Default Risk Premium and Asset Prices

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Abstract

We estimate a standard structural model of credit risk to draw insights about the premium demanded by investors for bearing default risk, using data on credit default swaps and market capitalization. We pin down the daily market value of assets for a set of non-financial firms and uncover cross-sectional heterogeneity in terms of the magnitude and time variation of the premium. By exploring the link between asset and default risk premia, we show that this heterogeneity closely depends on the relationship between the firm-specific market values of the assets and the business cycle.

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1. Introduction

This paper analyzes the premium demanded by investors for bearing the risk of corporate default. Investors do not price assets only on the basis of the actual default risk of the firm, but they build in an extra return that compensates for the risk they are bearing. The premium arises from the wedge between the market valuation of the firm's default risk, measured by the risk-neutral default probability, and the actual default risk of the firm, measured by the actual or real-world default probability (Hull et al. (2005)).

Specifically, we define the premium as the ratio between the risk-neutral and the real-world default probabilities, as in Driessen (2005) and Berndt et al. (2018). The premium provides a reward for the expected loss in case of default, evaluated using the real-world default probability and multiplied by a factor equal to the default risk premium. Driessen (2005) finds that investors multiply the actual default probability by a factor of approximately 6 for pricing corporate bonds, and Berndt et al. (2018) argue that investors price twice the expected default loss evaluated under the actual default probability.

We estimate daily default risk premia for a set of non-financial firms and document that the premium is substantially time-varying. We confirm that the premium is higher than 1 for the vast majority of the firms. For these firms, we uncover a positive relationship between the premium and the market value of equity and a negative relationship between the premium and credit default swap (CDS) spreads. In addition, we find that both the default premium and the slope of the term structure of the premium are pro-cyclical. However, we show that a significant fraction of firms display a default risk premium lower than 1. For these firms, we document a negative relationship between the premium and the market value of equity and a positive relationship between the premium and CDS spreads, and find that the slope of the term structure of the premium is more volatile, less cyclical and generally negative.

We rationalize our findings by studying the link between the default risk premium, the asset risk premium, and the business cycle. We provide empirical evidence that firms displaying a default premium lower than 1 exhibit a negative asset risk premium and that the market value of their assets is negatively correlated with the business cycle. Therefore, these firms offer an opportunity to hedge a systematic risk factor, such as the business cycle. This justifies a negative asset risk premium and a default premium below 1, as it happens with counter-cyclical and defensive stocks or for the income-hedging motive of stock market participation.[1]

We study the default risk premium through the lens of a simple structural model of credit risk. We adopt a first-time passage model in which the firm defaults as soon as the value of its assets falls below a default boundary, following the seminal papers of Black and Cox (1976) and Longstaff and Schwartz (1995). This framework embeds features and stylized facts of various bankruptcy codes in which bondholders can trigger default and extract value when certain financial ratios fall below certain specified boundaries.

We estimate the model with a non-linear Kalman filter using readily available firm-level data from the credit and stock markets, such as the market value of equity and CDS spreads. The Kalman filter allows us to exploit all of the information contained in the term structure of CDS spreads. Data about the pricing of CDS traded on different maturities provide multiple information that we can use to pin down the daily dynamics of the market value of assets, which is generally unobservable because several debt items are usually either not traded on the market or traded very infrequently.

The market value of assets is typically computed as the sum of the market value of equity and a proxy of the market debt based on book value; data on the latter is often released infrequently. Moreover, the book value may not incorporate new information in a timely manner and is subject to discretionary management. In addition, substantial debt items may be left out of the book and corporate statements are not easy to interpret (Giesecke

[1]Bonaparte et al. (2014) show that individuals are willing to invest in stocks when their labor income shocks are negatively correlated with stock market returns; that is, investment in stocks offers the opportunity to hedge income risk.

(2006)).[2]

Our approach can be applied to firms for which equity and CDS prices are available, even if debt prices are not observable or debt instruments are traded infrequently, as is common for non-defaulting firms. For each firm, we pin down the daily time series of the market value of assets and estimate the default boundary. We report that, on average, the firm defaults when the value of its assets is equal to 76% of the face value of the debt. The default boundaries are generally concentrated at between 60% and 80% of the face value of the debt and between 40% and 60% of the implied market value of the assets. Our estimates of the default barrier are in line with those of Wong and Choi (2009) and slightly higher than those of Perlich and Reisz (2007) and Davydenko (2012). As in Perlich and Reisz (2007), we depict a negative relationship between the default boundary and asset volatility.

Not surprisingly, we show that the vast majority of firms generate rates of return on their assets that are higher than the risk-free rate, thus displaying positive asset risk premia. However, we estimate negative asset risk premia for a considerable fraction of firms — around 20% of our sample. Using principal component analysis (PCA), we point out that the market value of assets of these firms is negatively correlated with the main latent common factor, which is highly correlated with the US GDP. The economic insight of this result is simple but important: when the market value of a firm's assets is negatively correlated with the business cycle, that firm presents an opportunity to hedge a systematic risk factor, and thus displays a negative asset risk premium. We label these firms as counter-cyclical or defensive. In contrast, we show that the implied asset value strongly co-moves with the US GDP when the market value of the firm's assets is positively correlated with the main common factor obtained from the PCA. We refer to these firms as pro-cyclical.

[2]Duffie and Lando (2001) discuss why it is extremely difficult, in practice, for investors in the secondary market for corporate bonds to observe corporate asset values due to noisy and delayed accounting reports or barriers to monitoring. In several scandals (e.g., Enron, WorldCom, and Tyco), the level of assets and liabilities on corporate statements was misrepresented by management.

Next, we link this result to our firm-specific estimates of the default risk premium. We show that the premium is generally higher than 1 and pro-cyclical for pro-cyclical firms, while the premium is lower than 1 and generally counter-cyclical for defensive firms. Time variation in the premium can be associated, in fact, with opposite responses from CDS spreads and equity value. On the one hand, the default premium may increase because the risk-neutral default probability increases more than the real-world default probability. As a consequence, we expect an increase in the CDS spread and a drop in the equity value due to a higher default risk, and thus a positive (negative) relationship between the premium and CDS spreads (equity). The default premium, however, may also increase because the real-world default probability decreases more than the risk-neutral default probability. In this case, we expect to observe a drop in the CDS spread and an increase in the equity value.

We show empirically that the first type of relationship between default risk premium and market data characterizes firms displaying a premium below 1. Importantly, we highlight that firms displaying a default premium lower than 1 are the same firms for which we estimate negative asset risk premia and negative correlations between the market value of their assets and the business cycle. We document, instead, a positive (negative) relationship between the premium and equity value (CDS spreads) for pro-cyclical firms. We show that the correlation between default risk premium and the market value of equity (CDS spreads) switches from negative (positive) to positive (negative) when looking at firms with default premium greater than 1. The sample correlation between default premium and the market value of equity (CDS spreads) is on average equal to 0.64 (-0.29) across pro-cyclical firms and equal to -0.82 (0.56) across defensive firms.

We further investigate the cross-sectional heterogeneity using sector classification. We find that the default risk premium is on average below 1 for defensive sectors and is clearly above 1 for cyclical sectors. Overall, we show that compared to firms from cyclical sectors, firms from defensive sectors display lower or even negative correlations with the main common factor that is highly correlated with the GDP, lower asset risk premia and lower default risk premia. Thus, the heterogeneity across firms that we highlight in terms of asset risk premium, relationship with the business cycle, and default risk premium is consistent with the usual definitions of cyclical and defensive sectors.

The rest of the paper is organized as follows. In the next Section, we position our paper in the extant literature. In Section 3, we present the underlying model. We next bring the model to the data using the methodology described in Section 4. In this Section, we also describe the data. We report our estimation results in Section 5. We focus on the default risk premium in Section 6 and link the premium to both equity and CDS data in Section 7. Section 8 concludes this paper and introduces directions for future research.

2. Related Literature

Our paper speaks to the broad literature that seeks to estimate unobservable firm fundamentals using market-based data and a structural approach. Structural models of credit risk have been widely used to study corporate risky debt and default risk since the pioneer model of Merton (1974), in which default occurs when the firm fails to pay back its debt at maturity. The important advantage of adopting a structural framework is that the model provides simple pricing equations relating the observable data, such as equity and CDS prices, to the unobservable firm fundamentals, such as the market value of the firm's debt and assets.

We adopt a simple and standard structural model, namely the first-time passage model, in which the default occurs as soon as the value of the assets crosses below a default boundary, as in Black and Cox (1976) and Longstaff and Schwartz (1995). We assume a constant and exogenous barrier, so the equity is equivalent to a down-and-out call option written on the value of the corporate assets. As in Perlich and Reisz (2007), we assume that the boundary is constant and strictly positive but no greater than the book value of the liabilities. Perlich and Reisz (2007) and Wong and Choi (2009) discuss why the results of Brockman and Turtle (2003), who report values of the default boundary above the nominal value of the debt, are misleading and driven by debt mispricing. Brockman and Turtle (2003), in fact, proxy the market value of debt with the book value.

The model is simple and very tractable when applied to the data. Notwithstanding several simplifying assumptions and a parsimonious set of parameters, we highlight that the model fits the data well. The model-implied market value of assets that we pin down with our method is highly positively correlated with the equity value and negatively correlated with the CDS spreads. Moreover, a measure of distance-to-default based on our model's estimation results negatively and significantly predicts both CDS spreads and equity returns.

Several papers have attempted to estimate the first-time passage model (Brockman and Turtle (2003), Perlich and Reisz (2007), Wong and Choi (2009), Forte and Lovreta (2012)). Recently, Du et al. (2019) and Huang et al. (2020) proposed a GMM estimation of a barrier-dependent structural model using information from both CDS and equity markets. We estimate the model with a non-linear Kalman filter and, for each firm, reconstruct the daily time-series of the unobservable market value of its assets using information coming from two liquid markets. CDS spreads and equity value represent the observable data that we use to infer the dynamics of the state variable and to estimate the unknown parameters. In the Kalman filter, the prediction of the current value of the unobservable variable is immediately updated once new information becomes available, and the impact of the new information on the update of this prediction is determined endogenously. The structural model, meanwhile, provides the pricing equations that play the role of the measurement equations in the Kalman filtering procedure based on a simple state-space model, while the pricing errors depend on the structural parameters and are used to construct the likelihood function. Inference on the state variable and estimation of the parameters, then, are jointly executed.

We report values of the default boundary around 76% of the face value of debt. Our estimates are slightly higher than those of Perlich and Reisz (2007) and Davydenko (2012), who estimate a default boundary around 66% of the debt value. Perlich and Reisz (2007) include a few bankrupt firms in their sample, while Davydenko (2012) provides results on a sample of defaulting firms, prior to the time of default, for which debt prices are observable.

We also contribute to the literature that studies the premium demanded by investors for bearing default risk, which captures remarkable attention in both the asset pricing and credit risk literature. Hull et al. (2005) compute the ratio between the risk-neutral and the real-world default probabilities by using corporate bond spreads and historical data. Driessen (2005) estimates a default intensity model to derive the default risk premium implied by corporate bond spreads and rating-based default probabilities. Berndt et al. (2018) estimate the premium using 5-year CDS spreads and expected default frequency data from KMV, while Diaz et al. (2013) use a wider term structure of CDS spreads on European firms. We study the default risk premium using data on CDS traded on multiple time horizons and market capitalizations, which we employ as observable variables to estimate the structural model.

