specifically propose a decomposition of inequality based on the Coefficient of Variation (CV), an inequality measure that is empirically flexible, and normatively interpretable in the context of equality of opportunity analysis.

The first reason for selecting the CV is its mathematical simplicity and clear interpretability. As a scale-invariant measure of dispersion, the CV permits meaningful comparisons of inequality across different populations and time periods. Moreover, the CV captures subtle differences in the distribution of outcomes, making it particularly sensitive to structural factors relevant to the EOp framework. Second, the decomposition structure of the CV naturally lends itself to identifying between-group (circumstance-driven) and within-group (effort-driven) components of inequality. Indeed, the squared CV belongs to the family of additively decomposable Generalized Entropy measures (with sensitivity parameter 2), thus allowing a precise partitioning of total inequality into between-group and within-group terms. The CV-based decomposition thus provides a measure that is close to the ideal type for analyzing inequality of opportunity, since it directly disentangles the inequality due to circumstances from that due to individual effort.

Our contribution is both theoretical and empirical. First, we establish a framework for decomposing the CV into within-group and between-group components, demonstrating its applicability to well-known normative definitions of equality of opportunity. In particular, we explicitly link the ex-ante definition (focusing on mean outcomes across circumstance groups) with the ex-post interpretation (focusing on equality conditional on relative effort quantiles), providing clear theoretical foundations for both approaches. This unified setting allows us to bridge the gap between these two prominent normative perspectives and clarify their empirical implementation (Fleurbaey & Peragine, 2013; Ramos & Van de Gaer, 2016), showing how each of them encompasses different ways in which unfair inequality is present. Second, we critically discuss the advantages and limitations of focusing solely on means versus incorporating distributional differences and offer practical guidance for empirical analyses. We also demonstrate empirically, using EU-SILC data (2005, 2011, and 2019), how our decomposition method facilitates some policy insights by clearly distinguishing inequality arising from circumstances and inequality resulting from individual effort.

By partitioning the population according to observable circumstances, we are able to isolate the share of disparity that may be attributed to factors that are beyond the control of individuals. For this reason, in the ex-ante approach, the within-group component of inequality is considered to reflect 'legitimate' disparities due to effort or other personal decisions, while the between-group component of inequality is interpreted as the 'unfair' aspect that policy should try to minimize or eradicate. By contrast, the ex-post perspective posits that individuals exerting the same relative effort (at the same within-type percentile) should achieve identical outcome distributions, regardless of their circumstances.

Consistent with the ethos of opportunity egalitarianism, our mean-based decomposition allows policymakers to pinpoint and target the outcome differences caused by pre-determined circumstances, while treating the remaining within-group variation as inequality arising from legitimate sources, primarily effort (and, in practice, certain preferences), although whether innate talent or deep preferences are matters of luck is debated (e.g., Arneson, 1989).[1] In this paper, we follow standard empirical practice by focusing only on clearly exogenous circumstances (gender, migrant status, parental education, parental occupation) and do not include talent or preferences, precisely due to normative controversy and data limitations. Accordingly, our definitions of types are conservative, and our EOp estimates represent lower bounds on unfair inequality. This approach is not only easy to implement and interpret, but also directly connected to the ethical principle that individuals should not be judged by factors for which they are not responsible. We provide a comprehensive methodological contribution to the literatures on inequality measurement and equality of opportunity (e.g., Shorrocks, 1982; Mookherjee and Shorrocks, 1982; Fields, 2003; Mussard et al., 2003; Cowell and Fiorio, 2011).

We also examine whether choosing a different inequality index (e.g., CV versus Theil or MLD) can influence the ranking of countries or policy scenarios in terms of equality of opportunity. Lastly, we discuss the interpretation of these decompositions in terms of their theoretical foundations and their policy implications for addressing inequality. One of the objectives of this study was to demonstrate, using empirical observations, that movements in the distribution can be decomposed into the fair and unfair components of inequality – a differentiation which has important implications for the design of equity enhancing policies.

Our empirical analysis on EU-SILC microdata for 16 European countries delivers three main results. First, total dispersion $C(F)$ differs markedly across countries and moves only moderately over time. The Baltics and Poland (e.g., Lithuania, Latvia, Estonia) sit at the top

[1] Indeed, some theorists in the luck-egalitarian tradition as Arneson (1989) argue that innate talents and deep preferences are themselves largely the product of brute luck or social background, and therefore should not be automatically attributed to individual responsibility.

of the cross-country distribution of $C(F)$, while Sweden, Belgium, Slovenia, and Austria lie at the bottom. Over time, most countries display a mild hump around the middle wave and a stabilization by 2019. *Second*, the ex-ante share of inequality attributable to circumstances is relatively stable and, in most cases, modest; however, there is clear heterogeneity in levels: countries with lower $C(F)$ tend to have a larger fraction of it concentrated in mean differences across types, whereas in high-dispersion countries a larger portion arises within types. *Third*, variance-based ex-post diagnostics reveal patterns that are not visible from means alone. In particular, ex-post equality of opportunity generally declines or remains contained in several countries (e.g., France), but rises or rebounds in others (e.g., Austria and Sweden), indicating growing heterogeneity in the outcome risks borne by equal-effort individuals across circumstance groups.

The remainder of the paper is organized as follows. Section 2 reviews the relevant literature. Section 3 sets out our mean-based decomposition methodology and clarifies its link to normative notions of equality of opportunity. Section 4 develops a micro-based model that jointly rationalizes the ex-ante and ex-post perspectives, while Section 5 formalizes the normative definitions of Equality of Opportunity – $EOp1$, $EOp2$, and $EOp3$. Section 6 describes the EU-SILC dataset and the sampling strategy. Empirical results, together with their policy implications, are presented in Section 7. Finally, Section 8 concludes and outlines directions for future research.

## 2 Literature Review

In the last decades, Equality of Opportunity (EOp) has emerged as an interesting analytical framework to understand and tackle inequality. Starting from Roemer's influential paper (Roemer, 1998) and the conceptual differentiation between *circumstances* and *effort*, the literature has widely investigated the normative implications of this distinction, both theoretically and empirically (see, e.g., Roemer and Trannoy, 2016; Ramos and Van de Gaer, 2016; Ferreira and Peragine, 2015). The methodologies to measure inequality of opportunity have developed into two main approaches: the *ex-ante* and *ex-post* differentiation (Fleurbaey & Peragine, 2013). The 'ex-ante' approach advocated in particular by Van de Gaer (1993) examines groups that are defined by circumstance and checks for mean outcomes equality across these groups (Lefranc et al., 2009; Checchi & Peragine, 2010). The *ex-post* perspective, in contrast, refers to eliminating inequalities among individuals exerting equal effort (Roemer, 1998; Pistolesi, 2009). Both perspectives reveal different aspects of the inequality of opportunity, but they are quite different not only in terms of how to measure inequality of opportunity but also regarding their normative implications (Fleurbaey & Peragine, 2013).

Parametric regression approaches are a common empirical method for quantifying EOp. Bourguignon et al. (2007) developed influential regression based decompositions to estimate circumstances inequality in Brazil. In the same vein, Ferreira and Gignoux (2011) introduced a series of parametric indices of opportunity inequality which are both theoretically grounded and robust, and have been successfully used by many researchers in comparative international studies (Brunori et al., 2013; Hufe et al., 2018). These studies typically report high levels of heterogeneity in the share of inequality explained by circumstances,which generally falls between 20% and 50% depending on the context (Ferreira & Gignoux, 2011; Marrero & Rodriguez, 2013).

Besides parametric approaches, we have seen the rise of non-parametric decompositions. Pioneering work by Shorrocks (1982) and Shorrocks (1984) developed important methods for a decomposition of inequality in within- and between-group terms. Dagum (1997) and Deutsch and Silber (1997) have since refined these methods for the subgroup inequality decompositions. More recently, Abatemarco (2010) generalized such methods directly to the EOp setting, measuring it as between-group inequality in circumstances. When it comes to policy analyses, non-parametric decompositions are particularly intriguing because of their natural interpretation and their robustness to specification assumptions.

In terms of indices used, the literature predominantly employs the Gini coefficient (Dagum, 1997) and Theil indices (Fields, 2003). Although widely used, the Gini coefficient cannot be decomposed in an additive manner due to the overlapping terms which restricts its application in comprehensive inequality breakdowns (Cowell & Fiorio, 2011). Theil indices, on the other hand, are decomposable but sensitive to distributional tails (Mussard et al., 2003). One of the main advantages of mean-based decomposition lies in its simplicity and relevance to policy. This is a pathway to isolating between-group disparities, a direct lens through which to examine systemic inequalities that are the result of events beyond individual control. Mookherjee and Shorrocks (1982) highlighted that such decompositions can thus help to analyze the relative importance of structural versus individual effort in driving inequality. This is similar to equality of opportunity, aiming to equalize the playing field by correcting differences based on circumstances.[2]

Despite the richness of this literature, the Coefficient of Variation (CV) is still one of the less widespread measures, although it is well suited to the analysis of inequality. The CV is a unit-free measure that can be compared among groups with different average levels and it is one of the Generalized Entropy class (parameter 2) and hence it possesses the property of exact additive decomposability (Sorensen, 2002). These properties render the CV especially appropriate to decompose between-group (circumstance-driven) and within-group (effort-driven) inequality, which are in line with the ex-ante equality of opportunity concept (Checchi & Peragine, 2010; Van de Gaer, 1993). Furthermore, the coefficient of variation simply represents the total variance on a normalized (scale-invariant) scale rather than a conceptually distinct measure. Accordingly, our empirical approach is fundamentally variance-based; the CV merely rescales variance in a way that facilitates direct comparisons across groups and remains consistent with the Generalized Entropy family.

Our work directly addresses this methodological gap. We offer a simple and coherent approach to the measurement of inequality of opportunity according to CV, where the roles of circumstances and effort are clearly separated. Unlike standard parametric methods (Ferreira & Gignoux, 2011; Bourguignon et al., 2007), our non-parametric CV-based decomposition avoids specification biases and provides an intuitive measure of between-group inequality. In addition, when compared to other decomposable indices, such as the MLD, the CV offers a higher degree of robustness to differences in scale and distributional tails. By clearly delineating circumstance-driven inequalities, it offers a transparent benchmark for equity-enhancing interventions, reinforcing the moral imperative embedded in opportunity-egalitarian policy frameworks (Roemer, 1998; Pignataro, 2012).

[2] However mean-based decomposition approach also has limitations. Considering only differences in means does not account for the benefits of within-group variability that could overlook group differences (Cowell, 1980). Similarly, Foster (1983) pointed out that methods based just on means may fail to capture the distributional aspects of inequality when the distributions are overlapping across subpopulations.

## 3 A Mean-based Approach

This section offers a complete derivation and explanation of the inequality of opportunity (EOp) specific variance-based decomposition of the Coefficient of Variation (CV). Rather, we fully decompose total inequality into two specific types of inequality: (i) *between-group inequality*, which results from differences in outcomes that are due to difference among groups (such as socio-economic status or gender), and (ii) *within-group inequality*, which represents variations attributable directly to other residual factors within each group. Here, we define and justify a robust and complete interpretation of Pearson's correlation ratio $\eta^2$ as a non-parametric alternative to the regression-based $R^2$ typically employed in empirical analyses. In addition, we highlight the conceptual and empirical benefits to using the CV instead of alternative decomposable indices like Theil or mean-log-deviation (MLD), and a comparison to existing methodologies, e.g., Checchi and Peragine (2010); Ferreira and Gignoux (2011).

Given a differentiable distribution function $F$ composed of $m$ distinct subpopulations, we can express $F$ as a weighted sum of these subpopulation distribution functions:

$$F = \sum_{i=1}^{m} f_i F_i,$$

where $f_i$ represents the proportion of the $i$-th subpopulation, and $F_i$ denotes the differentiable distribution function of the $i$-th subpopulation. Let $Y$ be the outcome variable with distribution $F$; denote its mean by $\mu = \mathbb{E}[Y]$ and its variance by $\mathrm{Var}(Y)$. Similarly, for each subgroup $C_i$ (with distribution $F_i$), let $\mu_i = \mathbb{E}[Y \mid C_i]$ and $\mathrm{Var}(Y_i)$ be the mean and variance of outcomes for individuals in group *i*. The variance of the overall distribution $F$ can be decomposed into two components: *i)* the *within-group* variance accounts for the variability within each subpopulation.

The *within-group* variance (i.e. the information spread within groups) is obtained by summing the variances of the subpopulation weighted by their sample sizes, while the *between-groups* variance depicts how different subpopulations means $\mu_i$ are compared to average population mean $\mu$. This reflects the average deviation of each subpopulation mean from the overall mean, weighted by their respective proportion in the total population. Formally, this decomposition is written as:

$$\mathrm{Var}(Y) = \sum_{i=1}^{m} f_i \mathrm{Var}(Y_i) + \sum_{i=1}^{m} f_i (\mu_i - \mu)^2 = \mathrm{Var}_{within} + \mathrm{Var}_{between}, \quad (1)$$

where the first term on the right side is the within-group variance, while the second term is between-group variance. This decomposition is especially beneficial for the conduct of statistical analyses featuring mixture distributions, as it enables researchers to disentangle the sources of variability in the data. Knowledge of within-group and between-group variances helps in determining if subpopulations are homogenous or heterogeneous in the overall distribution.