Berndt et al. (2018) report a default risk premium of around two. Consequently, they argue that investors price twice the expected default loss evaluated under the actual probability measure. Driessen (2005) finds that investors multiply the actual default probability by a factor close to 6 for pricing corporate bonds. We report a median value of the premium around 2 and an average value around 8. Importantly, we show that while the vast majority of firms display premia higher than 1, a significant fraction of firms is characterized by premia lower than 1. We point out that this cross-sectional heterogeneity closely depends on the relationship between the firm-specific implied market values of assets and the business cycle. We also show that firms displaying premia below 1 belong to sectors that are typically considered counter-cyclical or defensive, while we report that on average premia on default risk are substantially above 1 for firms in pro-cyclical sectors.

Huang and Huang (2012) calibrate different credit risk models with corporate bond spreads and default data from rating agencies and find that the premium decreases with credit quality as well as after crisis periods. Huang and Huang (2012) and Berndt et al. (2018) show that the premium is substantially time varying. We confirm a remarkable time variation of the default risk premium. In particular, we highlight cross-sectional heterogeneity in the dynamics over time of the premium. We note that the premium is pro-cyclical, co-moves with market capitalization and is negatively correlated with CDS spreads for pro-cyclical firms. We document an opposite relationship between the premium and equity and CDS prices, however, for a considerable fraction of our sample, in particular those firms with a market value of assets that is negatively correlated with the business cycle. Moreover, we show that the term structure of the premium is generally upward-sloping for pro-cyclical firms and downward-sloping for defensive firms.

Finally, we build on the literature that relates asset and default risk premia. Drawing insights from a Merton model, Friewald et al. (2014) argue that the excess rate of return on the assets depends on both the risk-neutral and the real-world default probabilities. We explore the link between asset and default risk premium using our estimation results of a first-time passage model. We highlight that firms display negative asset risk premia and default risk premia lower than 1 when the market value of their assets is negatively correlated with the business cycle.

3. The Model

We define the firm i as an entity financed with equity, of market value $S_{i,t}$ at time t, and a zero-coupon bond with face value F_i maturing at time T. Let $V_{i,t}$ be the value of the i-th firm's assets and $Z_{i,t}$ its risky zero-coupon bond value at time t. The following condition then holds for every point in time t and for every firm i:

$$V_{i,t} = S_{i,t} + Z_{i,t}.$$

The value of the i-th firm's assets follows a geometric Brownian motion on the filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t : t \geq 0\}, \mathcal{P})$:

$$dV_{i,t} = \mu_{V_i} V_i dt + \sigma_{V_i} V_i dW_{i,t},$$

where μ_{V_i} and σ_{V_i} are the firm-specific $\mathcal{P}$-drift and volatility constant coefficients and $W_{i,t}$ is a standard Brownian motion under the physical probability measure $\mathcal{P}$.

We define the market value of leverage as $L_{i,t} = \ln\left(\frac{F_i}{V_{i,t}}\right)$, which follows an arithmetic Brownian motion:

$$dL_{i,t} = \mu_{L_i} dt - \sigma_{L_i} dW_{i,t}, \tag{1}$$

where

$$\mu_{L_i} = -\left(\mu_{V_i} - \frac{1}{2}\sigma_{V_i}^2\right) \tag{2}$$

is the $\mathcal{P}$-leverage drift coefficient, and $\sigma_{L_i} = \sigma_{V_i}$.

For pricing securities, we adopt a first-time passage framework, as in Black and Cox (1976) and Longstaff and Schwartz (1995), in which the firm defaults as soon as the assets value crosses from above an exogenous barrier, which is constant up to maturity. We denote the default barrier by C_i, with $C_i < F_i$; that is, the barrier lies below the face value of the debt. The default condition may represent bankruptcy covenants, which allow bondholders to trigger a default when the value of the corporate assets falls below pre-specified levels. Therefore, default may occur either any time τ before debt maturity T if the assets' value crosses below the default boundary, with probability

$$\begin{aligned} PD_{i,t}^{\mathcal{Q}}(\tau < T) &= \Phi\left(\frac{K_i + L_{i,t} - \left(r - \frac{1}{2}\sigma_{L_i}^2\right)(\tau - t)}{\sigma_{L_i}\sqrt{(\tau - t)}}\right) \\ &\quad + \exp\left((K_i + L_{i,t})\left(\frac{2r}{\sigma_{L_i}^2} - 1\right)\right)\Phi\left(\frac{(K_i + L_{i,t}) + \left(r - \frac{1}{2}\sigma_{L_i}^2\right)(\tau - t)}{\sigma_{L_i}\sqrt{(\tau - t)}}\right), \end{aligned} \tag{3}$$

or at the debt maturity T, if the firm is not able to pay back the outstanding debt F_i at time T even though the assets' value never crossed the default boundary before T. Then, the total probability of default is given by:

$$PD^{Q}_{i,t}(\tau \leq T) = 1 - \Phi\left(\frac{-L_{i,t} + \left(r - \frac{1}{2}\sigma^2_{L_i}\right)(\tau - t)}{\sigma_{L_i}\sqrt{(\tau_t)}}\right) \\ + \exp\left((K_i + L_{i,t})\left(\frac{2r}{\sigma^2_{L_i}} - 1\right)\right)\Phi\left(\frac{(2K_i + L_{i,t}) + \left(r - \frac{1}{2}\sigma^2_{L_i}\right)(\tau - t)}{\sigma_{L_i}\sqrt{(\tau - t)}}\right), \quad (4)$$

where Φ stands for the cumulative distribution function of a standard normal variable, and $K_i = \ln\left(\frac{C_i}{F_i}\right)$. Since the default boundary is below the face value of the debt, K_i is always negative. The larger the absolute value of K_i, the larger the distance between the face value of the debt and the default barrier is. We obtain (3) and (4) from the equations for the early and total bankruptcy risks of Perlich and Reisz (2007), respectively, and using $L_{i,t} = \ln(F_i/V_{i,t})$ and $K_i = \ln(C_i/F_i)$.

Similarly, we obtain from Perlich and Reisz (2007) the pricing equation for the equity value. The firm equity is equivalent to a down-and-out European call option written on the value of the firm's assets, with strike price F_i and maturity T. The equity value $S_{i,t}$ is given by:

$$S_{i,t} = \frac{F_i}{e^{L_{i,t}}}\Phi(d_1) - F_i e^{-r(T-t)}\Phi(d_1 - \sigma_{L_i}\sqrt{(T-t)}) - \frac{F_i}{e^{L_{i,t}}}\exp\left((K_i + L_{i,t})\left(\frac{2r}{\sigma^2_{L_i}} + 1\right)\right)\Phi\left(d_1^{C_i}\right) \\ + F_i e^{-r(T-t)}\exp\left((K_i + L_{i,t})\left(\frac{2r}{\sigma^2_{L_i}} - 1\right)\right)\Phi\left(d_1^{C_i} - \sigma_{L_i}\sqrt{(T-t)}\right), \quad (5)$$

where

$$d_1 = \frac{-L_{i,t} + \left(r + \frac{1}{2}\sigma^2_{L_i}\right)(T-t)}{\sigma_{L_i}\sqrt{(T-t)}}, d_1^{C_i} = \frac{(2K_i + L_{i,t}) + \left(r + \frac{1}{2}\sigma^2_{L_i}\right)(T-t)}{\sigma_{L_i}\sqrt{(T-t)}}.$$

4. Model Estimation

We now bring the model to the data. First, we formulate the model in state-space form to apply our estimation methodology (Section 4.1), which we then briefly describe (Section 4.2). We estimate the model using a non-linear Kalman filter in conjunction with maximum likelihood by combining information from the credit and equity markets. Specifically, we collect data on credit default swap (CDS) spreads to extract implied default probabilities as well as data on market capitalization to proxy equity value. We shortly describe the data (Section 4.3) and how we use the data to draw inference the model parameters and the unobservable variable. In particular, we explain how we extract probabilities of default from CDS spreads (Appendix A).

4.1. State-Space model

We use a state-space representation of the model to relate the observable data, such as equity value and probabilities of default, to the state variable — the market value of leverage — and the model parameters. As is typical in state-space models, we assume both equity value and probabilities of default are observed with noise:

$$\tilde{PD}^{\mathcal{Q}}_{i,t}(\tau) = g\left(L_{i,t}; \mu_{L_i}, \sigma_{L_i}, K_i, \tau\right) + \epsilon_{i,t}, \tag{6}$$

and

$$\tilde{S}_{i,t} = h\left(L_{i,t}; \mu_{L_i}, \sigma_{L_i}, K_i, F_i, \tau\right) + u_{i,t}, \tag{7}$$

where g and h are two non-linear functions. The structural model provides the functional forms of g and h, which are given by the pricing equations (3)-(4) and (5), respectively. More precisely, the non-linear function g is defined by (3) when $\tau < T$ and by (4) when $\tau = T$, respectively. Equations (3)-(4) describe the non-linear relationship between the probability of default and leverage, model parameters, and time-horizon; equation (5) describes the non-linear relationship between equity value and leverage, model parameters, and time-horizon. In practice, the observed default probability — the probability of default implied by the CDS spread — is equal to the 'true' default probability plus a random measurement error denoted by $\epsilon_{i,t}$. Similarly, the observed market capitalization is equal to the 'true' equity value plus a random observation error denoted by $u_{i,t}$. We allow the measurement errors for both the default probability and equity value to vary across firms. For each firm i, we assume that the measurement errors regarding the default probabilities vary across time horizons, follow a multivariate normal distribution with a zero mean and diagonal covariance matrix, and are homoskedastic with firm-specific variance R_i. We also assume that the observation error $u_{i,t}$ is normally distributed, with zero mean and variance ω_i.

In the state-space model, the firm's leverage, $L_{i,t}$ is the latent state variable, evolving over time according to the following transition equation:

$$L_{i,t+\delta t} = f(L_{i,t}) + \eta_{i,t+\delta t},$$

where f is a linear function, δt denotes the discrete time-step, and $\eta_{i,t+\delta t}$ is the transition error. We derive the discrete-time version of equation (1) to describe the dynamics of the state variable:

$$L_{i,t+\delta t} = L_{i,t} + \mu_{L_i}\delta t + \eta_{i,t+\delta t}, \tag{8}$$

where $\eta_{i,t+\delta t} = \sigma_{L_i}(W_{i,t+\delta t} - W_{i,t}) \backsim \mathcal{N}(0, Q_i)$, and $Q_i = \sigma^2_{L_i}\delta t$.

In the structural model, the leverage drift and volatility, as well as the default boundary, are unobservable. In addition, the model assumes a very simplified debt structure: the firm issues only a zero-coupon bond with face value F_i. In reality, the corporate debt structure is usually characterized by several different instruments and maturities. As a consequence, F_i is not observable from the balance-sheet data. Moreover, the variances of the measurement errors are unobservable. We denote by θ_i the vector of the firm-specific unobservable parameters that we target in our estimation:

$$\theta_i = \{\mu_{L_i}, \sigma_{L_i}, K_i, F_i, \omega_i, R_i\}.$$

The market value of leverage at each point in time is also unobservable. Then, $L_{i,t}$ and θ_i form the set of unknown quantities that we jointly estimate for each firm i.

4.2. Kalman filter and Quasi-Maximum Likelihood Estimation

We estimate the model parameters and the dynamics of the state variable using maximum likelihood in conjunction with a non-linear Kalman filter to account for the non-linearity of the relationships between the observable measurement variables and the latent-state variable. Here, we describe the Extended Kalman filter, which is the simpler and straightforward extension to deal with non-linearities when the non-linear function is continuous and differentiable. In Appendix D, we briefly describe the Unscented Kalman filter, which makes it possible to handle non-linear as well as discontinuous relationships between the observable measurement variables and the latent state variable. We run the Unscented Kalman filter to corroborate the estimation results obtained using the Extended Kalman filter. We report results on the Unscented estimation in table D.7.