To express this in terms of the Coefficient of Variation (CV), we divide both sides by $\mu^2$:

$$\frac{\mathrm{Var}(Y)}{\mu^2} = \sum_{i=1}^{m} f_i \, \frac{\mu_i^2 \, \mathrm{Var}(Y_i)}{\mu^2 \mu_i^2} \; + \; \sum_{i=1}^{m} f_i \, \frac{(\mu_i - \mu)^2}{\mu^2}.$$

with $\mu_i^2$ appearing in both the numerator and the denominator of the first term. This adjustment allows us to express each component relative to the squared means, thereby standardizing the terms and facilitating the subsequent steps in our analysis.

Recognizing that the CV can be expressed as

$$C(F) = \frac{\mathrm{Var}(Y)}{\mu^2}, \qquad C(F_i) = \frac{\mathrm{Var}(Y_i)}{\mu_i^2},$$

we can rewrite the equation as:

$$C(F) = \sum_{i=1}^{m} f_i C(F_i) \left(\frac{\mu_i}{\mu}\right)^2 + \sum_{i=1}^{m} f_i \left(\frac{\mu_i - \mu}{\mu}\right)^2.$$

Letting $\alpha_i = \frac{\mu_i}{\mu}$, the equation simplifies to:

$$C(F) = \sum_{i=1}^{m} f_i C(F_i) \alpha_i^2 + \sum_{i=1}^{m} f_i (\alpha_i - 1)^2, \tag{2}$$

Expanding $(\alpha_i - 1)^2$, we get:

$$(\alpha_i - 1)^2 = \alpha_i^2 - 2\alpha_i + 1.$$

Substituting back, we have:

$$C(F) = \sum_{i=1}^{m} f_i C(F_i) \alpha_i^2 + \sum_{i=1}^{m} f_i (\alpha_i^2 - 2\alpha_i + 1).$$

Distributing $f_i$ and combining terms:

$$C(F) = \sum_{i=1}^{m} f_i C(F_i) \alpha_i^2 + \sum_{i=1}^{m} f_i \alpha_i^2 - 2 \sum_{i=1}^{m} f_i \alpha_i + \sum_{i=1}^{m} f_i.$$

Furthermore, considering that $\sum_{i=1}^{m} f_i \frac{\mu_i}{\mu} = 1$ and $\sum_{i=1}^{m} f_i = 1$, we have

$$C(F) = \sum_{i=1}^{m} f_i C(F_i) \alpha_i^2 + \sum_{i=1}^{m} f_i \alpha_i^2 - 2 \times 1 + 1.$$

Expanding and rearranging yields the decomposition clearly:

$$C(F) = \underbrace{\sum_{i=1}^{m} f_i C(F_i)\alpha_i^2}_{C(F)_{within}} + \underbrace{\left(\sum_{i=1}^{m} f_i \alpha_i^2 - 1\right)}_{C(F)_{between}}. \tag{3}$$

This equation breaks down total CV into a *within-type* share (characterizes residual inequalities in terms of individual choices, for example, individual effort) and a *between-type* share (specifies inequalities tied to differences in circumstances).

However, because *C*(*F*) normalizes total variance by the overall mean, the within-group term in Eq. 3 involves the squared relative mean $\alpha_i^2 = (\mu_i/\mu)^2$ for each type (rather than a simple weight). Consequently, we should note that the above breakdown is not a symmetric Oaxaca-type decomposition of variance; it should be understood as the natural CV-specific analogue. This nuance underscores that the underlying inequality measure remains the variance itself, with the CV simply rescaling it for convenient, scale-neutral comparisons across populations.

This is quantified by the share of total CV explained by the between-type component, termed $\eta^2$ (the Pearson correlation ratio), which can be defined as:

$$\eta^2 = \frac{C(F)_{between}}{C(F)} = \frac{\sum_{i=1}^{m} f_i(\mu_i - \mu)^2}{\mathrm{Var}(Y)}. \tag{4}$$

The index $\eta^2$ directly quantifies the fraction of total variance (and total CV) attributable exclusively to circumstances, so that the ex-ante inequality of opportunity can be read either on the variance scale or on the CV scale. Importantly, it is equivalent to the traditional $R^2$ from an ANOVA regression of outcomes[3] on condition group dummies – commonly used in the EOp literature (e.g., Bourguignon et al., 2007; Ferreira and Gignoux, 2011). The advantage of $\eta^2$ is that it is fully non-parametric in nature, does not rely on any functional form assumptions needed by some regression-based approaches. See Appendix A for a structured comparison and additional discussion on the coefficients adopted in the EOp literature.

Compared to other decomposable inequality measures frequently used, such as the Theil index or the Mean Log Deviation (MLD) highlighted by Checchi and Peragine (2010) and Ferreira and Gignoux (2011), the CV is preferable here for several reasons: (i) it is dimensionless, enabling direct comparisons across different contexts; (ii) its decomposition precisely mirrors a variance decomposition structure, making its interpretation more intuitive and directly analogous to familiar regression analyses; and (iii) unlike the Theil or MLD indices, the CV is less sensitive to extremes and tail behavior, thus providing stable and interpretable policy guidance.

The decomposition in Eq. 3 provides a clear indication to policy makers about how much inequality is due to different types of inequality which allows for a more parsimonious set of

[3] This variance decomposition is closely related to the *Analysis of Variance* (ANOVA), a classical statistical method extensively used to partition observed variability into distinct, interpretable sources. More explicitly, total variance is decomposed into two primary quantities: (i) the *between-group* variance, which represents variability across group means; and (ii) the *within-group* variance, which reflects variability within groups around their own means (Fisher, 1938). This decomposition has been used as a basic analytic tool in many areas to assess the percentage of variance among outcomes that is explained by factors affecting subpopulations defined by categorical circumstances (in our case) versus an unsystematic variation due either to individual differences or random effects (Scheffe, 1959).

information to correct unequal wealth distribution to a large extent. The impact of any type *i* on CV overall can be directly computed as:

$$\frac{f_i C(F_i)\alpha_i^2 + f_i(\alpha_i^2 - 1)}{C(F)},$$

## 4 A Micro-based Structure of the EOp Model

To frame our inequality decomposition within the normative theory of equality of opportunity (EOp), we adopt the canonical model formalized in Roemer (1998) and further developed by Fleurbaey and Peragine (2013) and Ferreira and Peragine (2015).

We consider a finite population of individuals indexed by $k = 1, \ldots, n$. Each individual *k* is characterized by an outcome $Y_k \in \mathbb{R}$ (e.g., income) and belongs to a single circumstance group $C_i$, where $i = 1, \ldots, m$. The set C $= \{C_1, \ldots, C_m\}$ contains all mutually exclusive and exhaustive types, each defined by a unique configuration of morally arbitrary traits such as parental background, gender, or place of birth. While individuals belong to a fixed circumstance group, they exert a personal effort level $e_k$, which is an observable realization of effort-related behavior (e.g., study hours, occupational choice, or work intensity). Effort is not grouped or categorized a priori: rather, individuals are ranked *within their own type* based on their effort level. This ranking yields a type-specific distribution of effort and assigns each individual a *quantile position* – such as a percentile or decile – relative to others within the same type. More specifically, individuals exert an *absolute* effort level $e_k$ (e.g., hours worked, study time, job search intensity, ...). The normative premise here is that individuals exerting the same relative effort (same percentile within their type) are to be treated equally from a fairness standpoint, regardless of their type. This construction allows us to isolate what remains of inequality once individual responsibility (i.e., relative effort) is controlled for.

Following the *statistical solution* of Roemer (1998), effort is not compared in absolute terms but through each person's *rank within* their type. Let $F_{E \mid C_i}(\cdot)$ denote the empirical distribution function of *e* in type $C_i$. We define the *relative effort* of individual *k* as the within-type quantile

$$q_k := F_{E|C_i}(e_k) \in (0, 1], \quad \forall k \in C_i, \forall C_i \in \mathrm{C}.$$

Hence $q_k$ measures how much effort *k* contributes *relative* to peers who share the same circumstances. In particular, this definition implies that the absolute levels of effort may differ across types, but fairness comparisons are made conditional on equal relative effort. Indeed, someone at the $30^{\mathrm{th}}$ percentile of effort in $C_1$ is deemed normatively identical – in terms of exertion – to a person at the $30^{\mathrm{th}}$ percentile in $C_2$. Individuals who share the same relative effort percentile across different types constitute what Roemer (1998) calls an *effort tranche*, as opposed to a *type*, which groups individuals based on identical circumstances. Thus, whereas types cluster individuals according to morally arbitrary characteristics, tranches group individuals solely based on comparable relative effort. This distinction between types (circumstance-based groups) and tranches (effort-based groups) provides a clear theoretical framework that enables meaningful normative comparisons, underpinning both ex-ante and ex-post analyses of equality of opportunity.

This rank-based view makes cross-type comparisons meaningful in both ex-ante and ex-post analyses.[4]

**Assumption 1** (Effort comparability across types) For every circumstance type $C_i$ the mapping $e \mapsto F_{E\,|\,C_i}(e)$ is continuous and strictly increasing. Consequently, the relative effort ranks $q_k$ are uniformly distributed on (0, 1) *within each type*:

$$q_k \mid C_i \sim \mathrm{U}(0,1), \quad \forall\, k \in C_i,\ \forall\, C_i \in \mathrm{C}.$$

Effort tranches are then defined as partitions of the unit interval: let $0 = p_0 < p_1 < \cdots < p_G = 1$, the *g*-th tranche is

$$\mathcal{T}_g := \{\, k : q_k \in (p_{g-1},\, p_g] \,\}, \qquad g = 1, \ldots, G,$$

with probability $p_g - p_{g-1}$ in every type. Thus, individuals at the same quantile position across types belong to the same tranche, allowing meaningful cross-type comparisons.

The absolute effort distributions $e_k \,|\, C_i$ may differ across types; only the rank scale is assumed comparable.

Under Assumption 1, the relative-effort ranks $q_k$ are uniformly distributed on (0, 1) within each circumstance group $C_i$.[5] We model the individual production of outcomes by the function

$$Y_k = g\big(C_i, e_k\big),$$

where $g(\cdot)$ is strictly increasing in $e_k$ and may interact with circumstances. For a fixed type $C_i$, the transformation $e \mapsto g(C_i, e)$ turns the uniform rank $q \sim \mathrm{U}(0, 1)$ into an outcome distribution

$$F_i(y) = \Pr\{g(C_i, Q_i^{-1}(q)) \leq y\},$$

with $Q_i^{-1}(\cdot)$ the inverse rank-to-effort map in type *i*. Thus each $F_i$ is generated by the *same* rank distribution but filtered through a (possibly) type-specific reward schedule $g(C_i, \cdot)$.

Aggregating over types with population weights $f_i$ gives the overall outcome distribution introduced in Section 3:

[4] In both the ex-ante and ex-post frameworks, comparisons across circumstance groups (types) are conducted by conditioning on individuals' position in the distribution of effort within their type. In particular, in the case of the ex-post logic of Roemer (1998), comparisons across types are drawn conditionally on individuals' rank within their own type-specific effort distribution (tranche). This implies that two individuals – each from a different type – are considered equivalent in terms of effort if they occupy the same relative position (e.g., median or decile) within their respective type. Holding *relative effort* constant allows outcome differences to be attributed solely to circumstances.

[5] We emphasize that this assumption does not require the effort distribution $e_k|C_i$ to be identical across types, but only that its rank representation be well-defined and strictly increasing. This is crucial for constructing EOp2 as a condition over distributions $F_i$ derived from uniform ranks *q*.

$$F(y) = \sum_{i=1}^{m} f_i F_i(y).$$

Hence the micro-level model $g(C_i, e_k)$ connects directly to the macro-level mean-based decomposition: between-group inequality reflects systematic shifts in $g$ across $C_i$, while within-group inequality captures dispersion generated by heterogeneous $e_k$ within each type.

This structural link between the micro-level production function and the macro-level distribution highlights how inequality components arise from the interaction between effort and circumstances. It makes clear that any decomposition of inequality implicitly reflects assumptions about how effort is rewarded and how types influence outcomes. While we defer formal definitions of our EOp criteria to the next section, we emphasize here that the micro-level foundations are fully compatible with variance-based decomposition: individual outcomes derive from transformations of relative effort within types, and their group-level aggregation determines the global structure of inequality.

The ethical foundations are captured by two classic principles: (i) *compensation* – outcome differences due to circumstances should be neutralised; and (ii) *reward* – outcome differences reflecting higher effort are acceptable. These two principles define the ethical core of the EOp framework, yet they may be mutually incompatible in practice, as shown by Fleurbaey and Peragine (2013).[6]

Two dominant approaches to operationalizing equality of opportunity have emerged in the literature: the *ex-ante* and *ex-post* approaches. These differ fundamentally in the unit of comparison and the ethical interpretation of fairness.

- The *ex-ante* approach evaluates equality of opportunity by comparing expected outcomes across groups defined by circumstances. Formally, for two individuals *k* and *l* from circumstance groups $C_i$ and $C_j$, respectively, equality of opportunity is achieved if the mean outcomes are identical:

$$\mathbb{E}[Y_k \mid C_i] = \mathbb{E}[Y_l \mid C_j] \quad \text{for all } i, j.$$

This aligns precisely with our definition of *EOp*1 (mean equality) introduced previously, and directly corresponds to the between-group inequality component in Section 3.

- The *ex-post* approach, by contrast, requires equality of outcomes conditional on individuals exerting the same relative effort (tranche) across different circumstance groups. Formally, let $q_k, q_l \in (0, 1]$ denote the quantile ranks of effort for individuals *k* and *l*

[6] The *compensation* and *reward* principles cannot in general be jointly satisfied. As demonstrated by Fleurbaey and Peragine (2013), this normative tension reflects a deeper conflict between the ex ante and ex post conceptions of equality of opportunity. *Ex post compensation* requires that individuals who exert the same effort achieve equal outcomes, regardless of their circumstances. *Ex ante compensation*, by contrast, demands that the distribution of outcomes across circumstance groups be equalized – potentially at the cost of rewarding individuals with lower effort levels. This incompatibility is formalized through social ordering axioms: *Ex Ante Compensation*, which favors reducing inequality across groups defined by circumstances, and *Ex Post Compensation*, which favors equal treatment of individuals conditional on effort. Fleurbaey and Peragine (2013) prove that no social welfare ordering can satisfy both simultaneously. Therefore, any empirical or normative implementation of EOp must make an explicit choice between these two logics – or adopt a hybrid criterion that trades off between them.

within their respective groups $C_i$ and $C_j$. Ex-post equality of opportunity is achieved if:

$$\mathbb{E}[Y_k \mid q_k = q, C_i] = \mathbb{E}[Y_l \mid q_l = q, C_j] \quad \text{for all } i, j \text{ and each } q \in (0, 1].$$

This condition demands that individuals who exert the same relative effort should achieve identical expected outcomes, regardless of their circumstances. Moreover, under the assumption of uniform distribution of effort (Assumption 1), identical conditional distributions imply that the variance and higher-order moments of outcomes within each tranche are also uniform across circumstance groups. Thus, the ex-post approach constitutes a stringent fairness standard that EOp2 – equality of within-group variances – can approximate empirically.

While the ex-ante approach is widely used in practice due to its simplicity and normative transparency, the ex-post approach captures a deeper, more rigorous form of fairness conditional on effort. Hence, any empirical implementation of equality of opportunity must explicitly acknowledge the normative choice or trade-off involved in adopting one approach over the other.

## 5 EOp Definitions Under the Mean-based Approach

Having established the conceptual and empirical foundation of our decomposition, we now formally define three normative conditions that correspond to progressively stricter implementations of equality of opportunity under the ex-ante and the ex-post perspective.

*EOp*1 captures group-level average fairness (ex-ante compensation); *EOp*2 reflects uniform dispersion of outcomes across equal-effort individuals (a tractable proxy for ex-post fairness); and *EOp*3 is not a new ethical construct, but a combined criterion that jointly enforces the assumptions behind the previous two. This layered approach permits separate empirical testing and also allows researchers and policymakers to diagnose in which normative dimension – mean or variance – the main deviations from equality of opportunity arise.

**Definition 1** (*Equality of Opportunity of Degree 1*, *EOp1*) *Equality of opportunity of degree 1 (EOp1) is achieved if and only if average outcomes are identical across all circumstance groups. Formally:*

$$\mu_i = \mu \quad \text{for each type } i \in \{1, \ldots, m\}.$$

*Under this condition, no systematic outcome differences due to circumstances remain once outcomes are averaged within each group.*

When *EOp*1 holds, total inequality measured by the coefficient of variation *C*(*F*) reduces entirely to within-group disparities:

$$C(F) = \sum_{i=1}^{m} f_i C(F_i),$$

where $f_i$ is the population share of group $C_i$, and $C(F_i)$ denotes within-group inequality. Empirically, adherence to *EOp*1 can be directly assessed via the Pearson index $\eta^2$ (introduced in Section 3). A value of $\eta^2 = 0$ corresponds exactly to the perfect fulfillment of ex-ante equality of opportunity, while significant positive values indicate deviations due to systematic inequality between circumstance groups.

Formally, *EOp*1 explicitly corresponds to ex-ante compensation principle that directly focuses on the group-level fairness of expected outcomes without any explicit conditioning on effort quantiles. To account for variability in outcomes related to relative effort, we now introduce a second criterion, *EOp*2, which demands that the dispersion of outcomes can also be identical across groups.

**Definition 2** (*Equality of Opportunity of Degree 2*, *EOp*2) *EOp2 asserts that equality of opportunity is achieved if the within-group variability of outcomes – conditional on circumstances – is identical across all circumstance groups. Formally, EOp2 holds if:*

$$\mathrm{Var}(Y_i) = K \quad \text{for all } i \in \{1, \ldots, m\},$$

*where K is a constant, and* $\mathrm{Var}(Y_i)$ *denotes the variance of outcomes within the circumstance group* $C_i$

This definition implies an alternative normative stance than *EOp*1: whereas *EOp*1 requires the equality of average outcomes across groups, *EOp*2 demands that each group's outcomes exhibit the same degree of dispersion or variability. *EOp*2 thus offers an empirically tractable and normatively defensible proxy for effort-based fairness. It can be interpreted as a reduced-form, distribution-based test of whether the *reward structure of effort* is homogeneous across circumstance groups. Although it is not equivalent to the full tranche ex-post condition proposed in Checchi and Peragine (2010); Fleurbaey and Peragine (2013); Ramos and Van de Gaer (2016); Roemer and Trannoy (2016), equality of within-group variances serves as a practical proxy for checking whether effort-based outcome dispersion is uniform across circumstances.

Therefore, *EOp*2 is best viewed as a statistically convenient, albeit limited, approximation to the full reward principle. Empirically, this homoskedasticity – like condition is most defensible in settings with relatively similar institutional structures or cultural norms around effort – such as among European countries. In such contexts, it is plausible to assume a roughly common reward structure for effort across groups, making equal within-group variances a meaningful condition to test.

Using the variance decomposition introduced in Section 3, we recall that total inequality *C*(*F*) can be expressed as:

$$C(F) = \sum_{i=1}^{m} f_i C(F_i)\alpha_i^2 + \sum_{i=1}^{m} f_i(\alpha_i - 1)^2.$$

Under *EOp*2, since $\mathrm{Var}(Y_i) = K$ for all *i*, the first component becomes uniform across groups. Formally, we have:

$$\sum_{i=1}^{m} f_i C(F_i)\alpha_i^2 = \frac{K}{\mu^2}\sum_{i=1}^{m} f_i = \frac{K}{\mu^2}.$$

Thus, under the assumption of equal within-group variances:

$$C(F) = \frac{K}{\mu^2} + \sum_{i=1}^{m} f_i\alpha_i^2 - 1.$$

The decomposition of *C*(*F*) provides meaningful insights into how within-group and between-group variations contribute to the overall inequality:

- The term $\frac{K}{\mu^2}$ represents the *within-group component*, which is uniform across subpopulations due to the constant variance $K$, i.e.,

$$C(F)_{within} = \frac{K}{\mu^2}$$

- The term $\sum_{i=1}^{m} f_i\alpha_i^2 - 1$ captures the *between-group component*, reflecting disparities in subpopulation means ($\mu_i$) relative to the overall mean ($\mu$), i.e.,

$$C(F)_{between} = \sum_{i=1}^{m} f_i\alpha_i^2 - 1.$$

This simplified decomposition shows clearly that under *EOp*2, inequality differences across groups must originate entirely from differences in group means ($\alpha_i$), rather than differences in group-level variability. Deviations from this condition signal a scenario in which circumstances affect not only average outcomes but also the uncertainty or variability individuals have within types.

In summary, *EOp*2 captures an intermediate normative condition situated between the ex-ante and the full ex-post criteria. It demands fairness in the distribution of outcomes conditional on relative effort across circumstance groups, providing a stronger basis of fairness where the results are not only equal in expectation but also have equal rates and variances across groups. Rather, checking for both *EOp*1 and *EOp*2 helps policymakers identify not only differences in desired results, but also variations in the ways circumstances affect outcome uncertainty and variance.

**From Variance Decomposition to *EOp*2 Metrics** To derive explicit metrics for evaluating deviations from *EOp*2, we begin by revisiting the variance decomposition from Eq. 1:

$$\mathrm{Var}(Y) = \sum_{i=1}^{m} f_i\mathrm{Var}(Y_i) + \sum_{i=1}^{m} f_i(\mu_i - \mu)^2,$$

where the first component, $\sum_{i=1}^{m} f_i\mathrm{Var}(Y_i)$, captures within-group variance, and the second component, $\sum_{i=1}^{m} f_i(\mu_i - \mu)^2$, captures between-group variance.

To empirically measure how much observed within-group variances deviate from this uniformity ideal (*EOp*2), we propose two complementary metrics. First, we isolate the within-group variance component explicitly by removing the between-group variance term from both numerator and denominator:

$$\frac{\mathrm{Var}(Y) - \min_i \mathrm{Var}(Y_i) - \sum_{i=1}^{m} f_i(\mu_i - \mu)^2}{\mathrm{Var}(Y) - \sum_{i=1}^{m} f_i(\mu_i - \mu)^2}.$$

We can also employ a simpler and more direct formulation:

$$\mathcal{D}_{\mathrm{EOp2}} = \frac{\sum_{i=1}^{m} f_i \mathrm{Var}(Y_i) - \min_i \mathrm{Var}(Y_i)}{\sum_{i=1}^{m} f_i \mathrm{Var}(Y_i)}. \quad (5)$$

where the dispersion index $\mathcal{D}_{\mathrm{EOp2}}$ measures the relative difference between the average within-group variance and the smallest within-group variance, providing a straightforward interpretation of variability differences.

This metric takes values between 0 and 1. A value of 0 indicates perfect equality of within-group variances (*EOp*2 fully achieved), whereas values closer to 1 highlight significant deviations, reflecting substantial differences in how relative effort is rewarded across circumstance groups.

**Combining *EOp*1 and *EOp*2: The EOp3 Benchmark** We now introduce a third normative criterion – Equality of Opportunity of Degree 3 (*EOp*3) – which integrates *EOp*1 and *EOp*2 simultaneously:

**Definition 3** (Equality of Opportunity of Degree 3, *EOp*3) A society achieves equality of opportunity of degree 3 (*EOp*3) if both average outcomes and within-group variances are identical across all circumstance groups. Formally, *EOp*3 holds if, for every circumstance group $C_i$:

$$\mu_i = \mu \quad \text{and} \quad \mathrm{Var}(Y_i) = K, \quad \forall i \in \{1, \ldots, m\},$$

where $\mu$ and *K* are constant across all groups.
Under *EOp*3, the overall inequality measured by *C*(*F*) simplifies dramatically. Since both the means and variances are identical across groups, we have:

$$C(F) = C(F_i), \quad \forall i.$$

It forms a novel and integrated EOp standard encompassing the ex-ante as well as aspects of the ex-post perspective by Assumption 1. In that case *EOp*3 is fair both in terms of ex-ante expected outcomes and, conditional on effort, ex-post outcome structure.

To empirically assess how closely a society approaches *EOp*3, we propose a synthetic index combining deviations from *EOp*1 and *EOp*2:

$$\text{EOp Index} = \frac{\beta \sum_{i=1}^{m} f_i(\mu_i - \mu)^2 + (1-\beta)\left(\sum_{i=1}^{m} f_i \mathrm{Var}(Y_i) - \min_i \mathrm{Var}(Y_i)\right)}{\mathrm{Var}(Y)}, \quad (6)$$

with $\beta \in [0, 1]$ reflecting the relative importance placed on mean versus variance equality. A 0 value indicates the satisfaction of the *EOp*3 criterion, while a larger value denotes greater departures from this joint benchmark. Importantly, *EOp*3 does not introduce a new normative principle beyond *EOp*1 and *EOp*2, but instead serves as a composite benchmark integrating both criteria. The weighting parameter $\beta$ is a normative choice: here we use the EOp Index to illustrate how country rankings and trends change under different weights on mean vs variance equality, rather than to recommend any specific $\beta$ value. Our EOp Index is intended as an exploratory tool to visualize how country outcomes vary with different $\beta$ weightings of mean-based versus variance-based opportunity inequality, rather than to recommend a specific choice of $\beta$.

**Theoretical and Empirical Implications** In sum, the strength of this framework lies not in prescribing a specific fairness definition, but in enabling separate and transparent evaluation of two normative dimensions – group-level compensation and rank-based effort reward – that are often conflated in practice. By articulating *EOp*1 and *EOp*2 in variance-based terms and combining them analytically through *EOp*3, we provide a flexible and interpretable lens for assessing the multidimensional nature of opportunity inequality.

The definition of *EOp*3 takes into account both ex-ante and ex-post conditions, making it a more robust normative standard to measure equality of opportunity. To facilitate *EOp*3 from a policy standpoint will not only require a set of targeted interventions that remedy systemic disparities in average outcomes, but also those that address structural inequalities which affect outcome uncertainty within groups.

## 6 Data

Our empirical analysis is based on cross-sectional data from the European Union Statistics on Income and Living Conditions (EU-SILC) survey for the years 2005, 2011, and 2019. EU-SILC is the most suitable dataset for our study as it provides harmonized cross-sectional data on income and various socio-economic aspects, enabling a comprehensive comparison of individuals' characteristics across most European countries. We focus on these three waves because they include a special module on intergenerational mobility, providing crucial information on individuals' circumstances.

We include all available countries, excluding those missing at least one of the three surveys or lacking data on one of our variables of interest. As a result, our final sample comprises 16 European countries: Austria, Belgium, Czech Republic, Estonia, Greece, Spain, France, Ireland, Italy, Lithuania, Luxembourg, Latvia, Poland, Portugal, Sweden, and Slovenia. Although the EU-SILC data are designed to be harmonized across countries, residual differences in survey design, income definitions, and non-response can affect cross-country comparability. We therefore interpret cross-country differences in our EOp measures with appropriate caution.