4.2.1. Extended Kalman filter

For a Gaussian state-space model, under standard assumptions, the discrete Kalman filter is proven to be the minimum mean squared error estimator. However, in the case of a non-linear relationship between the measurement and the state variables, the classic linear Kalman filter is no longer optimal. One possible solution is to transform the non-linear measurement functions into linear equations by using the partial derivatives of the measurement functions with respect to the state variable around the current estimate of the state variable:

$$d_g = \frac{\partial g}{\partial L}(\hat{L}),$$

$$d_h = \frac{\partial h}{\partial L}(\hat{L}),$$

where d_g and d_h are the partial derivatives of the measurement functions g and h, respectively, with respect to the state variable, computed around the current estimate of the state $(\hat{L})$. We drop here the i subscript to lighten the notation.

The filter is initialized with arbitrary values for the state variable and the conditional state variance: $\{l_{t-1}, p_{t-1}\}$, with $t = 1$, where we use l_{t-1} and p_{t-1} to denote the priors for the state variable and variance. Then, we generate the prediction about the value of the state variable at t using the prior l_{t-1}, the state equation (8), holding the following expectation:

$$E[L_t] = \hat{l}_t = l_{t-1} + \mu_L \delta t,$$

where we use $\hat{l}_t$ to denote the expected value of the state variable at t based on information up to $t - 1$. We also form a prediction about the conditional state variance:

$$\hat{p}_t = p_{t-1} + Q.$$

We then generate a prediction about the value at t of the observable data using the predicted value of the state variable at t and the measurement functions g and h; that is, by computing equations (3), (4), (5) by using $\hat{l}_t$:

$$P\hat{D}_t^{\mathcal{Q}}(\tau) = g(\hat{l}_t, \tau), \tag{9}$$

$$\hat{S}_t = h(\hat{l}_t, T). \tag{10}$$

In practice, we compute the expectation of the measurement equations (6) and (7), respectively. We also compute the expected value of the observable data using the prior l_{t-1} in order to obtain numerical partial derivatives of the measurement functions with respect to the state variable:

$$\hat{d}_g = \frac{g(\hat{l}_t, \tau) - g(l_{t-1}, \tau)}{\hat{l}_t - l_{t-1}},$$

$$\hat{d}_h = \frac{h(\hat{l}_t, T) - h(l_{t-1}, T)}{\hat{l}_t - l_{t-1}},$$

and we collect derivatives in one vector, d.

We can now update the priors for the state variable and variance, using the new available information at time t. We observe the actual realizations of the data and compute the measurement errors:

$$e_{PD,t} = \tilde{PD}_t^{\mathcal{Q}}(\tau) - \hat{PD}_t^{\mathcal{Q}}(\tau),$$

$$e_{S,t} = \tilde{S}_t - \hat{S}_t.$$

We collect the measurement errors in one vector $\mathbf{e}_t$, which we then use to compute the posteriors for the state variable and variance, according to the following updating equations:

$$l_t = \hat{l}_t + J \cdot \mathbf{e},$$

$$J = \hat{p}_t \cdot d' \cdot (d \cdot \hat{p}_t \cdot d' + R)^{-1},$$

where R is the covariance matrix of the measurement errors and J is the Kalman gain, that is the weight assigned to the measurement errors in the updating of the prior for the state variable. The value of J depends on how reliable we consider the state prior and the actual observation of the data to be, where the (inverse of) reliability is expressed by the state and the measurement error variances, respectively. We also update the conditional state variance:

$$p_t = p_{t-1} \cdot (1 - J \cdot d).$$

Finally, we can use l_t and p_t as the priors for the next point in time and iterate the procedure over the entire time series. The model parameters that characterize the measurement and transition equations are estimated by maximum likelihood. At each point in time, the filter generates a vector of measurement errors $\mathbf{e}_t$ that depend on the unknown parameters collected in θ. By assuming independence and normality for $\{\mathbf{e}_t\}_{t=1}^{t^n}$, where t^n is the number of available data points, we can build a likelihood function and estimate the model parameters.

In summary, for each firm and each point in time, we generate a prediction of the default probabilities and equity value using the equations (9) and (10), respectively, for a given prior for the market value of leverage. The difference between the predicted and the observed values of the equity and default probabilities yields a set of measurement errors that is used to form a posterior about the market value of leverage. The posterior also becomes the prior for the state variable at the next point in time. The measurement errors are functions of the model parameters, which are estimated by maximum likelihood under the usual assumption of a Gaussian distribution. Thus, we reconstruct the time series of the market value of leverage for each firm in the sample and estimate the model parameters at the firm-level.

Next, we can finally pin down the dynamics of the value of the assets $V_{i,t}$ and the level of the default boundary C_i using the following equations:

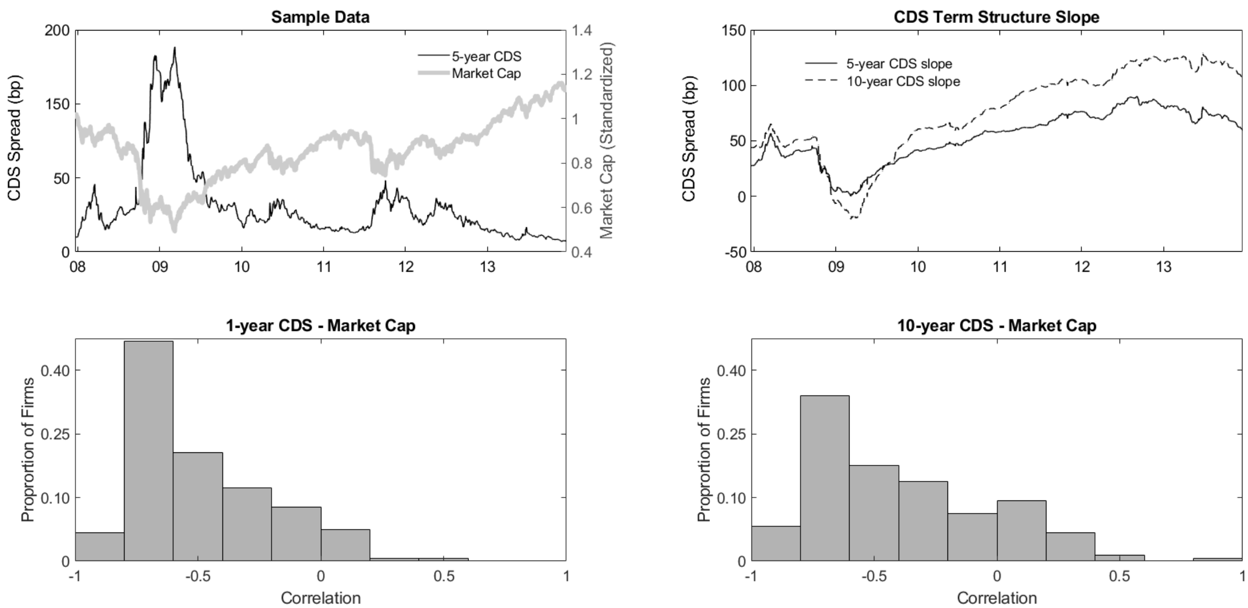

Figure 1. The Data: The figure summarizes daily data on market capitalization and CDS spreads between December 20, 2007, and December 19, 2013, for 164 firms. Data are obtained from Thomson Reuters. The top-left panel shows the average 5-year CDS spread (black line) and the standardized value of the market capitalization (gray line) across firms. CDS spreads are expressed in basis points. We compute the standardized value of the market capitalization for each firm as the ratio between the daily market capitalization and the value of the market capitalization on the first day of the time series, at the firm-level. The top-right panel shows the average 5-year (solid black line) and 10-year (dotted black line) CDS slopes across firms. We compute the daily 5(10)-year CDS slope as the difference between the daily 5(10)-year CDS spread and the 1-year CDS spread, at the firm-level. The bottom-left (right) panel shows the distribution of the firm-specific correlations between the 1(10)-year CDS spread and market capitalization.

$$V_{i,t} = \frac{F_i}{e^{L_{i,t}}}, \tag{11a}$$

$$C_i = e^{K_i} \cdot F_i. \tag{11b}$$

4.3. Data

We collect daily data on market capitalization (the product of the number of outstanding shares and the price) and CDS spreads for a sample of worldwide non-financial firms between December 2007 and December 2013 from Thomson Reuters. We collect data on

Summary Statistics					
	Mean	St.Dev.	p1	Median	p99
CDS(1Y)	55.13	80.59	4.97	31.00	430.92
CDS(3Y)	83.05	82.28	13.38	59.00	445.00
CDS(5Y)	109.07	84.38	23.60	85.08	458.96
CDS(10Y)	130.86	84.07	34.51	106.81	464.14
Market Cap	0.86	0.38	0.25	0.81	2.13
Equity Vol	0.36	0.10	0.18	0.35	0.97
CDS Slope(5Y)	53.94	43.28	-40.00	45.09	197.08
CDS Slope(10Y)	75.72	61.46	-75.74	67.79	260.47

Table 1. **Summary Statistics:** The table reports summary statistics of daily data on market capitalization and CDS spreads between December 20, 2007, and December 19, 2013, for 164 firms. Data are obtained from Thomson Reuters. We report the mean and standard deviation, the 1-st, 50-th, and 99-th percentiles for the 1-year, 3-year, 5-year, and 10-year CDS spreads, the standardized value of the market capitalization, and the 5-year and 10-year CDS slopes, respectively. We compute the standardized value of the market capitalization for each firm as the ratio between the daily market capitalization and the value of the market capitalization on the first day of the time series. We compute the equity return volatility (Equity Vol) as the standard deviation of the daily growth rate of the market capitalization at the firm-level and we report it in annual terms. We compute the daily 5(10)-year CDS slope as the difference between the daily 5(10)-year CDS spread and the 1-year CDS spread at the firm-level. CDS spreads and the CDS slope are expressed in basis points.

CDS traded on four different maturities: the 1-year, 3-year, 5-year, and 10-year, which are the most liquid on the CDS market. The availability of liquid data on the term structure of the CDS spreads is a key factor in successfully implementing our estimation methodology, because any different CDS maturity equips the state-space model with an additional and informative measurement equation. We provide details about CDS data in Appendix A, where we also explain how we extract probabilities of default from CDS spreads.

Our universe of firms is the set of reference entities listed on the Markit indexes, which include the most liquid companies in terms of CDS transactions. We refer to the iTraxx indexes for Australia, Japan and Europe, and the CDX North America - Investment Grade index for the United States.[3]

Next, we apply a set of filters to the initial sample. We use only reference entities with available data on CDS spreads from December 2007, the starting point of our time series. We exclude financial firms, which are characterized by a peculiar capital structure in terms of assets and liabilities. We also control for stale prices; that is, prices that do not change for at least five consecutive days, as in Friewald et al. (2014). Stale prices may create issues when we extract default probabilities from CDS spreads. In fact, a sufficiently large discrepancy in the number of active trading days across CDS traded on different maturities may produce non-monotone default probabilities for a given reference entity. Non-monotonicity appears when the default probability for the longer time horizon is lower than the default probability for the shorter time horizon, which is completely wrong.[4] We exclude companies with non-monotone default probabilities for more than 1% of observations for at least one maturity. In the end, our dataset contains 172 firms. We also remove 8 firms with no data on market capitalization. Our final dataset consists of 164 firms across four different regions: Australia (9), Japan (10), the United States (89), and Europe (56).