We restrict the sample to individuals aged 25 to 60, as some of the variables of interest are only available for this age range, and we exclude the top and bottom percentiles of the distribution to prevent results from being driven by outliers. Table 1 presents summary statistics on the equivalent disposable household income, which EU-SILC defines as

**Table 1** Descriptive statistics on the disposable household income

| | Obs. | Mean | Variance |
|---|---|---|---|
| Austria | 9,079,474 | 24,327.309 | 147,494,655.938 |
| Belgium | 12,245,225 | 23,238.079 | 114,573,401.199 |
| Czech Republic | 9,781,592 | 8,892.269 | 23,210,794.678 |
| Estonia | 1,307,309 | 8,389.368 | 45,117,103.884 |
| Greece | 12,770,572 | 11,176.692 | 43,146,534.536 |
| Spain | 62,944,188 | 15,568.914 | 81,782,157.574 |
| France | 67,923,616 | 22,485.090 | 141,197,609.409 |
| Ireland | 3,761,116 | 26,309.398 | 194,672,031.597 |
| Italy | 73,340,584 | 17,915.797 | 100,740,541.003 |
| Lithuania | 2,942,731 | 5,150.083 | 24,877,095.713 |
| Luxembourg | 641,687 | 37,261.026 | 383,848,865.692 |
| Latvia | 1,905,498 | 5,347.253 | 23,227,162.987 |
| Poland | 29,560,064 | 5,087.970 | 14,441,831.472 |
| Portugal | 13,742,852 | 10,511.646 | 41,861,622.874 |
| Sweden | 5,541,118 | 23,840.337 | 111,690,892.793 |
| Slovenia | 991,196 | 12,620.845 | 37,212,236.994 |

the total gross personal income for all household members. To account for differences in household size and composition, and thereby allow for meaningful comparisons of income levels across countries and population groups, we use the concept of equivalised disposable income. This measure is calculated as total net (i.e., disposable) household income divided by the number of 'equivalent adults' in the household, according to a standard equivalence scale. More precisely, we apply the modified OECD scale,[7] which assigns a weight of 1.0 to the first adult in the household, 0.5 to each additional person aged 14 or over, and 0.3 to each child under the age of 14. We treat this variable as the most suitable proxy for individual well-being. As expected, Luxembourg exhibits the highest average income, followed by Ireland and Austria. Conversely, the lowest average incomes, slightly above 5,000 euros, are observed in Lithuania, Poland, and Latvia.

As previously mentioned, we restrict our analysis to the three surveys that provide information on intergenerational mobility. We focus on four dimensions of inequality of opportunity: gender (equal to one if the respondent is female), migrant background (equal to one if the respondent was born abroad), parental education, and parental occupation. Similarly to Palmisano et al. (2022), parental education is classified into five categories: both parents have low education, or one parent has low education and no information is available for the other; at least one parent has completed medium education; only the father has attained tertiary education; only the mother has attained tertiary education; both parents have attained tertiary education. Parental occupation is also coded into five categories: both parents are not working, or one parent is not working and no information is available for the other; at least one parent is a blue-collar worker and the other is not working/unknown; the mother

[7] https://ec.europa.eu/eurostat/statistics-explained

is a white-collar worker and the father is a blue-collar worker or not working/unknown; the father is a white-collar worker and the mother is a blue-collar worker or not working/unknown; both parents are white-collar workers. Summary statistics on these variables are shown in Table 8 in Appendix B.[8]

In addition, for the ex-post analyses, we constructed tranches by jointly partitioning individuals according to all available circumstance variables. Given that our set of circumstances includes two dummy variables and two categorical variables with five levels each, this procedure results in 100 possible tranches. To ensure statistical reliability, we dropped tranches with fewer than three individuals; for consistency, these observations were also removed from the corresponding ex-ante analyses.[9] Each remaining tranche was then further divided into quartiles of the disposable household income distribution, which allowed us to conduct within-quartile comparisons across types. This step was necessary to operationalize our ex-post analysis in a tractable and comparable way.

Despite its advantages, EU-SILC has some limitations. Although EU-SILC is designed to be harmonized across countries, residual differences in survey design, income definitions, and non-response can affect cross-country comparability. Furthermore, key circumstance variables in our definition of types are missing or incomplete for certain country-year combinations. In particular, the coverage of parental occupation is limited in more recent survey waves (e.g., 2023), which is one reason we restrict attention to data up to 2019. Finally, because we lack consistent information on factors like parental income, wealth, or neighborhood characteristics, our measures of inequality of opportunity should be interpreted as lower-bound estimates of the total unfair inequality. We acknowledge these data constraints and consider their implications when interpreting the results.

## 7 Results

A central objective of the empirical exercise is to quantify departures from equality of opportunity in both the ex-ante and ex-post senses. From the ex-ante viewpoint, we decompose the Coefficient of Variation *C*(*F*) into a between-type component $C(F)_{\text{between}}$ and a within-type component $C(F)_{\text{within}}$, where types are defined by circumstance groups (Section 3). In this metric, $C(F)_{\text{between}}$ captures the dispersion attributable to differences in *circumstances* (reflecting unequal opportunities), whereas $C(F)_{\text{within}}$ represents residual dispersion consistent with individual *responsibility* (effort and idiosyncratic factors), consistently with the framework outlined in Definition 1.

For the ex-post perspective, we rank individuals within each type according to their relative effort and forming *effort tranches* (quantile ranks á la Roemer, 1998). In this alternative decomposition, the within-tranche component $C(F)_{\text{within}}$ tranches isolates the impact

[8] From an empirical point of view, there is a well-known trade-off between omitting relevant variables – biasing inequality of opportunity estimates downward-and over-partitioning, which can undermine statistical precision. Recent methodological contributions offer practical tools to mitigate this risk. In particular, Moramarco et al. (2024) introduce reward and compensation scores and an iterative algorithm, the 'Opportunity Tree', which refines the partition into types while respecting the two foundational EOp logics (compensation and reward). This approach is especially useful when only a limited set of observed circumstances is available, as it guides a principled refinement of type definitions without violating normative requirements.

[9] The number of dropped tranches varies depending on the type of analysis conducted, with a maximum of 610 tranches excluded in the country-year analysis.

**Table 2** Decomposing *C*(*F*) under both Ex-ante and Ex-post approaches of Inequality of Opportunity by year

| | Ex-ante approach | | | Ex-post approach | |
|---|---|---|---|---|---|
| | *C*(*F*) | $C(F)_{within}$ | $C(F)_{between}$ | $C(F)_{within}$ | $C(F)_{between}$ |
| 2005 | 0.515 | 0.475 | 0.040 | 0.173 | 0.342 |
| 2011 | 0.463 | 0.441 | 0.023 | 0.158 | 0.306 |
| 2019 | 0.410 | 0.388 | 0.022 | 0.135 | 0.275 |

of unequal opportunities, while the between-tranche component $C(F)_{\text{between}}$ captures the residual inequality of *responsibility*. Note that $C(F)_{between}$ tranches in the ex-post approach adopted in Table 2 is based on the same coefficient of the between component under Section 5, while considering the tranches in place of the types. In this case, the tranches correspond to four groups defined by the quartiles of effort in the distribution, rather than to the types under Definition 1. The same structure is adopted for $C(F)_{within}$.

**Ex-ante vs. Ex-post Measurement** Table 2 summarizes these results by reporting these decompositions. Total inequality declines from $C(F) = 0.515$ (2005) to 0.410 (2019), a reduction of about $20\%$. In ex-ante terms, the between component falls only slightly in absolute terms, but substantially in relative terms (from 0.040 to 0.022, $-45\%$), while the within component decreases more markedly in absolute terms, but less in relative terms (from 0.475 to 0.388, $-18.3\%$). Conditioning on relative effort, the within-tranche inequality – the ex-post unfair component – is small in levels and declines notably (from 0.173 to 0.135, about $-22\%$). Its share in total inequality declines from $0.173/0.515 \simeq 33.6\%$ (2005) to $0.135/0.410 \simeq 32.9\%$ (2019), suggesting a slight improvement in ex-post fairness: individuals at the same effort quantile look slightly more alike across types. By contrast, the ex-post between component (inequality across effort tranches, i.e., responsibility) remains large though it also decreases in level (from 0.342 to 0.275, about $-19.6\%$).

Read together, the two methods deliver a coherent picture of the opportunity and responsibility split. Ex-ante, only a small part of *C*(*F*) is attributed to between-type differences, pointing to marginal deviations from *EOp*1, that recede over time. Ex-post, most inequality is due to differences *across effort tranches* (responsibility), while the *unfair* dispersion *within tranches* across types is both small and falling. This pattern associates an improvement in ex-post fairness (smaller $C(F)_{\text{within}}$ under tranches) with a small ex-ante between component, a situation that is possible even if the two criteria answer different normative questions.

**Ex-post Decomposition by Effort Tranches** Following the results in Table 2, Table 3 breaks the *ex-post* diagnostic into tranches of the within-type effort rank. Since we focus on the four-way partition of the distribution, from this point onward we shall simply refer to these

**Table 3** Ex-post approach: $C(F)_{within}$ by effort quantiles

| | Q1 | Q2 | Q3 | Q4 |
|---|---|---|---|---|
| 2005 | 0.005 | 0.016 | 0.027 | 0.125 |
| 2011 | 0.005 | 0.012 | 0.021 | 0.120 |
| 2019 | 0.003 | 0.005 | 0.015 | 0.112 |

quantiles as quartiles (or effort tranches). By construction, the four entries for each year sum to the ex-post within term reported in Table 2.[10] Three patterns emerge clearly:

1. *Monotonic concentration at high effort.* In every year the tranche $Q4$ (top relative effort) accounts for the bulk of ex-post opportunity inequality: its share of the ex-post within term rises from about 72% in 2005 (0.125/0.173) to 76% in 2011 (0.120/0.158) and to 83% in 2019 (0.112/0.135). By contrast, the bottom tranches $Q1$–$Q3$ together account for only 28%, 24%, and 17% of the ex-post within term in 2005, 2011, and 2019, respectively. This monotonic profile indicates that *conditional on equal relative effort*, between-type inequality in outcomes is substantially larger at the top of the effort distribution. In EOp terms, the reward schedule to effort appears more heterogeneous across types precisely where relative effort is highest (glass-ceiling/network effects, segmented high-return opportunities, etc.).
2. *Aggregate decline of ex-post within inequality.* The ex-post within term falls from 0.173 (2005) to 0.158 (2011) and 0.135 (2019). All quartiles decline in levels, with particularly sharp reductions in $Q1$–$Q3$ (e.g., $Q2$: $0.016 \rightarrow 0.012 \rightarrow 0.005$; $Q3$: $0.027 \rightarrow 0.021 \rightarrow 0.015$). This suggests a gradual equalization of outcomes across types for low-to-mid relative effort ranks, consistent with policies and institutions that compress returns at the lower/middle part of the distribution.
3. *Rising share at the top despite level declines.* Although $Q4$ decreases in level ($0.125 \rightarrow 0.120 \rightarrow 0.112$), its share of the ex-post within term increases, because the lower tranches compress faster. Thus, violations of ex-post fairness (unequal outcomes across types at the same effort rank) are increasingly concentrated among the most diligent individuals. This is precisely where EOp2-type diagnostics under Definition 2 are most informative: equalizing the variance of outcomes across types at the same $q$ would target the locus where ex-post unfairness is largest.

In turn, Table 3 shows that, even when average (ex-ante) differences across types are limited, substantial inequality of opportunity may persist *conditional on equal relative effort* and is disproportionately borne by high-effort tranches. This highlights the complementarity of the two perspectives and the policy relevance of targeting heterogeneity in returns to effort across circumstance groups at the top of the effort distribution.

**Cross-country Evidence** Table 4 reports, for each country, the total coefficient of variation $C(F)$ and the results for both approaches in terms of opportunity and responsibility components. Overall levels of inequality, $C(F)$, are highest in the Baltics (Lithuania 0.833, Latvia 0.749, Estonia 0.616) and lower in Northwestern Europe (Sweden 0.194, Belgium 0.210, Slovenia 0.227, Austria 0.241). Against this backdrop, the *ex-ante* picture indicates that *differences in type means* account for a small fraction of total inequality in most countries. The *ex-post* reading reveals a much sharper cross-country contrast. In Northwestern Europe, the residual unfair dispersion across types at equal effort is at an average level: the ratio $C(F)_{\text{within}}/C(F)$ is around 35-39% in Belgium (0.075/0.210), Austria (0.085/0.241), Ireland (0.095/0.274), and France (0.101/0.270), and remains below 35% in Spain, Greece, and Italy, reaching a minimum 31% in Sweden. By contrast, ex-post unfair dispersion is large in

[10] For example, in 2005: $0.005 + 0.016 + 0.027 + 0.125 = 0.173$, which equals $C(F)_{within}$ under the ex-post approach in Table 2. The same identity holds for 2011 and 2019.