Finally, we use the sovereign bonds curve constructed by Bloomberg for Australia, Japan, the United States, and Europe to proxy the term structure of the risk-free rate. The European curve is the result of the aggregation of the triple-A sovereign bonds issued by France, Germany, and Holland. The inclusion of the best-rated government bonds from very safe and solid countries guarantees that we are truly considering rates of return on risk-less assets. This curve coincides with the Euro area yield curve computed exclusively on AAA-rated

[3]The Markit iTraxx and CDX indices are constructed every six months according to specified criteria and selection rules which determine the eligibility of an entity to be a constituent of the index. We refer to the list 20 for the iTraxx indexes and the list 21 for the CDX index. All the lists were issued in September 2013. We do not include firms from the CDX North America - High-Yield index because no such firms have liquid CDS data from the beginning of the time period covered in the analysis and continuing through the entire term structure of CDS considered in the paper.

[4]A CDS downward-sloping term structure may arise when the company is perceived to be riskier in the short term than in the long term. However, this curve must be not too steep.

central government bonds provided by Thomson Reuters.

We summarize the data in table 1 and figure 1. In the top panels of figure 1, we plot the dynamics of the average market capitalization across firms. In particular, we plot the standardized value of the market capitalization that we obtain by simply dividing, for each firm, the daily market capitalization by the value on the first day of the time series. We undertake this step to guarantee comparability across regions and to highlight the dynamics in the sample period. Market capitalization tumbles during the 2008-2009 crisis, drops again during the 2011 crisis, and recovers the initial value only at the end of the sample period. We also plot the average 5-year CDS spread, which displays an opposite pattern to that of market capitalization. The dynamics of the 1-year, 3-year, and 10-year CDS spreads are very similar to those of the 5-year CDS spread, which is usually considered the most liquid and traded maturity.

In the top-right plot, we document strong cyclicality in the slope of the CDS term structure: the difference between the longest tenor (10-year) and the shortest tenor (1-year) CDS spreads is, on average across firms, highly correlated with the average market value of equity. This result is likely due to the highly negative correlation between market capitalization and the short-term CDS spread. In the bottom panels, in which we plot the distribution of the firm-specific correlations between market capitalization and CDS spreads, we show that the negative correlation between market capitalization and CDS spreads is stronger for the short maturity CDS than for the longer maturity CDS. For comparability with Han et al. (2017), we compute the CDS term structure slope using the 5-year maturity as well; this exhibits a similar pattern to the 10-year slope, and we report an average close to Han et al. (2017) but with a smaller standard deviation.

5. Estimation Results

We combine the default probabilities extracted from the CDS spreads along with the market value of equity to compose the set of observable variables, thus combining information from the credit and equity markets to estimate the unobservable dynamics of the market value of leverage and the model parameters. Because we observe the CDS spreads over four time horizons (1-year, 3-year, 5-year, and 10-year), we have five measurement equations in the state-space model, at each time t and for each firm i: the market capitalization and the CDS-implied default probabilities over the four time horizons. In the state-space model, we use the default probabilities rather than the CDS spreads to improve tractability and to speed up the estimation process. However, the two approaches are ultimately equivalent due to the one-to-one mapping between the CDS spreads and the corresponding implied default probabilities (Appendix A).

We report here the main estimation results using the entire time series of data and the Extended Kalman filter. We begin this section with an assessment of the overall model fit (Section 5.1). We next present our estimation results, providing evidence about the relationship between the default boundary and volatility (Section 5.2) and the asset drift (Section 5.3). We finally focus on the asset risk premium and rationalize our estimates of the asset drift using Principal Component Analysis (Section 5.4).

In the Appendix, we report estimation results when splitting the time series into two separate time windows and when using the Unscented Kalman filter on the full data sample. In Appendix C, we show that our parameter estimates are generally stable across varying time periods (table C.6). In Appendix D, we show that our model estimation is robust to an alternative filtering technique, as the results obtained using the Unscented Kalman filter are equivalent to those obtained using the Extended Kalman filter.

5.1. Model Fit

We summarize the estimation results in table 2, in which we report summary statistics of the parameter estimates and provide a breakdown of the model fit. We offer graphical evidence of our estimation results and model fit in figures B.10 and B.11 (Appendix B).

We compute for each firm the correlation between the implied market value of the assets

Model Parameters

Parameter Estimates	Mean	St.Dev.	p1	Median	p99
Volatility	0.12	0.05	0.06	0.11	0.27
Drift(Lev)	-0.03	0.04	-0.18	-0.03	0.07
Drift(Ass)	0.04	0.05	-0.06	0.04	0.21
Barrier/Debt	0.76	0.07	0.51	0.78	0.88
Barrier/Assets	0.53	0.14	0.12	0.55	0.78

State Variable

Correlation with Implied Assets	Mean	St.Dev.	p1	Median	p99
CDS(1Y)	-0.60	0.24	-0.85	-0.62	0.28
CDS(3Y)	-0.60	0.23	-0.88	-0.63	0.19
CDS(5Y)	-0.55	0.28	-0.89	-0.59	0.48
CDS(10Y)	-0.30	0.40	-0.89	-0.35	0.65
Equity	0.92	0.10	0.42	0.96	0.99

Model Fit

Correlation with Implied Data	Mean	St.Dev.	p1	Median	p99
CDS(1Y)	0.60	0.30	-0.25	0.70	0.95
CDS(3Y)	0.67	0.26	-0.16	0.75	0.98
CDS(5Y)	0.66	0.25	-0.34	0.71	0.97
CDS(10Y)	0.54	0.36	-0.40	0.63	0.96
Equity	0.99	0.01	0.93	0.99	1

Table 2. Estimation Results: The table reports results from the structural model estimation, using daily data on market capitalization and CDS spreads between December 20, 2007, and December 19, 2013, for 164 firms. For each firm, we estimate the leverage volatility (σ_L) and drift (Drift(Lev) $= \mu_L$), the (log)-barrier-to-debt ratio ($K = \ln(C/F)$), and the face value of debt (F), using a non-linear Kalman filter in conjunction with maximum likelihood. We also compute the asset drift (Drift(Ass) $= \mu_V$) using equation (2). We report results about the default barrier in terms of the face value of debt ($\exp(K) = C/F$) where C is the nominal value of the default barrier, as well as in terms of the value of the assets (C/V). We compute the nominal value of the default barrier C and the value of the assets V using equations (11b) and (11a), respectively. Here in particular, we use the firm-specific average assets value over time. The top panel reports summary statistics of the model parameters estimates. In the middle panel, we report summary statistics of the firm-specific correlations between the observed data and the model-implied market value of the assets estimated with the non-linear Kalman filter. In the bottom panel, we report summary statistics of the firm-specific correlations between the observed data and the corresponding model-implied data. We obtain the model-implied data from equations (3), (4), (A.2) and (5), using the estimated value of the state variable and model parameters. We report the mean, the standard deviation, and the 1-st, 50-th, and 99-th percentiles.

and the observable data, such as the equity value and the CDS spreads. We obtain the implied market value of the assets from equation (11a), using our estimates of the state variable (the market value of leverage) and the estimated model parameters at the firm level. As expected, the correlation between implied assets and market capitalization is positive and close to 1, and the correlation between implied assets and CDS spreads is highly negative but weaker when considering longer maturities.

We report similar patterns for the correlations between actual data and model-implied data. For each firm and each point in time, we obtain the model-implied equity value and CDS spreads using our estimates of the state variable and the model parameters at the firm level. We document correlations close to 1 between actual and model-implied market capitalization, as well as correlations between actual and model-implied CDS spreads that are highly positive, though lower, for the longer time horizons.

5.1.1. Distance-to-Default

In addition, we compute a measure of the Distance-to-Default (DtD) using our estimates of the model parameters and state variable. The DtD expresses the distance between the current market value of the corporate assets and the default point. In particular, the DtD measures how many standard deviations the current market value of the assets is from the default point, which is the default boundary in the specific structural model adopted in this paper. We build on the standard DtD formula adopted in the Moody's KMV credit risk model, using our firm-specific estimates of the default boundary rather than the face value of debt:

$$DtD_{i,t}(\tau) = \frac{\ln\left(\frac{V_{i,t}}{C_i}\right) + \left(\mu_{V_i} - \frac{1}{2}\sigma_{V_i}^2\right)(\tau - t)}{\sigma_{V_i}\sqrt{\tau - t}}.$$

By using

$$K_i = \ln\left(\frac{C_i}{F_i}\right); L_{i,t} = \ln\left(\frac{F_i}{V_{i,t}}\right); \mu_{L_i} = -\left(\mu_{V_i} - \frac{1}{2}\sigma^2_{V_i}\right); \sigma_{L_i} = \sigma_{V_i},$$

we can express the DtD in terms of our model parameters and state variable as follows:

$$DtD_{i,t}(\tau) = \frac{-L_{i,t} - K_i - \mu_{L_i}(\tau - t)}{\sigma_{L_i}\sqrt{\tau - t}} \tag{12}$$

Thus, the DtD reflects information from both the credit and equity markets when using our estimates of the model parameters and state variable obtained from CDS spreads and market capitalization. Next, we test the relationship between the Distance-to-Default and the observable data using a simple regression analysis. We perform a panel regression of both CDS spreads and equity returns, defined as the growth rate of market capitalization, over the lagged DtD, which is the value of the DtD one day earlier. Controlling for both region and year effects, we find that the DtD negatively and significantly predicts both CDS spreads and equity returns, in line with the intuitive negative relationship between the DtD and both CDS spreads and expected equity returns. Indeed, a larger DtD signals that the current market value of the corporate assets is further from the default boundary and, therefore, the firm is more worthy and safer. As a consequence, we expect lower CDS spreads and lower equity returns. We report the results in table B.5 (Appendix B).

5.2. Default Boundary and Volatility

We document that the default barrier is equal to 76% of the face value of debt and 53% of the assets value on average. Our estimates of the default boundary are in line with those of Wong and Choi (2009), but lower than those of Brockman and Turtle (2003), which likely suffer from significant bias due the use of the book value of debt in the barrier estimation. Moreover, our default boundary estimates are slightly higher than those of Leland and Toft (1996), Perlich and Reisz (2007), and Davydenko (2012), who report mean barrier-to-debt ratios equal to 66%, a result likely due to the lower volatility that characterizes our sample