**Table 4** C(F) by countr y

| | Ex-ante Approach | | | Ex-post Approach | |
|---|---|---|---|---|---|
| | $C(F)$ | $C(F)_{within}$ | $C(F)_{between}$ | $C(F)_{within}$ | $C(F)_{between}$ |
| Austria | 0.241 | 0.214 | 0.027 | 0.085 | 0.157 |
| Belgium | 0.210 | 0.181 | 0.028 | 0.075 | 0.135 |
| Czech Republic | 0.321 | 0.249 | 0.072 | 0.139 | 0.182 |
| Estonia | 0.616 | 0.503 | 0.113 | 0.260 | 0.356 |
| Greece | 0.338 | 0.311 | 0.027 | 0.112 | 0.226 |
| Spain | 0.345 | 0.297 | 0.048 | 0.119 | 0.226 |
| France | 0.270 | 0.246 | 0.025 | 0.101 | 0.169 |
| Ireland | 0.274 | 0.238 | 0.036 | 0.095 | 0.179 |
| Italy | 0.318 | 0.286 | 0.032 | 0.102 | 0.217 |
| Lithuania | 0.833 | 0.662 | 0.171 | 0.418 | 0.415 |
| Luxembourg | 0.276 | 0.232 | 0.044 | 0.113 | 0.162 |
| Latvia | 0.749 | 0.597 | 0.153 | 0.351 | 0.398 |
| Poland | 0.496 | 0.384 | 0.112 | 0.231 | 0.265 |
| Portugal | 0.380 | 0.329 | 0.050 | 0.148 | 0.232 |
| Sweden | 0.194 | 0.173 | 0.021 | 0.060 | 0.134 |
| Slovenia | 0.227 | 0.189 | 0.038 | 0.090 | 0.137 |

the Baltics and in Poland: Lithuania reaches around $50\%$ (0.418/0.833), Latvia $47\%$, Poland $47\%$ and Estonia $42\%$. In these settings, even among individuals exerting the same relative effort, types differ markedly in the spread of outcomes, indicating more pronounced departures from the *EOp*2 ideal of homogeneous returns to effort.

Taken together, the two approaches pinpoint where unfair opportunity gaps reside. In countries such as France, ex-ante unfairness is high (circumstances strongly shift group means), but the ex-post diagnostic is lenient: conditional on effort tranche, type-specific dispersions look similar. Policy in these contexts should thus prioritize *leveling means across circumstance types* (e.g., early-life interventions, baseline schooling quality, regional equalization) more than harmonizing returns to effort.

In Spain, the pattern is the opposite: while ex-ante shares are comparatively lower in proportion, ex-post unfair dispersion is high – at equal effort, types still face very different outcome variability. Here, policies should include instruments that *equalize the reward structure of effort across types* (e.g., anti-discrimination enforcement, transparent grading and hiring standards, access to risk-mitigating public services).

In sum, the *C*(*F*)-scale decomposition shows that both ex-ante inequality of opportunity (gaps in type means) and the ex-post diagnostic (dispersion across types at equal effort) cleanly separates Northwestern Europe from the Baltics and Poland. The use of both approaches allows one to diagnose whether unfairness stems primarily from *mean differ-*

**Table 5** Person-level statistics and the measure of *EOp*2 dispersion

| | $\eta^2$ | $\mathcal{D}_{\mathrm{EOp2}}$ |
|---|---|---|
| 2005 | 0.079 | 0.869 |
| 2011 | 0.049 | 0.712 |
| 2019 | 0.055 | 0.587 |

*ences across types* (under *EOp*1) or from *heterogeneous returns to effort across types* (under Assumption 1 and *EOp*2), thereby potentially guiding targeted policy responses.

**Pearson $\eta^2$ and the EOp-dispersion Index $\mathcal{D}_{\mathbf{EOp2}}$** Table 5 reports two complementary diagnostics aligned with our framework. First, the *Pearson index* $\eta^2$ measures the ex-ante share of total dispersion attributable to circumstances; values closer to zero indicate that mean differences across circumstance types are small (closer to the *EOp*1 benchmark), whereas larger values reveal substantive ex-ante gaps in group means. Second, the *dispersion EOp* index summarizes how similar the within-type dispersions are across circumstance groups, capturing our variance-based *EOp*2 notion.

First, in line with the country patterns documented in Tables 2 and 4, $\eta^2$ is significantly lower in magnitude compared to $\mathcal{D}_{\text{EOp2}}$, indicating that inequality may reside slightly within types rather than between them; in the ex-ante view, deviations from *EOp*1 are present but limited. Second, the dispersion index $\mathcal{D}_{\text{EOp2}}$ is typically larger than the corresponding $\eta^2$ for the same country, pointing to more pronounced ex-post frictions: even where average outcomes across types are relatively close, the spread of outcomes conditional on effort differs across circumstance groups. This configuration suggests that policy conclusions based solely on ex-ante diagnostics may be overly optimistic: countries with low $\eta^2$ but a sizable $\mathcal{D}_{\text{EOp2}}$ achieve near-parity in group means while still offering unequal predictability in incomes for individuals exerting the same relative effort (a failure of *EOp*2).

**Evidence on Ex-post and Ex-ante Components** Figure 1 complements this picture on the ex-ante side by plotting $C(F)_{between}$, the between-type component that reads as inequality due purely to circumstances (our *EOp*1 diagnostic). Trends are heterogeneous. Luxembourg and Portugal display clear declines in between-type dispersion, consistent with shrinking mean gaps across circumstance groups. Ireland and Spain rise moderately over time, whereas Italy and France are comparatively flat. Sweden shows a pronounced increase from a very low initial level, and Lithuania also turns upward after 2011. Latvia, in contrast, drifts down-

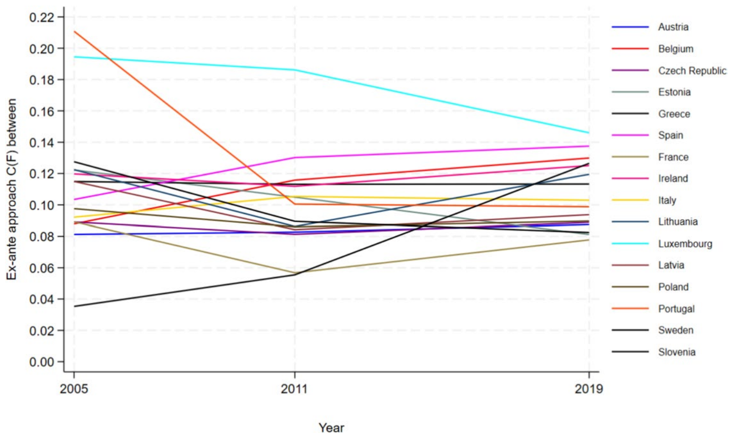


**Fig. 1** Ex-ante approach: evolution of $C(F)_{\text{between}}$ by country and year. *Notes:* $C(F)_{\text{between}}$ captures mean gaps across circumstance-defined types (ex-ante inequality of opportunity). Lower values indicate smaller between-type differences (closer to the *EOp*1 benchmark)

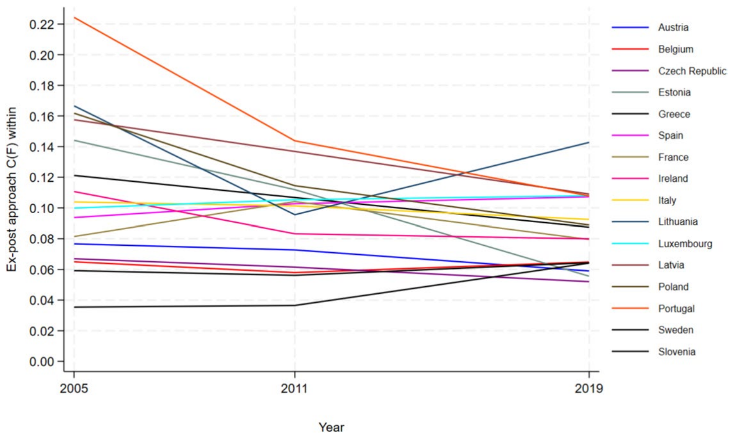


**Fig. 2** Ex-post approach: evolution of $C(F)_{\text{within}}$ by country and year. *Notes:*$C(F)_{\text{within}}$ measures within-tranche inequality. Lower values indicate more homogeneous rewards to the same relative effort across circumstance types (closer to the *EOp*2 benchmark)

ward. These paths indicate that some countries compressed mean circumstance gaps, while others saw an expansion of ex-ante differences despite contemporaneous movements in the ex-post dimension.

Figure 2 traces the ex-post rule of equality of opportunity by plotting $C(F)_{within}$. Because tranches hold relative effort constant in the Roemer sense (same within-type rank), a decline in $C(F)_{within}$ signals more equal treatment of equal *relative* effort across circumstance types, i.e., progress toward *EOp*2. A broad pattern is visible: many countries experienced a marked reduction in ex-post dispersion from 2005 to 2011, followed by milder and more heterogeneous movements thereafter. Portugal exhibits the clearest improvement, falling from the highest initial level to values close to the cross-country median by 2019, consistent with a substantial compression of outcome variability conditional on effort. Estonia and several Western European economies also trend downward, indicating increasingly uniform rewards to effort across types. Two moderate exceptions are Lithuania and Sweden: Lithuania dips until 2011 and then rebounds, while Sweden rises from a low base throughout the period. By 2019 there is some convergence in $C(F)_{within}$, although levels remain higher in several Southern and Baltic countries than in the Nordics and parts of Central Europe.

Taken together, the two panels show that progress on opportunity equality can arise along different margins and need not be synchronized. For instance, Portugal improved on both fronts – lower $C(F)_{within}$ and lower $C(F)_{between}$ – suggesting simultaneous gains toward *EOp*2 (more even rewards to equal effort) and *EOp*1 (smaller average gaps between types). Sweden, by contrast, combines a rise in ex-post dispersion with a sharp increase in ex-ante between-type inequality. Countries with roughly stable $C(F)_{between}$ but declining $C(F)_{within}$ achieved improvements primarily by harmonizing the reward structure conditional on effort rather than by compressing average circumstance gaps.

**Evidence from Definitions 1 and 2** Figure 3 tracks, for four illustrative countries, the evolution of overall dispersion *C*(*F*) (left axis, blue), the ex-ante deviation *EOp*1 (left axis, red),

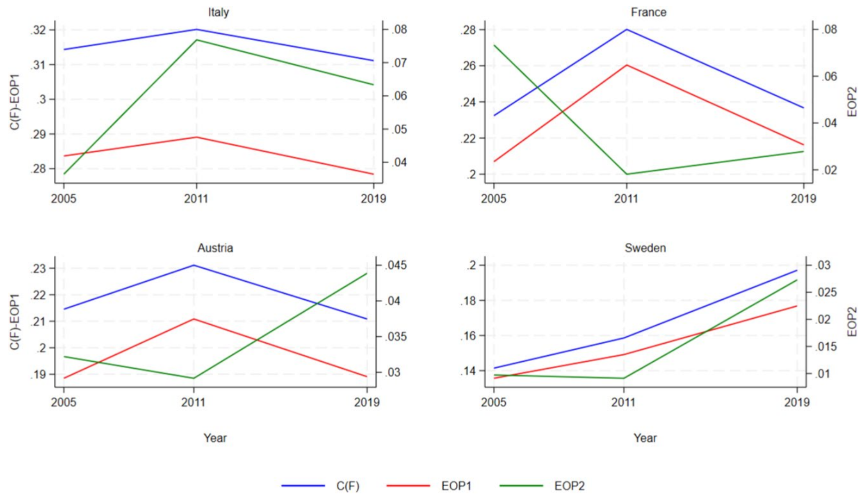


**Fig. 3** $C(F)$, $EOp1$, and $EOp2$ for some countries: Italy, France, Austria, Sweden

and the variance-based deviation $EOp2$ (right axis, green). The four cases were chosen to represent distinct regimes of inequality dynamics, while results for all the considered countries together are reported in Appendix C.

For Italy, overall dispersion is fairly stable and mildly hump-shaped. The ex-ante component ($EOp1$) moves in parallel, indicating that changes in mean gaps across circumstance types account for most of the small movements in $C(F)$. By contrast, the variance-based diagnostic ($EOp2$) spikes in the middle year and then recedes, suggesting a temporary widening and subsequent partial re-alignment of reward structures for equal relative effort across types. The Italian pattern is therefore one where ex-post inequality of opportunity remains a relevant margin even when average gaps are modest. France displays a different configuration: the rise and subsequent decline of $C(F)$ is mirrored by $EOp1$, while $EOp2$ stays low and comparatively flat. This points to changes in average outcomes between types as the key driver of total inequality, with little evidence of systematic divergence in the dispersion faced by equal-effort individuals across types. Austria exhibits a re-balancing over time. Overall dispersion increases and then falls, with the ex-ante share ($EOp1$) peaking at the midpoint, while $EOp2$ first dips and then rises markedly. The late increase in the variance-based diagnostic signals growing heterogeneity in outcomes among equal-effort tranches across types, even as average gaps narrow. The Austrian case illustrates how an exclusive focus on means can understate emerging ex-post disparities. Sweden shows a broadly monotonic increase in all three series. Both mean differences across types and dispersion conditional on tranche intensify, though from relatively low initial levels. This joint movement suggests a generalized widening of inequality of opportunity, simultaneously along the ex-ante and variance-based margins.

Figure 4 reports the joint $EOp$ index in Eq. 6 for the same four countries under alternative normative weights $\beta \in \{0.2, 0.4, 0.6, 0.8\}$. Lower $\beta$ puts more emphasis on variance equalization across types (the $EOp2$ dimension), higher $\beta$ on mean equalization (the $EOp1$ dimension). In Italy the index is consistently higher when $\beta$ is small and its time profile is

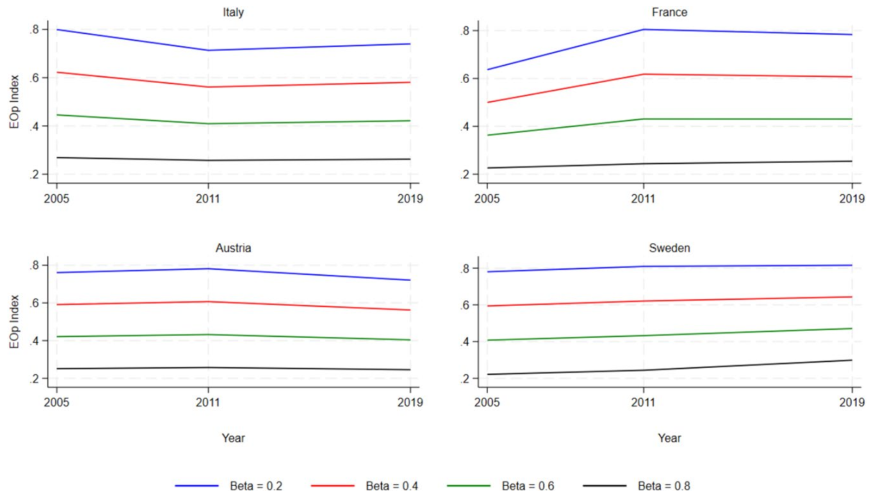


**Fig. 4** EOp Index under different $\beta$ values over years for some countries: Italy, France, Austria, Sweden

driven by the variance component, confirming that the main residual fairness concern lies on the ex-post margin. France improves between the first two waves across all $\beta$ and then exhibits a mild reversal, with the gain most apparent at higher $\beta$, again consistent with movements concentrated on ex-ante gaps. Austria is non-monotonic: improvements when means receive more weight ($\beta$ large) are offset by worsening when variance receives more weight ($\beta$ small), in line with the late increase in $EOp2$ observed above. Sweden worsens across the board, and the deterioration is strongest for low $\beta$, indicating that the expansion of within-type dispersion across equal-effort tranches is the dominant concern. Results for all European countries together are reported in Appendix D.

## 8 Concluding Remarks

In this paper we proposed a new methodology to estimate and explain EOp using a variance-based decomposition method, taking the coefficient of variation (CV) as the cornerstone measure. Our model combines two normative conditions – $EOp1$ and $EOp2$ based on Roemer (1998) canonical theory. This creates a clear distinction between inequality that arises from morally irrelevant factors beyond people's control and inequality due to their choices (and the relative efforts of people).

Our emphasis on variance as the fundamental statistical scale underlying all three EOp definitions aligns with classical decompositions in the inequality literature and preserves interpretability. We rely on the Coefficient of Variation because it preserves the variance-based nature of our framework while making the analysis empirically comparable across countries and over time. Its scale-invariance ensures that differences in mean income do not distort cross-country rankings, and its standardized form allows a consistent interpretation of inequality levels and their decomposition. In this sense, the CV is not an auxiliary choice but a suitable normalized expression of variance for a multi-country, multi-period EOp setting.

A key theoretical breakthrough is the explicit link between the ex-ante view of opportunity equality (*EOp*1), which looks at how average outcomes are equal across sets of circumstances (types), and the ex-post view (*EOp*2). In particular, *EOp*2 calls for consistency in the diversity of results within groups. People with the same amount of effort (tranche) should experience the same level of uncertainty and outcome dispersion, independently of their initial conditions. We also combined the first two definitions into a third one (*EOp*3), thus requiring the means and variances of all groups to be equal at the same time. This is a challenging but useful standard.

Rather than advocating for *EOp*3 as a separate ethical ideal, we treat it as a diagnostic benchmark. By combining *EOp*1 and *EOp*2 in a synthetic index, we are able to explore how different normative priorities – i.e., the weight placed on mean versus variance fairness – affect empirical rankings and trends.

Using EU-SILC data, our empirical analysis demonstrates the methodological robustness and practical relevance of this unified CV-based approach. Results highlight significant variation in inequality of opportunity across European countries. While the between-group inequality component – captured by the Pearson index $\eta^2$ – remains stable over time, its magnitude varies substantially across countries. Austria exhibit relatively higher levels of between-group inequality, signaling substantial structural disparities, while Sweden shows notably lower levels, indicating more homogeneous opportunity structures.

An essential insight derived from our empirical evaluation is the policy significance of *EOp*2: by analyzing the uniformity of within-group variances, policymakers can identify whether reward structures associated with effort is equitable across circumstance groups. Differences from equality in within-group variances indicate heterogeneity in the (un)certainty of outcomes people face with similar relative effort, suggesting a veil of hidden structural inequalities not captured by group means. Hence our weighting scheme that includes *EOp*1 and *EOp*2 provides a timely form of targeted instrument for policymakers to intervene in cases of unfair inequality using both mean-oriented and variance-oriented perspective.

Importantly, our approach has clear policy implications as well. When huge between-group disparities are detected, this suggests that interventions should be geared towards tackling structural discrimination through affirmative action, earmarked education directed at specific groups, and support mechanisms to allow underrepresented or disadvantaged communities to achieve a fairer playing field. Conversely, large negative deviations from equal within-group variances indicate the presence of circumstances that inject risk into ordinary lives and thus call for group-level policies that provide a degree of consistency and predictability of outcomes for comparable levels of relative effort across groups.

We also emphasize that expressing *EOp*1 and *EOp*2 on a shared variance-based scale enables a transparent diagnostic of where fairness violations are located: in average outcomes across types or in the structure of rewards to relative effort. This clarity is essential for designing precise and normatively coherent interventions.

Nonetheless, our results suggest that we should be cautious about assessing equality of opportunity along only a single dimension. Even countries exhibiting a low level of between-group inequality (*EOp*1 satisfied) may have a large within-group variability, underscoring the importance of considering both *EOp*1 and *EOp*2. As such, future research should continue to refine empirical applications, including the use of longitudinal data where available, and examining how institutional contexts shape opportunity inequality through its mean and variance components.

Ultimately, our work offers both theoretical clarity and empirical specificity concerning the dual normative focus required by studies of equality of opportunity. It lays fertile ground for follow-up research, underscoring the necessity of novel responses to systemic inequities and meritocratic rules of economic reward across diverse socioeconomic contexts.

## Appendix A Comparative Properties of $\eta^2$ with EOp Measures

Here we clarify how the variance share $\eta^2$ in Eq. 4 relates to regression-based $R^2$, and how our CV-based decomposition compares with other decomposable indices in the Generalized Entropy family, notably the Mean Log Deviation (MLD) and the Theil index. Using a single variance metric aligns the ex-ante share ($\eta^2$) with the mean-based CV decomposition in Section 3.

Our aim here is to clarify its central role in aligning standard ANOVA-based decompositions with our CV-based framework. This allows us to express both EOp1 (mean-based inequality) and EOp2 (variance-based dispersion) using a unified statistical language.

By the law of total variance,

$$\eta^2 \;=\; \frac{\sum_{i=1}^{m} f_i(\mu_i - \mu)^2}{\mathrm{Var}(Y)} \;=\; \frac{\mathrm{Var}\big(\mathbb{E}[Y \mid C]\big)}{\mathrm{Var}(Y)}$$

is the share of total dispersion on the variance scale that is attributable to circumstances $C$ (the between-group component), with the complement $(1 - \eta^2)$ equal to the within-group share, resonating with the ex-ante view of factors beyond individual control. The same logic applies whenever groups are defined by any observed partition – be it by circumstances or, in tranche-based analyses, by relative-effort strata – so that the between term captures systematic differences across groups, while the within term reflects residual dispersion within them.

Two properties are central for cross-country EOp work: (i) scale invariance (rescaling any outcome $Y$ by any positive constant leaves $\eta^2$ unchanged), and (ii) direct ex-ante interpretability, because $C$ is the only source behind the between component.

Running an OLS regression of $Y$ on a full set of mutually exclusive and exhaustive circumstance dummies (no other covariates), the fitted values are exactly the group means $\mu_i$ and the explained sum of squares is exactly the between-group variance. In that specification,

$$R^2 \;=\; \eta^2.$$

Thus, $\eta^2$ reproduces the familiar 'share explained by circumstances' from Roemer-type regressions, but without functional-form choices or controls. When parametric models add functional forms, interactions, or controls, the reported $R^2$ becomes specification-dependent, whereas $\eta^2$ remains a model-free benchmark anchored in the same between/within identity used in Section 3.

**Partial and Nested Decompositions** Because $\eta^2$ is built from the law of total variance, it extends naturally to multiple circumstance dimensions through nested partitions and partial contributions (e.g., gender, migration status, and their interaction), preserving additivity on the variance scale. This makes the mapping from 'which circumstances matter?' to 'how much they explain?' transparent and comparable to the CV terms used throughout the paper.

**Alignment with the CV Decomposition and EOp Conditions** Recall that $C(F) = \mathrm{Var}(Y)/\mu^2$ and the CV decomposition in Section 3 follows as:

$$C(F) = \underbrace{\sum_{i=1}^{m} f_i C(F_i)\, \alpha_i^2}_{C(F)_{within}} + \underbrace{\sum_{i=1}^{m} f_i (\alpha_i - 1)^2}_{C(F)_{between}}, \quad \alpha_i = \mu_i/\mu.$$

Dividing both terms by *C*(*F*) shows that

$$\frac{C(F)_{between}}{C(F)} = \frac{\mathrm{Var}\big(\mathbb{E}[Y \mid C]\big)}{\mathrm{Var}(Y)} = \eta^2,$$

so your ex-ante EOp share is numerically identical whether read on the CV scale or on the raw variance scale.

This alignment also justifies our use of the CV: while our decomposition is rooted in the variance scale, the CV enables normalized cross-context comparison, and the interpretation of $\eta^2$ as the ex-ante share remains intact.

**Path-dependence and Shapley-Shorrocks Attributions** Another point of additive decompositions is path-dependence (with the exception of the MLD): the contribution of each factor (e.g., parental background, gender, migration status) can vary depending on the order in which factors are removed from the inequality calculation. To address this, the literature has proposed Shapley-Shorrocks decompositions, which attribute contributions by averaging over all possible orderings (Shorrocks, 1982; Mookherjee & Shorrocks, 1982). Our variance-based framework offers a parallel solution: the law of total variance permits nested partitions that yield partial $\eta^2$'s, decomposing $\mathrm{Var}(\mathbb{E}[Y|C])$ into additive shares of each circumstance dimension and their interactions. These partial $\eta^2$'s are directly interpretable on the variance scale, thereby preserving comparability with the CV decomposition and avoiding arbitrary path-dependence.

This decomposition is fully compatible with our empirical framework, as it allows for robust attribution of responsibility to each circumstance dimension while remaining consistent with the logic behind $EOp1$. Our approach thus integrates normative and statistical considerations in a single interpretable structure.

**Comparison with the Generalized Entropy Family** The squared CV belongs to the Generalized Entropy family with parameter $\alpha = 2$,

$$GE(2) = \tfrac{1}{2}\, C(F),$$

and inherits exact additive decomposability by subgroups. Within the GE family, sensitivity to the upper tail increases with $\alpha$; hence *GE*(2) (and *C*(*F*)) puts relatively more weight on large deviations above the mean than $GE(1)$ (Theil) or $GE(0)$ (MLD). This can be normatively desirable when high-end disparities across circumstances are of particular concern.

For reference, the between-group components of MLD and Theil are

$$MLD_{\text{between}} = \sum_i f_i \ln\left(\frac{\mu}{\mu_i}\right), \qquad Theil_{\text{between}} = \sum_i f_i \frac{\mu_i}{\mu} \ln\left(\frac{\mu_i}{\mu}\right).$$

These are log-scaled divergences: their 'between over total' ratios can be reported as robustness checks, but they do not read as variance shares and are not directly aligned with the variance-based $EOp2$ condition. By contrast, $\eta^2$ is bounded in [0, 1], scale invariant, and directly interpretable as a variance share, making it particularly transparent alongside our CV-based framework.

Our CV-based measure therefore offers a coherent normative lens for both ex-ante and ex-post fairness diagnostics. While MLD and Theil provide valuable distributional perspectives, only $C(F)$ permits simultaneous comparability across units and transparent mapping to $\eta^2$, which is critical for EOp analysis.

Table 6 summarizes the main trade-offs across metrics discusses here.

**Comparing Ex-ante and Ex-post Components Across Indices** Table 7 reports, for three standard decomposable indices, two separate decompositions of the same outcome distribution: the ex-ante component (between types defined by circumstances) and the ex-post component (within tranches of relative effort). Because these columns arise from distinct partitions of the population, the ex-ante and ex-post entries are **not** complements and must not be added. Moreover, absolute magnitudes are index-specific (variance scale for $C(F)$; entropy/log scales for MLD and Theil), so cross-row level comparisons are not meaningful; the informative reading is within each column.

Indeed, across all three metrics the between-type signal is modest. The variance-based $C(F)$ yields a between component of 1.035, while the entropy measures are definitively lower (0.019 for MLD and 0.018 for Theil). The tight clustering of MLD and Theil suggests that average gaps across circumstance groups are limited and not concentrated exclusively in either tail of the distribution. The somewhat larger figure under $C(F)$ is consistent with its connection to $GE(2)$ (greater weight on large deviations) and with the variance scale capturing upper-tail spreads more prominently. In EOp terms, these values indicate only

**Table 6** Summary comparison for EOp applications

| Metric | Scale | Group-additive | Ex-ante share | Tail sens. |
|---|---|---|---|---|
| $\eta^2$ (variance share) | Variance | Yes (ANOVA) | Yes ($\in [0,1]$) | Neutral |
| $C(F)$ (CV; $2\,GE(2)$) | Var./$\mu^2$ | Yes | $C_{\text{between}}/C = \eta^2$ | Upper ($\alpha = 2$) |
| Theil ($GE(1)$) | Entropy/log | Yes | Ratio-specific | Upper (mod.) |
| MLD ($GE(0)$) | Entropy/log | Yes | Ratio-specific | Bottom |
| $R^2$ (type dummies) | Variance | Yes ($= \eta^2$) | Yes (that spec.) | Spec.-dep. |

**Table 7** Comparisons across metrics

| Metric | Ex-ante (Between types) | Ex-post (Within tranches) |
|---|---|---|
| $C(F)$ | 0.035 | 0.162 |
| MLD | 0.019 | 0.093 |
| Theil | 0.018 | 0.082 |

moderate departures from the ex-ante benchmark (*EOp*1), i.e., mean outcomes across types are relatively close.