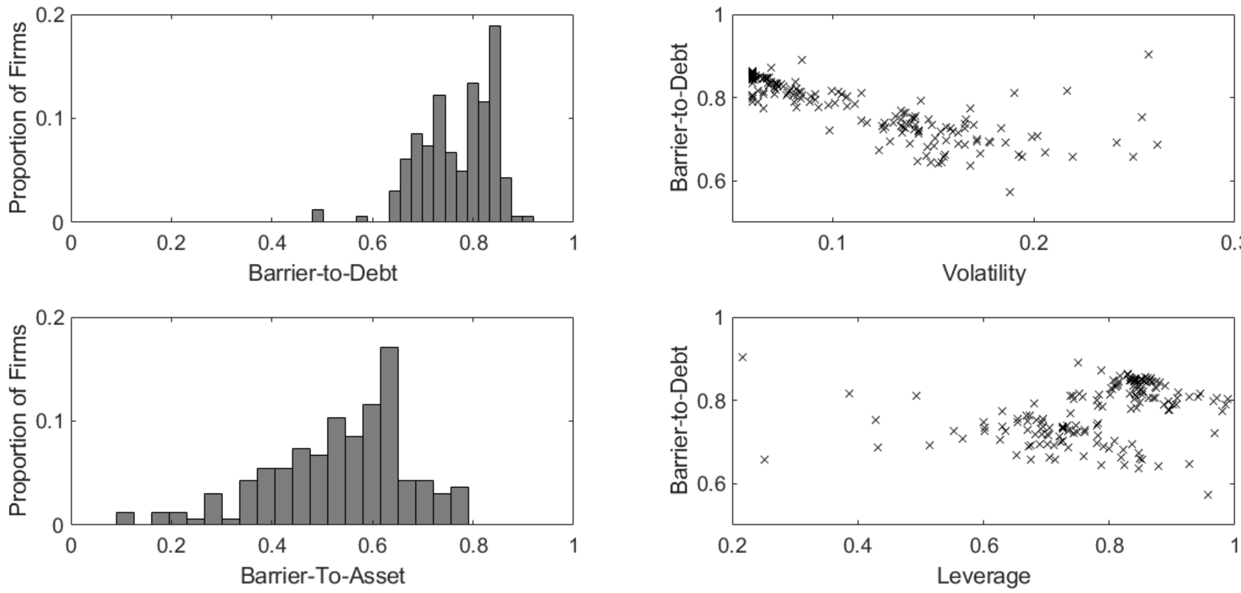

Figure 2. Default Boundary: The figure shows the distribution of the firm-specific barrier-to-debt ratios (top-left panel) and barrier-to-assets ratios (bottom-left panel). We compute the barrier-to-debt ratio as ($\exp(K) = C/F$) and the barrier-to-assets ratio as (C/V). We estimate K as well as the face value of debt F using the non-linear Kalman filter in conjunction with maximum likelihood, and we compute the nominal value of the default barrier C and the assets value V using equations (11b) and (11a), respectively. Here in particular, we use the firm-specific average assets value over time. In the top-right panel, we relate the barrier-to-debt ratio to the estimated firm-specific volatility (σ_L). In the bottom-right panel, we relate the barrier-to-debt ratio to the average leverage over time, for each firm. Leverage is defined as ($\exp(L)$), where $L = \ln(F/V)$ is the state variable in the state-space model (Section 4.1).

firms compared to those of Perlich and Reisz (2007). In fact, we estimate a mean annual volatility equal to 12%, while they report a mean annual volatility equal to 43%. However, Perlich and Reisz (2007) use a different approach to back out the implied volatility and include bankrupt firms in their sample. As a result, their point estimates of asset volatility span a larger interval of values.

In figure 2, we display the distribution of the default boundaries across firms, which are mostly concentrated between 60% and 80% of the face value of debt (top-left panel) and between 40% and 60% of the implied market value of the assets (bottom-left panel). As in Perlich and Reisz (2007), we document a clear negative relationship between default

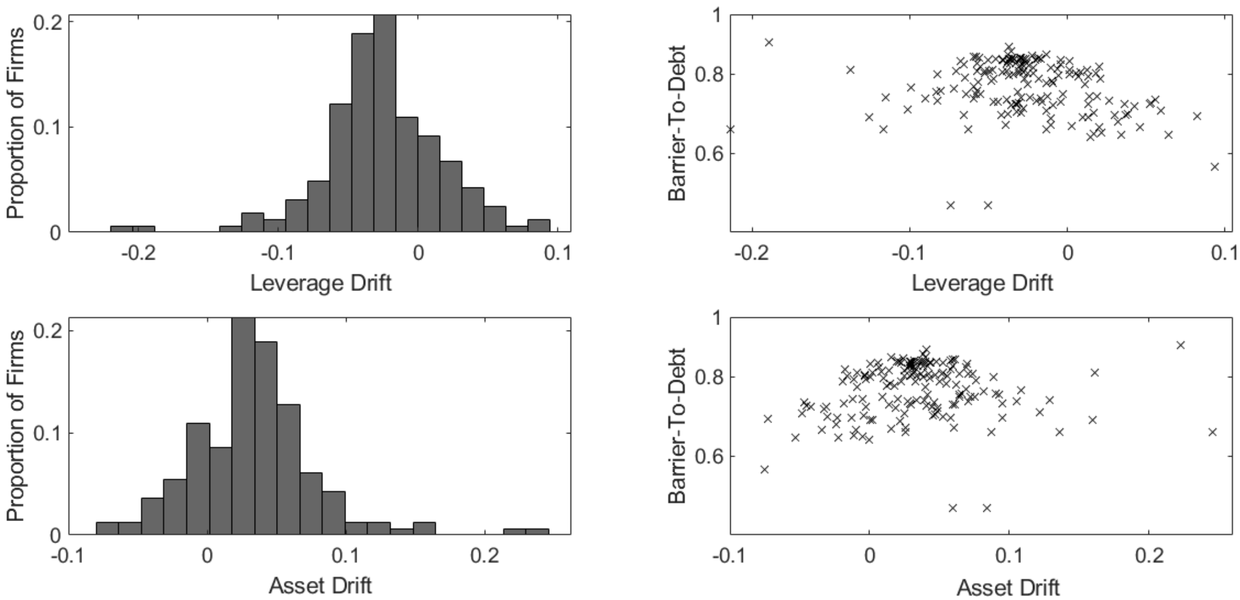

Figure 3. Drift: The figure shows the distribution of the firm-specific leverage drifts (μ_L, top-left panel) and asset drifts (μ_V, bottom-left panel) obtained from equation (2), using μ_L and σ_L estimated with the non-linear Kalman filter in conjunction with maximum likelihood. In the top (bottom)-right panel, we relate the firm-specific leverage (asset) drifts to the barrier-to-debt ratios. We compute the barrier-to-debt ratio as ($\exp(K) = C/F$). We estimate K and the face value of the debt F using the non-linear Kalman filter in conjunction with maximum likelihood, and we compute the nominal value of the default barrier C using equation (11b).

boundary and asset volatility (top-right panel). Despite using different approaches and datasets, our point estimates of the barrier-to-debt ratio are close to those of Perlich and Reisz (2007) within intervals of values of asset volatility. For instance, we estimate an average barrier equal to 82% of the face value of debt for firms with annual volatility lower than 10% and an average barrier equal to 71% of the debt for firms with annual volatility between 10% and 20%. Similar to Perlich and Reisz (2007), we also depict a positive relationship between the default boundary and the market value of leverage (bottom-right panel), which is intuitive because lenders require a stronger protection when the firm is more levered.

5.3. The Drift

We estimate on average an annual leverage drift equal to -3% and a corresponding annual asset drift equal to 4%. Our sample estimates of the firm-specific asset (leverage) drifts are mainly positive (negative), which is intuitive because the asset risk premium is typically positive for most of the firms. Moreover, when we estimate the model by splitting the time series into two time windows, we find that the asset (leverage) drift is generally higher (lower) in the second period (i.e., 2011-2013) than in the first period (i.e., up to the end of 2010). This result may help explain both the lower default risk priced in the CDS spreads and the increasing market capitalization — particularly from 2010 onward.

We chart the distribution of the firm-specific leverage and asset drifts in figure 3. We report a positive (negative) asset (leverage) drift for most of the firms. Specifically, we estimate a positive (negative) μ_{V_i} (μ_{L_i}) for 80% (76%) of our sample firms. In figure 3, we relate the leverage drifts and asset drifts to the firm-specific barrier-to-debt ratios. While we do not find any clear relationship between leverage drifts and default boundaries, we uncover a prevailing positive relationship between asset drifts and boundaries — after controlling for volatility, since $\mu_{V_i} = -(\mu_{L_i} - 0.5\sigma^2_{L_i})$.

Because the asset drift expresses the expected return on the assets of the i-th firm, we expect μ_{V_i} to be generally positive across firms and μ_{L_i} to be generally negative across firms. However, the proportion of firms displaying negative (positive) asset (leverage) drifts, and thus negative risk premia on the market value of their assets, is remarkable. Perlich and Reisz (2007) discuss a similar issue in their estimation results regarding physical asset drift, but prefer to adopt a proxy of the asset drift based on the approach of Huang and Huang (2012).

In this paper, however, we tackle this intriguing result thoroughly. First, unlike Perlich and Reisz (2007), we note that our estimates of the asset drift, when splitting the time series, are substantially stable across time periods. Second, we point out that a negative asset risk premium can be justified if the investment in the firm offers a hedging opportunity with respect to an aggregate risk factor, such as the business cycle, as is seen with negative beta stocks in standard market models. As a consequence, firms may exhibit negative asset risk premia when they represent potential hedging opportunities with respect to a systematic factor priced in the cross-section of the asset returns. We then proceed to empirically test this simple intuition.

5.4. Principal Component Analysis

We perform PCA to extract the main latent factors that drive the dynamics of the implied market value of leverage and assets of our sample firms. Because the implied market value of assets is simply a log-transformation of the market value of leverage, there is no difference between performing PCA on the assets value versus performing it on the leverage. We present our results in figure 4.

First, when decomposing the variance-covariance matrix of the implied market value of our sample firms' assets, we find that very few factors explain most of the total variability (top-right panel). More specifically, only the first three principal components (PC) are necessary to explain around 90% of the overall variability. We focus in particular on the first PC, which is the main latent factor in terms of explained variance (greater than 60%), and we study to what extent the first PC is related to an aggregate risk factor, such as the business cycle. We achieve this by collecting data on the US GDP released quarterly.[5]

PCA allows to depict how the overall variability in a panel of data is spread across different (independent) latent sources, but it does not in principle provide any clear economic insight. In figure 4 (top-left panel), we show that the first PC is highly correlated with the US GDP. As such, we can state that the main latent factor driving the dynamics of the implied market value of assets of our sample firms, which we pin down with our method,

[5]Data on GDP and other macroeconomic indicators are publicly available on the website of the Federal Reserve of St. Louis.