Instead, within-tranche dispersion is materially larger in all indices: 0.162 for $C(F)$, 0.093 for MLD, and 0.082 for Theil. While levels are not comparable across rows, the consistent ordering $C(F) > \text{MLD} > \text{Theil}$ points to a robust pattern: conditional dispersion among equally ranked effort tranches is sizeable whatever the metric, and it is especially visible when large deviations receive more weight. That MLD (bottom-sensitive) exceeds Theil (more upper-tail sensitive) suggests that ex-post inequality is not solely a top-tail phenomenon: dispersion among equal-effort individuals is also salient in the lower part of the distribution. In EOp terms, these findings highlight a pronounced ex-post component (*EOp*2): even when types are held fixed, outcomes for equal relative effort remain widely dispersed.

It seems that the three indices deliver a consistent message: structural differences across types (ex-ante) are present but comparatively limited, whereas within-tranche heterogeneity (ex-post) is substantial. This motivates a dual policy focus: mean-equalizing interventions across circumstance groups to address *EOp*1, coupled with measures that harmonize the reward structure for similar relative effort to improve *EOp*2.

## Appendix B Descriptive Statistics

Table 8 reports descriptive statistics for the outcome variable (household income) and for the main circumstance proxies used to define types in our EOp analysis. With the exception of income, all variables are binary indicators. Women represent 52.3% of the pooled sample. This near balance suggests that gender-defined types will be sufficiently populated to support precise ex-ante comparisons. The share of foreign-born individuals is 11.2%.

**Table 8** Descriptive statistics

| | Obs. | Mean | Variance |
|---|---|---|---|
| Household income | 308,478,816 | 15,593.151 | 128,057,889.045 |
| Female | 308,478,816 | 0.523 | 0.250 |
| Born Abroad | 308,478,816 | 0.112 | 0.099 |
| Parents low educ | 308,478,816 | 0.695 | 0.201 |
| Parents medium educ | 308,478,816 | 0.163 | 0.123 |
| Mother high educ | 308,478,816 | 0.036 | 0.030 |
| Father high educ | 308,478,816 | 0.058 | 0.055 |
| Parents high educ | 308,478,816 | 0.047 | 0.043 |
| Parents don't work | 308,478,816 | 0.069 | 0.066 |
| Parents blue collar | 308,478,816 | 0.209 | 0.164 |
| Mother white collar | 308,478,816 | 0.311 | 0.227 |
| Father white collar | 308,478,816 | 0.086 | 0.072 |
| Parents white collar | 308,478,816 | 0.326 | 0.210 |

Given the well-documented association between migration background and economic outcomes across European contexts, this variable is expected to be an informative circumstance dimension in the between-type component.

The distribution of parental schooling is skewed toward the lower end: $69.5\%$ of respondents report low parental education, while $16.3\%$ fall in the medium category. The indicator for Parents high education takes value 1 for a relatively small fraction ($4.7\%$), and the separate parental indicators for mother high education $3.6\%$ and father high one $5.8\%$ confirm the rarity of high-educated parental backgrounds in the pooled data. This composition anticipates potentially non-negligible ex-ante disparities if returns to parental education translate into systematic mean gaps across types. At the same time, the relatively small high-education group implies that precision for that type may be more sensitive to sampling error in country-year cells.

Regarding family labour status, $6.9\%$ of observations report non-working parents, and $20.9\%$ report a blue-collar background. White-collar backgrounds are comparatively more frequent at the household level ($32.6\%$), with the separate parent indicators for mother and father, showing respectively $31.1\%$ and $8.6\%$. Because these variables are not exclusive, the household-level indicator will typically sit between the maximum of the two parental shares and their sum, as observed here. Cross-country classification differences in occupational taxonomies may affect these levels, but the pattern points to a sizeable fraction of respondents growing up in non-manual households, a circumstance that can underpin mean income differences across types in the ex-ante decomposition.

Household income displays the expected high dispersion, with an average of $15,593.151$ monetary units and a large variance ($128,057,889.045$). This heavy-tailed dispersion is precisely why we adopt variance-based tools (the CV and its within/between decomposition): they are scale-invariant and map directly into our $\eta^2$ share and $EOp2$ diagnostics. The descriptive dispersion reported here provides the baseline against which we subsequently partition inequality into between-type (ex-ante) and within-type (variance-based, ex-post) components.

About the robustness and representativeness of type definitions, the sample composition ensures sufficient granularity for each circumstance variable. All types are constructed from combinations of these binary or categorical proxies, and tranches are further defined conditional on effort ranks within types. Our balanced distribution of gender and coverage across parental background categories makes both the ex-ante and ex-post decompositions empirically reliable. The observed heterogeneity in circumstance variables provides a solid empirical foundation to interpret inequality patterns using the $C(F)$ and $\eta^2$ frameworks, as well as to apply the $EOp1$ and $EOp2$ diagnostics without relying on restrictive functional assumptions.

## Appendix C Decomposition of $C(F)$, $EOp1$, and $EOp2$ - all European Countries

This appendix provides a country-by-country reading of the joint evolution of overall dispersion $C(F)$ (left axis, blue) and our two opportunity diagnostics: the ex-ante benchmark $EOp1$ (left axis, red), capturing mean differences across circumstance types, and the variance-based ex-post measure $EOp2$ (right axis, green), capturing dispersion within tranches

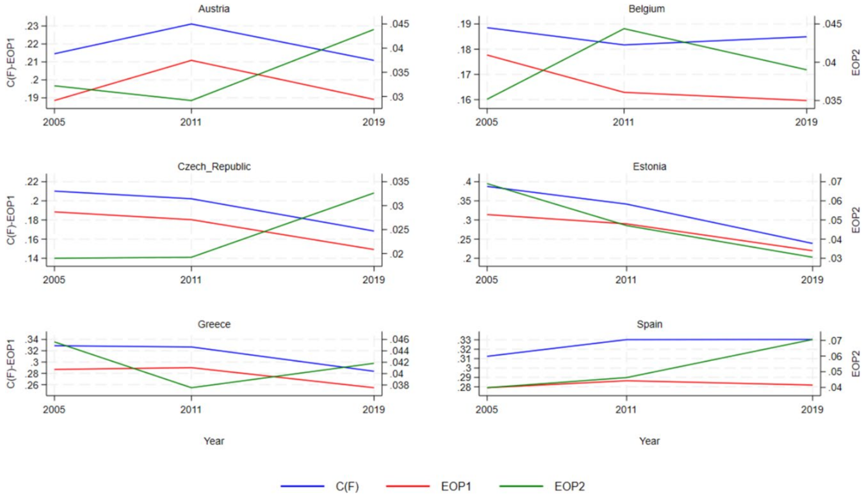


**Fig. 5** Time paths of $C(F)$ (left axis), $EOp1$ (left axis), and $EOp2$ (right axis): Austria, Belgium, Czech Republic, Estonia, Greece, Spain

of relative effort. Figures 5-7 report three snapshots (2005, 2011, 2019) for sixteen European countries.

Starting with Fig. 5, Estonia shows a broad-based decline: $C(F)$, $EOp1$, and $EOp2$ all fall from 2005 to 2019, indicating improvements both in mean equality across types and in the homogeneity of outcomes conditional on type and relative effort. The Czech Republic also exhibits a reduction in $C(F)$ and $EOp1$, but $EOp2$ increases by 2019; in this case, convergence in group means coexists with a rise in within-tranche dispersion. Austria displays a hump in overall dispersion with $C(F)$ and $EOp1$ peaking around 2011, while $EOp2$ ends higher in 2019, pointing to a growing contribution of ex-post heterogeneity in the most recent year. Belgium changes only modestly: $EOp1$ drifts down and $EOp2$ follows a mild hump pattern, so the structure of inequality remains fairly stable. Greece records gradual reductions in $C(F)$ and $EOp1$ with a slight U-shape in $EOp2$, consistent with slow convergence of type means and broadly stable conditional dispersion. Spain shows a slight increase in $C(F)$ with nearly flat $EOp1$ and a clear rise in $EOp2$, suggesting that the recent gains in total dispersion are driven mainly by ex-post differences among equal-effort individuals rather than by mean gaps across types.

Turning to Fig. 6, France exhibits a hump-shaped profile: $C(F)$ and $EOp1$ rise to 2011 and then fall, while $EOp2$ drops sharply between 2005 and 2011 and stabilizes at a lower level. The temporary peak in overall dispersion is thus primarily an ex-ante phenomenon. Ireland moves in the opposite direction: $C(F)$, $EOp1$, and $EOp2$ all decline, revealing a simultaneous improvement on both margins of opportunity. Italy remains relatively stable overall; $C(F)$ and $EOp1$ peak in 2011 and then recede, whereas $EOp2$ ends above its 2005 level, indicating that conditional dispersion plays a larger role at the end of the period than at the beginning. Lithuania shows a pronounced U-shape in all three series, with a notable rebound in $EOp2$ by 2019; this points to growing heterogeneity among individuals exerting similar relative effort. Luxembourg experiences a gradual increase in $C(F)$ and $EOp1$ with a mild decline in $EOp2$, suggesting that widening mean gaps across types are the main driver

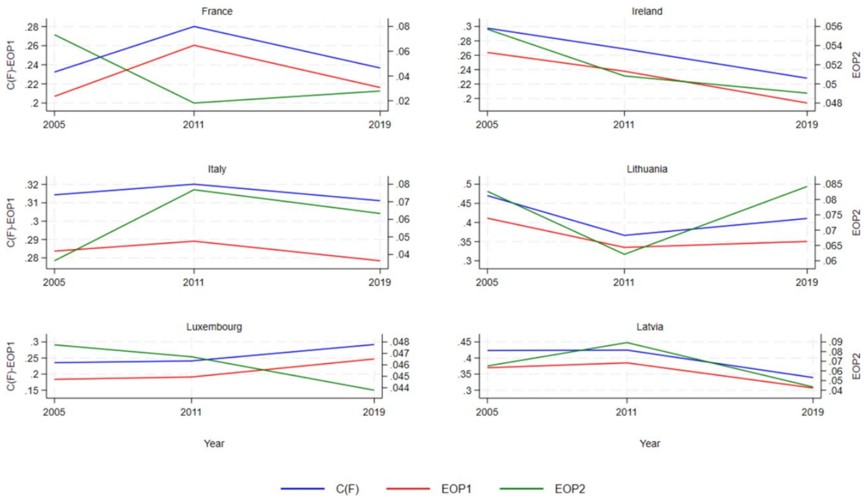


**Fig. 6** Time paths of $C(F)$ (left axis), $EOp1$ (left axis), and $EOp2$ (right axis): France, Ireland, Italy, Lithuania, Luxembourg, Latvia

there. Latvia displays a hump around 2011 in both $C(F)$ and $EOp1$ followed by declines, while $EOp2$ trends downward throughout, consistent with improvements concentrated on the ex-post margin.

Finally, Fig. 7 highlights four distinct cases. Poland shows a marked and monotone decrease in $C(F)$, mirrored by declines in both $EOp1$ and $EOp2$; the reduction in overall dispersion is therefore shared by ex-ante and ex-post components. Portugal follows a very similar path, with concerted falls in all three series. Sweden is the outlier: $C(F)$, $EOp1$, and $EOp2$ all increase, indicating that both widening mean differences across types and growing

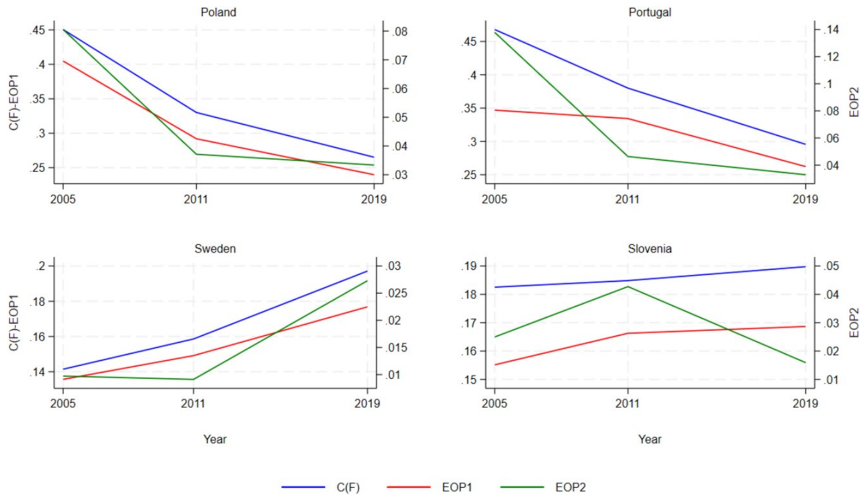


**Fig. 7** Time paths of $C(F)$ (left axis), $EOp1$ (left axis), and $EOp2$ (right axis): Poland, Portugal, Sweden, Slovenia

within-tranche dispersion contribute to higher overall inequality. Slovenia records small increases in $C(F)$ and $EOp1$ by 2019, while $EOp2$ rises to 2011 and then falls below its mid-period level; in that setting, the mild growth in total dispersion appears to be driven mainly by between-type differences.

From a general perspective, we can confirm that movements in overall dispersion can be traced to either margin: in several countries, $C(F)$ changes while $EOp1$ and $EOp2$ move in opposite directions, underscoring the need to monitor both ex-ante and ex-post conditions. Second, improvements on both margins are observable (Estonia, Ireland, Poland, Portugal), whereas Sweden offers a clear counterexample with simultaneous deterioration. Third, in countries such as Luxembourg and Slovenia the ex-ante margin dominates recent changes, while in Spain and, to a lesser extent, Austria and Italy the ex-post margin becomes comparatively more salient. This heterogeneity in drivers is precisely what the dual reading of $EOp1$ and $EOp2$ is designed to reveal.

## Appendix D Measurement Under EOp Index

Figures 8-10 display the EOp index defined in Eq. 6 for four values of the weight parameter $\beta \in 0.2, 0.4, 0.6, 0.8$ across the years 2005, 2011, and 2019, covering all 16 European countries. Two regularities are immediate and informative. First, in every country the curves are *monotonically ordered* by $\beta$ (the index is largest when $\beta = 0.2$ and smallest when $\beta = 0.8$), indicating that variance-based deviations ($EOp2$) are systematically larger than mean-based deviations ($EOp1$). Second, temporal movements are generally moderate, but not uniform: several countries display clear directional changes, while others remain remarkably stable. Together, these patterns validate the usefulness of a joint metric: it reveals both the relative weight of $EOp1$ versus $EOp2$ (the vertical spread across $\beta$) and the direction of change through time (the slope for each $\beta$).

**Poland, Portugal, Sweden, Slovenia (Fig. 8)** For *Poland* the index is high and remarkably stable across all $\beta$, with a slight rise between 2005 and 2011 and near-constancy thereafter; the wide vertical gap between $\beta = 0.2$ and $\beta = 0.8$ signals that $EOp2$ deviations dominate the joint assessment. *Portugal* shows a small increase for low $\beta$ and near-flat or mildly declining trajectories for higher $\beta$, pointing to mild improvements when mean differences are weighted more, and little change when the focus remains variance-based. *Sweden* displays a clear deterioration across *all* $\beta$, with the gap between curves widening over time; this indicates that both mean- and variance-based fairness receded, with $EOp2$ playing an increasing role. *Slovenia* exhibits a U-shaped profile: the index falls from 2005 to 2011 and rises again by 2019; the ordering across $\beta$ persists throughout, so $EOp2$ remains the larger source of deviation.

**France, Ireland, Italy, Lithuania, Luxembourg, Latvia (Fig. 9)** *France* improves notably from 2005 to 2011 (across all $\beta$) and then stabilizes; both EOp1 and $EOp2$ contributions ease, with a persistent dominance of $EOp2$ (vertical spread remains wide). *Ireland* is strikingly stable at all weights, suggesting persistent opportunity structures with limited reallocation across types or tranches. *Italy* improves around 2011 and partially reverses by 2019; the gap across $\beta$ stays large, indicating that variance-based heterogeneity within types continues to

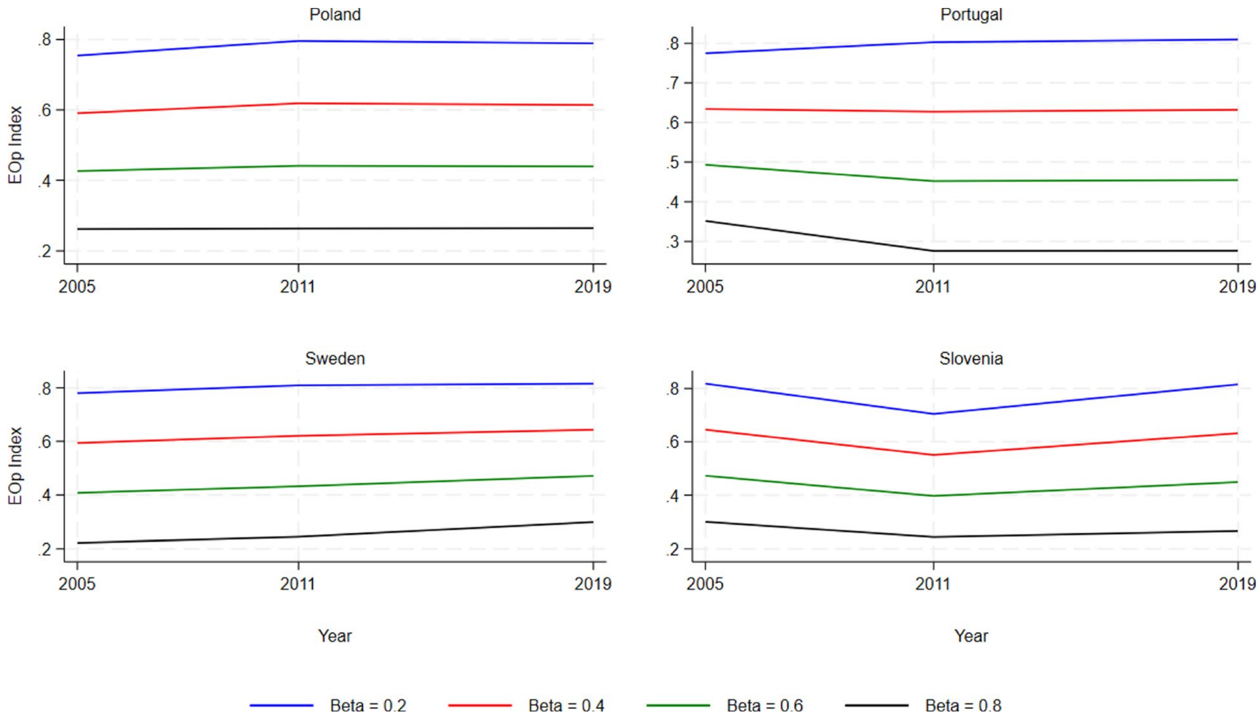


**Fig. 8** EOp Index by $\beta$ (Poland, Portugal, Sweden, Slovenia). Lines correspond to $\beta \in \{0.2, 0.4, 0.6, 0.8\}$ (blue, red, green, black). Lower values indicate conditions closer to EOp

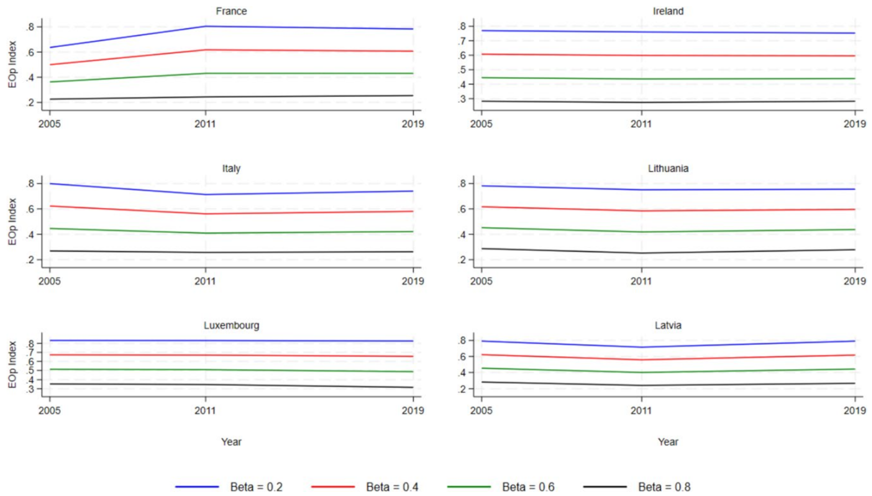


**Fig. 9** EOp Index by $\beta$ (France, Ireland, Italy, Lithuania, Luxembourg, Latvia). Lines correspond to $\beta \in \{0.2, 0.4, 0.6, 0.8\}$ (blue, red, green, black)

drive the diagnosis. *Lithuania* follows a mild U-shape (decline then slight rebound), again with *EOp*2 larger than EOp1 throughout. *Luxembourg* is one of the most stable cases in the sample; despite small declines for higher $\beta$, the four curves remain nearly parallel, implying a steady balance between mean and variance components. *Latvia* is broadly flat with a slight uptick by 2019; the vertical ordering remains pronounced.

**Austria, Belgium, Czech Republic, Estonia, Greece, Spain (Fig. 10)** *Austria* shows a gentle improvement by 2019 at every $\beta$, with slightly tighter spacing between curves; both *EOp*1 and *EOp*2 deviations fall, and variance-based gaps shrink marginally. *Belgium* moves in the opposite direction, with a mild deterioration across *all* weights, consistent with a worsening in both mean and variance dimensions. *Czech Republic* displays one of the clearest improvements: the joint index declines monotonically for every $\beta$, and the gap between curves narrows somewhat, suggesting that gains are not confined to means but extend to the dispersion conditional on type. *Estonia* is essentially flat, with very modest movement and persistent *EOp*2 dominance. *Greece* is also notably stable over the full window, indicating that neither mean gaps nor variance spreads shifted much. *Spain* shows small changes only, with a slight easing at higher $\beta$ by 2019, again under a stable ordering across weights.

As a general perspective, the consistent ordering across $\beta$ confirms that, in this sample and period, variance-based deviations (*EOp*2) are the principal obstacle to joint equality of opportunity: even where mean gaps are modest, conditional dispersion within types remains sizable. Second, improvements – when they occur – are rarely confined to one dimension: the clearest success cases (e.g., Czech Republic; to a lesser extent Austria and France) show progress across *all* weights, pointing to concurrent reductions in both *EOp*1 and *EOp*2 components. Third, several countries are highly persistent (Ireland, Luxembourg, Greece, Estonia), underscoring that institutional configurations may lock in the balance between mean and variance dimensions of opportunity for long stretches of time.

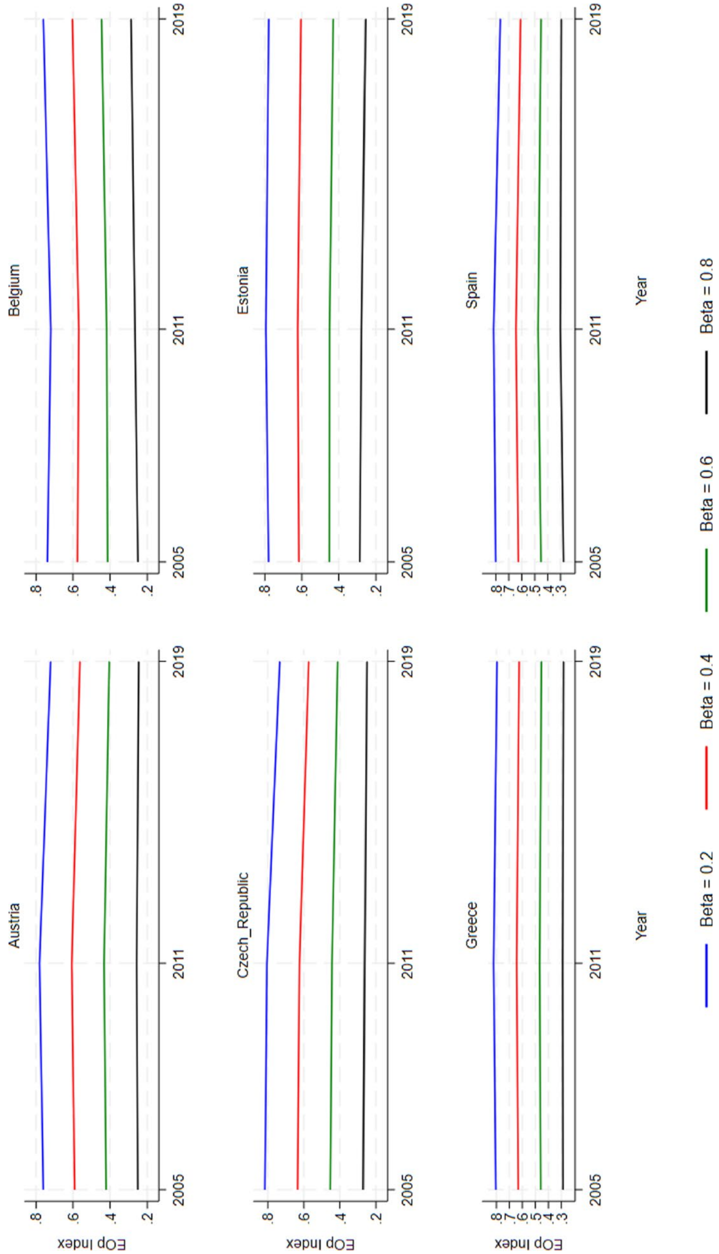


**Fig. 10** EOp Index by $\beta$ (Austria, Belgium, Czech Republic, Estonia, Greece, Spain). Lines correspond to $\beta \in \{0.2, 0.4, 0.6, 0.8\}$ (blue, red, green, black)

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**Data Availability** This study uses microdata from the European Union Statistics on Income and Living Conditions (EU - SILC), accessed under a restricted data agreement within the framework of the project Inequality of Opportunity in Europe and the role of Education to Fight Multidimensional Deprivation. Data are not publicly available and were used in compliance with the terms and conditions set by Eurostat.

## Declarations

**Conflicts of interest** The authors have no financial or proprietary interests in any material discussed in this article.



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