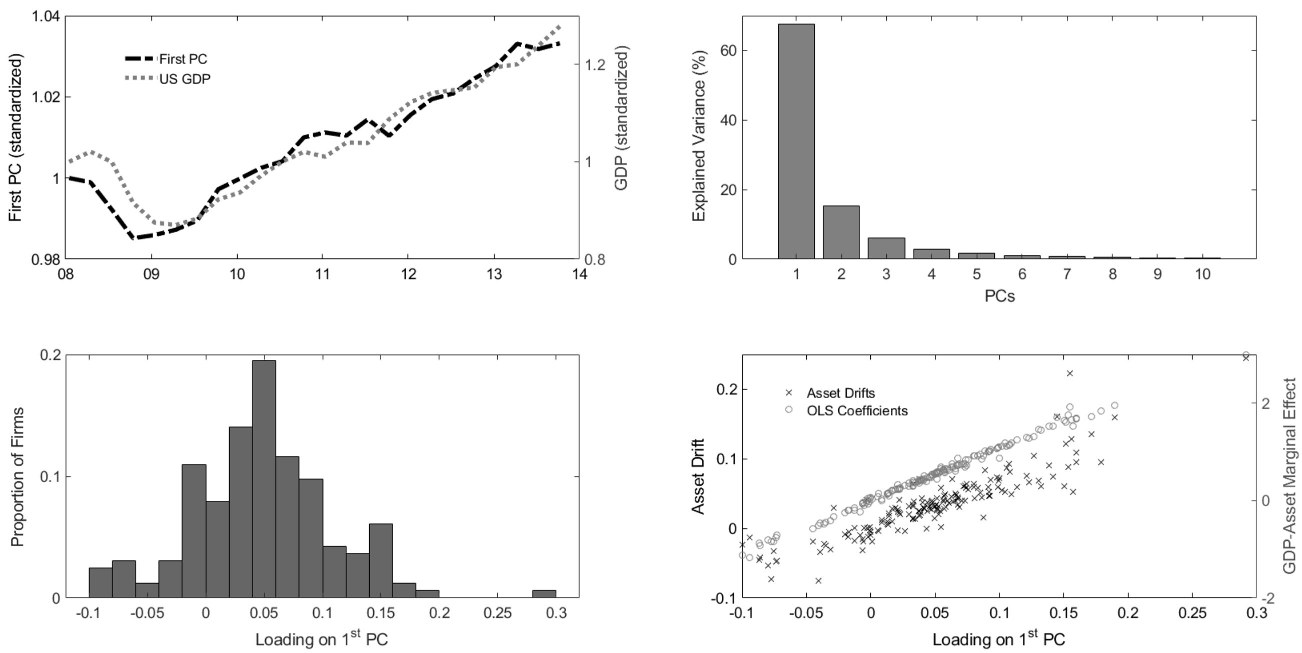

Figure 4. Principal Component Analysis: The figure displays results from Principal Component Analysis (PCA) of the variance-covariance matrix of the implied market value of assets across firms. We obtain the market value of assets from equation (11a) using the estimated firm-specific model parameters and state variable. In the top-left panel, we compare the first PC (black dashed line) with the US Gross Domestic Product (GDP) (gray dotted line) at quarterly intervals. We collect quarterly GDP data from the Federal Reserve of St. Louis between 2008 and 2013. In the plot, we standardize both variables using the value of the first PC and GDP on the first day of the time series. In the top-right panel, we report the eigenvalues associated to the first ten PCs in terms of proportion of explained variance. In the bottom-left panel, we plot the distribution of the firm-specific loadings of the implied market value of assets in the first PC. In the bottom-right panel, we relate these loadings to the firm-specific coefficients of the OLS regression of firm assets value over GDP at quarterly intervals (right axis, circle marker), as well as to the firm-specific asset drifts (μ_V, left axis, cross marker) obtained from equation (2) using μ_L and σ_L estimated with the non-linear Kalman filter in conjunction with maximum likelihood.

proxies the business cycle.[6]

[6]Because our sample firms are worldwide, we check whether the US GDP may work as a good proxy of the World GDP, by using annual data from the World Bank database. We find that the correlation between annual real World GDP and annual real US GDP is equal to 0.98 over the entire available time-series of data (1960-2020) and equal to 0.96 on the sample time-series. Hence, the US GDP well proxies the World GDP and thus the worldwide business cycle. Nevertheless, we use the US GDP in our empirical analysis because data about the US GDP are promptly released on a quarterly basis, whereas data about the World GDP are instead typically released with lower frequency and wider time lag.

Going one step further, we investigate the heterogeneity across firms. From PCA, we obtain firm-specific loadings in the first PC. The loading reveals how each firm-specific market value of assets is related to the first PC. The firm-specific loading in the first PC is in fact equivalent to a linear regression coefficient of the firm's assets value in the first PC. We plot the distribution of the firm-specific loadings and show that the vast majority of firms display positive correlation with the first PC (figure 4, bottom-left panel).

There is, however, a significant fraction of firms for which we obtain negative loadings in the first PC, revealing negative correlations with the business cycle. Importantly, we document a neat positive mapping between the firm-specific loadings in the first PC and the firm-specific drifts of the implied market value of assets (figure 4, bottom-right panel). In particular, firms presenting negative loadings in the first PC are those firms for which we report negative estimates of asset drift, and vice-versa. The economic insight of this result is simple but remarkable: firms that are negatively correlated with the business cycle, thus offering a hedging opportunity with respect to an aggregate risk factor, display negative asset risk premia. We corroborate this result by estimating a linear regression of the implied market value of assets at quarterly intervals over the GDP. We show that the firm-specific marginal effects of GDP on the implied market value of assets clearly correspond to the firm-specific loadings in the first PC, thus providing evidence that (i) the main latent common factor of the assets value dynamics is the business cycle and (ii) the assets value dynamics of firms that are negatively correlated with this common factor tend to be counter-cyclical. In figure 5, we plot the average implied assets value across firms with positive (dashed line) and negative (dotted line) loadings in the first PC against the GDP. We show that the implied assets value of firms with positive loadings in the first PC is highly correlated with the GDP, while the implied assets value of firms with negative loadings in the first PC displays the opposite pattern from the Great Financial Crisis onward.

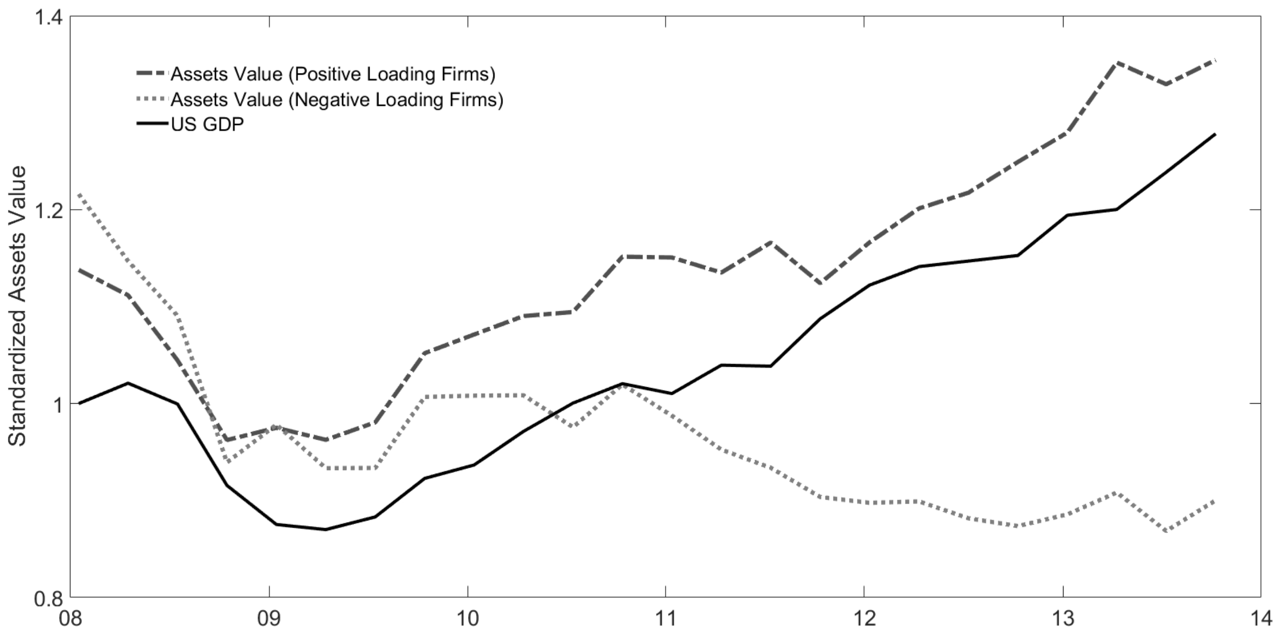

Figure 5. Implied Assets and GDP: The figure shows the average assets values at quarterly intervals across firms with positive (dashed line) and negative (dotted line) loadings in the first PC obtained from PCA on the variance-covariance matrix of the implied market values of assets. For each firm, we obtain the implied market value of assets from equation (11a) and using the state variable and model parameters estimated with the non-linear Kalman filter in conjunction with maximum likelihood. We also display the US GDP at quarterly frequency (solid line). Quarterly GDP data from 2008 and 2013 was collected from the Federal Reserve of St. Louis. We standardize all variables using the firm-specific values of the implied assets and of the US GDP on the first day of the time series, respectively.

6. Default Risk Premium

The default risk premium (DRP) is the wedge between the market valuation of default risk and the actual bankruptcy risk. The premium quantifies the compensation demanded by investors for bearing corporate default risk. Similar to Driessen (2005) and Berndt et al. (2018), we define the DRP as the ratio between the risk-neutral default probability and the real-world default probability. We compute the premium for each firm i, each point in time t, and each time-horizon τ as follows:

Summary Statistics

	Mean	St.Dev.	p1	Median	p99
DRP(1Y)	6.90	9.60	0.09	2.95	45.91
DRP(3Y)	8.61	13.49	0.10	2.93	65.61
DRP(5Y)	9.29	13.49	0.09	2.69	78.36
DRP(10Y)	8,77	16.09	0.10	2.07	82.91
Slope(5Y)	8,77	31.88	-1.32	-0.01	188.71
Slope(10Y)	7.91	63.91	-65.56	-0.21	295.86

DRP > 1 (**DRP** < 1)

	Corr-Equity	Corr-CDS	Loading	Drift(Lev)	Drift (Ass)
Mean	0.64 (-0.82)	-0.29 (0.56)	0.07 (-0.02)	-0.04 (0.03)	0.05 (-0.02)
Median	0.80 (-0.90)	-0.37 (0.65)	0.06 (-0.01)	-0.04 (0.02)	0.04 (-0.01)

Table 3. **Default Risk Premium**: The table reports results about the Default Risk Premium (DRP) computed for 164 firms between December 20, 2007, and December 19, 2013. We compute the DRP for each day t, each time horizon τ, and each firm i as $PD^{\mathcal{Q}}(\tau)/PD^{\mathcal{P}}(\tau)$. We obtain $PD^{\mathcal{Q}}(\tau)$ and $PD^{\mathcal{P}}(\tau)$ from equations (3)-(4), using our estimates of the model parameters and state variable, and then using μ_V to obtain $PD^{\mathcal{P}}(\tau)$ and the risk-free interest rate $r(\tau)$ to obtain $PD^{\mathcal{Q}}(\tau)$. We obtain μ_V from equation (2) using the estimated μ_L and σ_L. In the top panel, we report summary statistics of the DRP for different time horizons and the 5-year and 10-year slopes of the DRP term structure. We compute the daily 5(10)-year DRP slope as the difference between the daily 5(10)-year DRP and 1-year DRP at the firm-level. We report the mean and standard deviation, as well as the 1-st, 50-th, and 99-th percentiles. In the bottom panel, we report the mean and median of the firm-specific correlations between the 5-year DRP and market capitalization and between the 5-year DRP and 5-year CDS spread, and the mean and median of the firm-specific loadings in the first PC obtained from PCA on the variance-covariance matrix of the implied market value of assets, as well as the mean and median of the leverage and asset drifts (μ_L, μ_V), across firms with DRP > 1 and across firms with DRP < 1 (within brackets), respectively.

$$DRP_{i,t}(\tau) = \frac{PD^{\mathcal{Q}}_{i,t}(\tau)}{PD^{\mathcal{P}}_{i,t}(\tau)}, \tag{13}$$

where we obtain the risk-neutral default probability $PD^{\mathcal{Q}}$ and the real-world default probability $PD^{\mathcal{P}}$ from equations (3)-(4) using our model estimation results. Specifically, we plug in the equations (3)-(4) the market value of leverage, which we pin down with the Kalman filter, and the model parameters obtained by maximum likelihood estimation. The difference between the risk-neutral and the real-world default probabilities is given by the drift coefficient. We use the risk-free rate for the risk-neutral default probability and the firm-specific physical asset drift μ_V for the real-world default probability. We obtain μ_V from equation (2) using the estimated μ_L and σ_L at the firm-level. Finally, we compute the risk-neutral and real-world default probabilities over four time horizons denoted by τ, with $\tau = \{1, 3, 5, 10\}$-year. Consequently, we obtain the daily term structure of the default risk premium for each firm. For instance, $DRP_{i,t}(5)$ is the ratio between the probability that firm i defaults within 5 years, computed under the risk-neutral measure at time t, and the probability that firm i defaults within 5 years, computed under the real-world measure at time t.

We report that on average the DRP is higher than 1; this is expected when investors demand compensation for bearing default risk. We report summary statistics of our DRP estimates in table 3, in which we document similar numbers to those of Berndt et al. (2018) and Driessen (2005). We find that the risk-neutral default probability is higher than the real-world probability of default for around 74% of our sample firms (143 firms). This occurs when the expected return on the assets is higher than the risk-free rate (positive asset risk premium).

In figure 6 (top panels), we show that the premium varies substantially over time. In particular, we document that the DRP is pro-cyclical for firms displaying DRP higher than 1. Moreover, we plot the slope of the DRP term structure, which is the difference between the DRP computed for the longest time horizon (i.e., 10-year) and the DRP computed for the shortest time horizon (i.e., 1-year). The slope is generally positive and pro-cyclical for firms presenting DRP above 1 and negative and more volatile for firms presenting DRP below 1, thus signaling a downward-sloping curve and lower premium at the longer time horizon.

6.1. Expected Loss and Implied Spread

The DRP has an important and straightforward economic meaning in terms of corporate bond spread and price. We obtain the price q at time t of a risky zero-coupon bond with maturity T and face value equal to 1, issued by a firm i that defaults when the value of its assets crosses below a barrier $b_i < 1$, as the expected discounted payoff under the risk-neutral

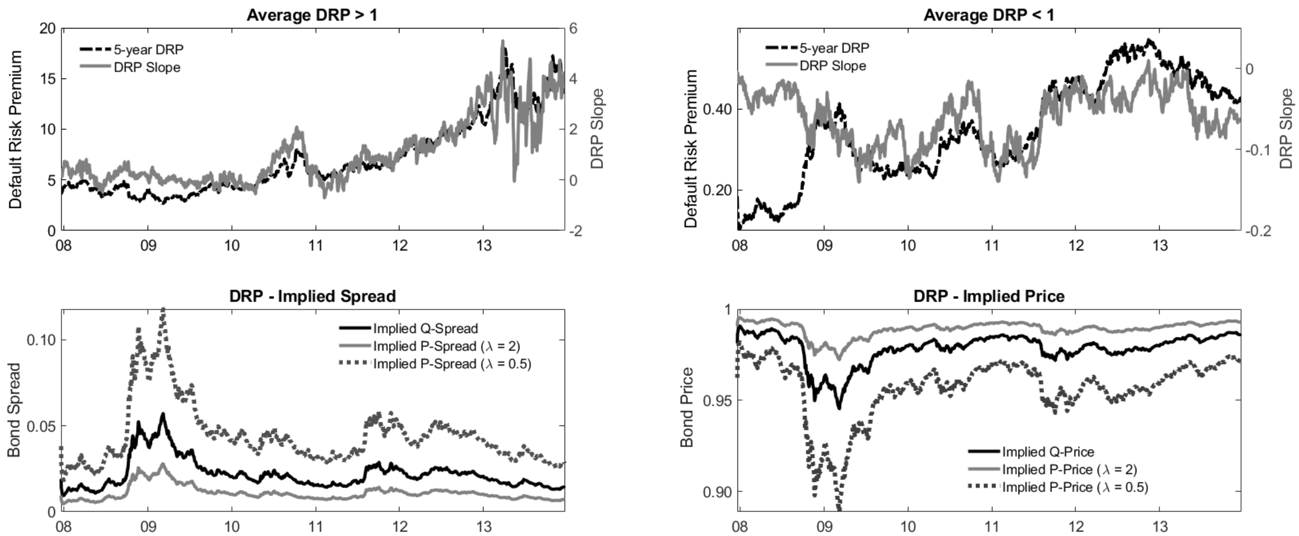

Figure 6. DRP and Implied Spreads: The figure displays results regarding the term structure of the Default Risk Premium (DRP) and the implied bond spreads. In the top-left (top-right) panel, we show the dynamics of the median 5-year DRP and DRP slope over the sample time series across firms with average DRP higher (lower) than 1. We plot the 5-year DRP using the black line and the slope using the gray line. We generate the slope for each firm and each point in time as the difference between the DRP computed at the longest (10-year) and the shortest (1-year) time horizons. In the bottom-left panel, we show the average implied bond spread across firms under the risk-neutral measure $\mathcal{Q}$ (black line) and the physical measure $\mathcal{P}$ when the DRP is either equal to 2 (solid gray line) or 0.5 (dotted gray line). In the bottom-right panel, we plot the corresponding implied bond prices. We obtain the firm-specific implied bond spreads and prices under $\mathcal{Q}$ from equation (15) using our model estimation results, and we obtain the implied bond spreads and prices under $\mathcal{P}$ from equation (16) using our model estimation results and using λ equal to either 2 or 0.5.

measure $\mathcal{Q}$:

$$q_{i,t} = \exp\left(-r_t(T-t)\right) \cdot E_t^{\mathcal{Q}}\left[(1 - I(t^* \leq T)) + b_i \cdot I(t^* \leq T)\right], \tag{14}$$

where t^* is the time of default, $I(t^* \leq T)$ is an indicator function with a value equal to 1 if the firm defaults, and zero otherwise, and b_i denotes the fraction of nominal debt value accruing to the bondholders if the firm defaults. We use the implication that the bondholders take over the firm as soon as the assets cross from above the barrier b_i and so collect a fraction of the debt value equal to the barrier. By computing the expectation, dropping subscripts, and assuming r_t=0 for simplicity without loss of generality, the bond price is:

$$q = \left[(1 - PD^{\mathcal{Q}}(T)) + b \cdot PD^{\mathcal{Q}}(T)\right] = 1 - PD^{\mathcal{Q}}(T) \cdot (1 - b).$$

In other words, the risky bond price is equal to the risk-less bond price minus the expected loss in case of default. Next, we compute the expected discounted payoff of the zero-coupon bond under the real-world measure $\mathcal{P}$:

$$p = \left[(1 - PD^{\mathcal{P}}(T)) + b \cdot PD^{\mathcal{P}}(T)\right]$$

Then, we can express the bond price q in terms of $PD^{\mathcal{P}}(T)$. Denoting by λ the T-default risk premium, that is the ratio between $PD^{\mathcal{Q}}(T)$ and $PD^{\mathcal{P}}(T)$, the bond price is:

$$q = 1 - \lambda PD^{\mathcal{P}}(T) \cdot (1 - b).$$

Thus, when λ is larger than 1, the market price of the risky bond is lower than the expected payoff computed under the real-world measure: the investor demands a premium in the form of the difference between the expected payoff and market price. As a straightforward consequence of this, we can compute the compensation demanded by the investor to bear the expected loss in case of default in terms of $PD^{\mathcal{P}}(T)$ and λ as follows:

$$1 - q = \lambda PD^{\mathcal{P}}(T) \cdot (1 - b) = EL^{\mathcal{Q}}.$$

Thus, the left-hand side is the compensation demanded by the investor in the form of a discounted price of the risky bond with respect to the risk-less bond. Therefore, $1 - q$ compensates, and is equal to, the expected loss in case of default, which is also equal to the expected loss evaluated using the real-world measure multiplied by a factor equal to the default risk premium:

$$EL^{\mathcal{Q}} = \lambda EL^{\mathcal{P}}.$$

When λ is larger than 1, $EL^{\mathcal{Q}}$ is greater than $EL^{\mathcal{P}}$, and the difference between $EL^{\mathcal{Q}}$ and $EL^{\mathcal{P}}$ is the extra return that the investors build in to compensate the risk they are bearing. For instance, when the default boundary is 0.75, the real-world default probability is 4%, and the default risk premium is 2, the investor prices at 0.98 a risky zero-coupon bond with nominal value 1 and expected payoff under the $\mathcal{P}$-measure equal to 0.99: the investor requires a compensation equal to 0.01 (100bps) in the form of a bond price discount with respect to the expected payoff under the $\mathcal{P}$-measure.

We now turn to the implied bond spread. We denote by $y_{i,t}^{\mathcal{Q}}$ the implied rate of return that prices at t the zero-coupon bond, issued by the i-th firm, with face value 1 and default boundary b_i under the risk-neutral probability measure:

$$q_{i,t} = \exp\left(-y_{i,t}^{\mathcal{Q}}(T-t)\right) = \exp\left(-r_t(T-t)\right) \cdot E_t^{\mathcal{Q}}\left[(1 - I(t^* \leq T)) + b_i \cdot I(t^* \leq T)\right].$$

By dividing both sides by the risk-less bond price and computing the expectation, we obtain the risky zero-coupon bond spread over the risk-less rate under the risk-neutral measure $\mathcal{Q}$ that solves the following equation:

$$\exp\left(-{y_{i,t}^{*}}^{\mathcal{Q}}(T-t)\right) = \left[(1 - PD_{i,t}^{\mathcal{Q}}(T)) + b_i \cdot PD_{i,t}^{\mathcal{Q}}(T)\right] = 1 - PD_{i,t}^{\mathcal{Q}}(T) \cdot (1 - b_i). \qquad (15)$$

We compute ${y_{i,t}^{*}}^{\mathcal{Q}}$, for each firm i and each day t using our model estimation results.[7] Similarly, we can obtain the risky zero-coupon bond spread over the risk-less rate and the

[7]Because we express the default boundary as a fraction of the debt value when defining the model parameter K_i, as a result b_i is simply equal to $\exp(K_i)$.

corresponding risky zero-coupon bond price under the real-world measure $\mathcal{P}$. Then, we can express ${y_{i,t}^*}^{\mathcal{P}}$ in terms of ${y_{i,t}^*}^{\mathcal{Q}}$ and the default risk premium:

$$
{y_{i,t}^*}^{\mathcal{P}} = -\ln\left(1 - \frac{1}{\lambda_{i,t}}(1 - \exp(-{y_{i,t}^*}^{\mathcal{Q}}))\right). \tag{16}
$$

We compute ${y_{i,t}^*}^{\mathcal{P}}$ and the price of the risky zero-coupon bond for each firm i and each day t using our firm-specific estimates of the default risk premium. We display our results in figure 6 (bottom panels). The average implied bond spread across firms under the risk-neutral measure $\mathcal{Q}$ is twice the corresponding $\mathcal{P}$-implied bond spread when the DRP is equal to 2, or one-half the corresponding $\mathcal{P}$-implied bond spread when the DRP is equal to 0.5. Similarly, we report the average implied bond price across firms under the risk-neutral measure $\mathcal{Q}$ and the real-world measure $\mathcal{P}$. We show that a DRP equal to 2 (0.5) maps to a bond $\mathcal{P}$-price around 3% (5%) higher (lower) than the bond $\mathcal{Q}$-price during the Great Financial Crisis and less than 1% (2%) higher (lower) in economic booms: investors valuate at 0.98 (0.90) a zero-coupon bond with a face value of 1 and market price of 0.95 during the crisis, when using the $\mathcal{P}$-measure and when the default risk premium is 2 (0.5).

Thus, for a given premium, the distance between the spreads and corresponding bond prices across the two different risk measures is more severe during crisis periods in which bond yields are higher due to higher default risk. Meanwhile, the DRP tends to be lower in crisis periods due to a substantial increase in the actual default risk measured by the $\mathcal{P}$-probability of default.

7. DRP and Asset Prices

The dynamics of the DRP over time is only driven by time variation of the state variable, that is the market value of leverage $L_{i,t}$. Therefore, the risk-neutral and the real-world probabilities of default in fact move in the same direction. However, as a ratio, the DRP may increase either due to the risk-neutral default probability (the numerator) increasing more than the real-world default probability (the denominator), or due to the real-world default probability decreasing more than the risk-neutral default probability. In the first case, we expect an increase in the CDS spread and a drop in the equity value due to a higher default risk. In the second case, instead, we expect a drop in the CDS spread and an increase in the equity value to a better firm credit condition. As a result, an increase in the DRP can be associated with opposing responses by CDS and equity prices.

Similarly, a decrease in the DRP can be associated with opposing dynamics in CDS and equity prices. The DRP may decrease either because the risk-neutral default probability increases but less than the real-world default probability, or because the real-world default probability decreases but less than the risk-neutral default probability. As before, we expect an increase in the CDS spread and a drop in the equity value in the first case, and a drop in the CDS spread and an increase in the equity value in the second case.

7.1. Numerical Analysis

In summary, the relationship between the premium and both the CDS spread and equity value may be either positive or negative. To better understand this relationship, we study numerically the sensitivity of the DRP to the market value of leverage, based on the facts that (i) the dynamics of the DRP is solely driven by time variation of the single state variable of our model and (ii) the model features a one-to-one mapping between the state variable and both equity value and the CDS spread.

In figure 7 we show that, *ceteris paribus*, the relationship between the default risk premium and the state variable depends on the difference between μ_V and r, and in particular on whether this difference is positive or negative.

The DRP decreases when the leverage increases if the DRP is higher than 1, and thus $\mu_V > r$. In this case, the DRP decreases because the risk-neutral default probability increases but less than its real-world counterpart. The DRP increases, instead, when the leverage increases, if the DRP is lower than 1 and thus $\mu_V < r$. In this case, the DRP increases because

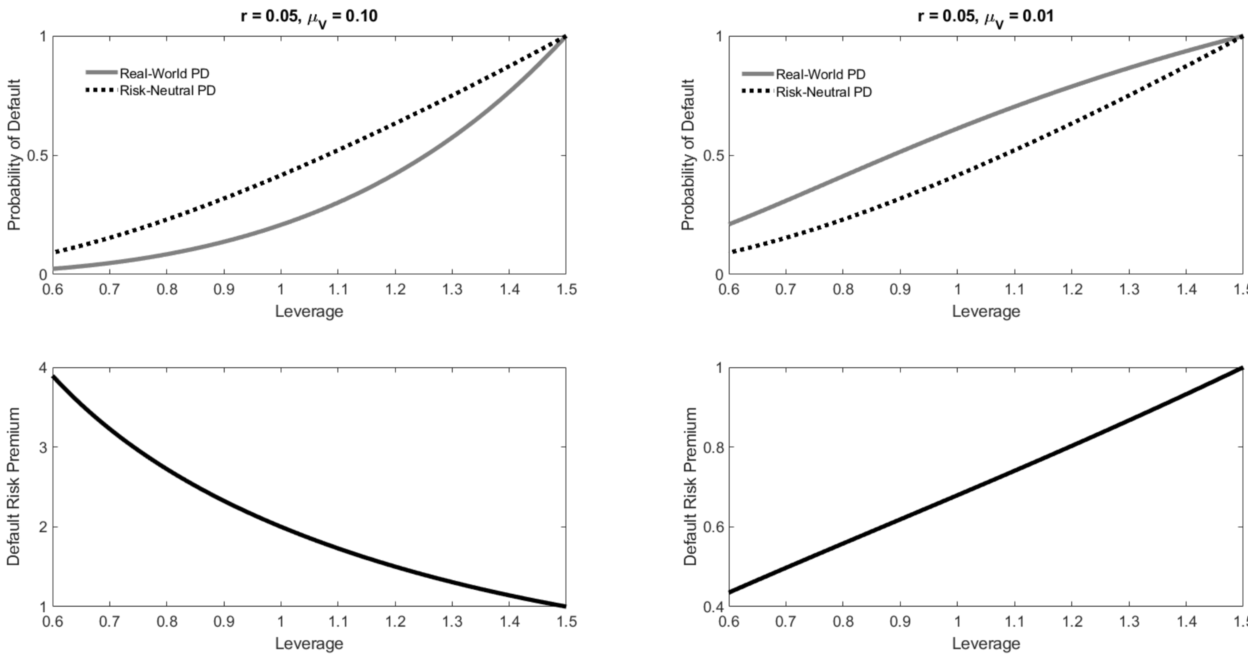

Figure 7. Numerical Analysis: The plots show the risk-neutral default probability (dotted black line), the real-world default probability (solid gray line), and the default risk premium (solid black line) when the market value of leverage takes on different values (x-axis) up to the default boundary. We express the leverage as ($\exp(L) = F/V$), where L is the state variable in the state-space model (Section 4.1). We compute the risk-neutral probability of default $PD^{\mathcal{Q}}$ using equation (4) and the risk-free rate r, and we compute the real-world probability of default $PD^{\mathcal{P}}$ using equation (4) and $\mu_L = -(\mu_V - 0.5\sigma_L^2)$. We then compute the ratio $PD^{\mathcal{Q}}/PD^{\mathcal{Q}}$ to obtain the DRP. The panels on the left display the results when $\mu_V = 0.10$ and the panels on the right display the results when $\mu_V = 0.01$. The value of the other model parameters are constant across the two cases and are as follows: $r = 0.05$, $\sigma_L = 0.2$, $K = -0.3$, τ=5-year.

the risk-neutral default probability increases more than the real-world default probability. In both cases, however, the CDS spread increases and the equity value drops. Therefore, the opposing DRP dynamics can be associated with equivalent responses by equity value and CDS spread, and vice-versa. Next, we study this prediction empirically.

7.2. Empirical Evidence

We test this simple prediction by computing the firm-specific sample correlations between the DRP and both equity value and CDS spread. We focus on the 5-year time horizon, which is usually the most liquid and traded maturity of the CDS. We rank the firms according to

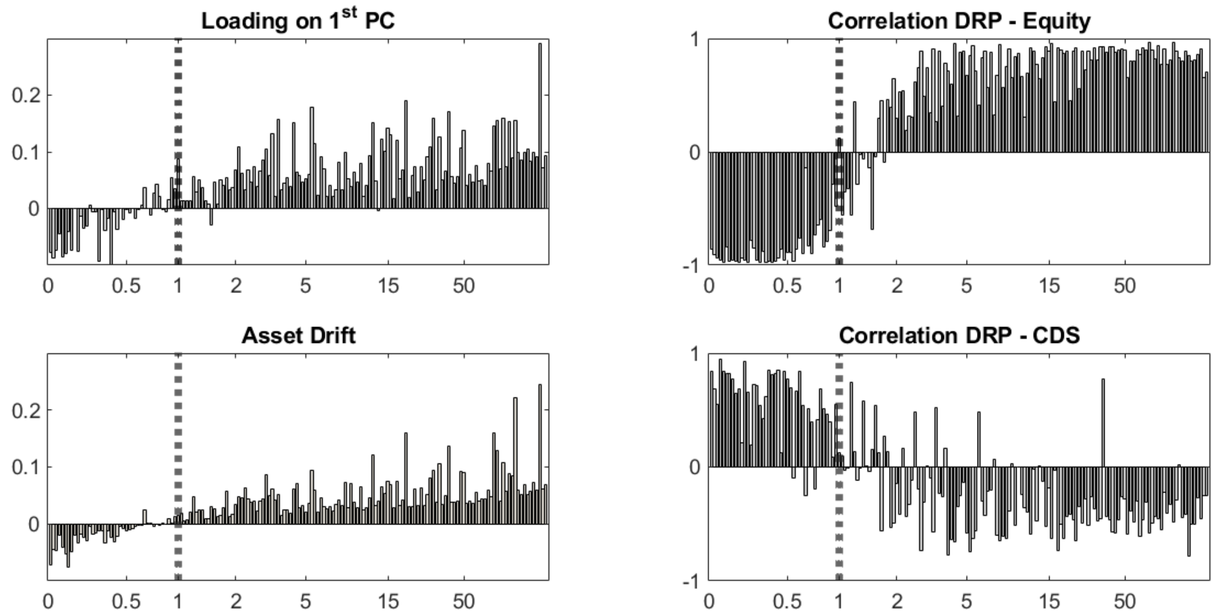

Figure 8. DRP Ranking: The figure shows the firm-specific loadings in the first PC obtained from PCA on the variance-covariance matrix of the implied market value of assets (top-left panel), correlations between daily default risk premium (DRP) and market capitalization over the sample time series (top-right panel), asset drifts μ_V obtained from $\mu_V = -(\mu_L - 0.5\sigma_L^2)$ using the estimated firm-specific μ_L and σ_L (bottom-left panel), and correlations between daily DRP and 5-year CDS spread over the sample time series (bottom-right panel). In the x-axis, we report the firm-specific average 5-year DRP over time. The firms are ranked in ascending order according to their DRP.

their average DRP over time and chart the correlations between the DRP and both equity value and CDS spread over the sample time series. We present our results in figure 8.

We show that the correlation between DRP and equity value switches from negative to positive when examining firms with average DRP over time higher than 1, and that the correlation between DRP and CDS spreads switches from positive to negative when looking at firms with average DRP over time higher than 1, with very few exceptions.

Overall, the correlation between DRP and market data generally flips when the average premium becomes greater than 1, in line with the theoretical relationship highlighted in the numerical analysis. When the DRP is below 1, in fact, an increase in the DRP is associated with a drop in the equity value and an increase in the CDS spread. When the DRP is greater than 1, meanwhile, an increase in the DRP is associated with an increase in the equity value and a drop in the CDS spread. In table 3, we show that the sample correlation between DRP and equity value is on average equal to 0.64 across firms with DRP higher than 1 and on average equal to -0.82 across firms with DRP lower than 1. The corresponding figures for the sample correlations between DRP and CDS spread are -0.29 and 0.56, respectively.

Importantly, we show in table 3 and figure 8 that firms with average DRP lower than 1 present negative asset risk premia and negative loadings in the first PC. Therefore, the counter-cyclical or defensive firms that offer hedging opportunities with respect to an aggregate risk factor, such as the business cycle, are those firms that pay out negative asset risk premia and display default premia below 1. Cyclical firms, on the other hand, pay out positive asset risk premia and display default premia above 1. For these firms, we predict a positive relationship between DRP and equity value and a negative relationship between DRP and CDS spread. In fact, in figure 6, we show that the DRP is pro-cyclical for firms with DRP higher than 1; that is, the DRP co-moves with the business cycle. When economic conditions later improve, we would expect a general reduction in corporate default risk and a higher valuation of the firm.

7.3. Sector Heterogeneity

We further investigate cross-sectional heterogeneity using sector classification. We adopt the usual Global Industry Classification Standard (GICS) and assign each firm to its corresponding sector. The complete list of sectors is sketched out in table 4. We are particularly interested in studying whether the link between asset and default risk premia is also found at the sector level, and to what extent the heterogeneity depicted across firms in terms of asset risk premium, relationship with the business cycle, and default risk premium is consistent with the traditional definition of cyclical and defensive sectors. To this end, we compute the average loading in the first PC obtained from PCA on the implied market value of assets for each sector. In addition, we compute the mean leverage drift, asset drift, and 5-year default

Sector	Loading	Lev Drift	Asset Drift	DRP	N
Energy	0.027	-0.009	0.013	0.745	9
Material	0.072	-0.033	0.042	8.552	17
Industrial	0.037	-0.017	0.024	5.839	28
Cons Services	0.080	-0.046	0.054	11.518	36
Cons Discretionary	0.073	-0.044	0.054	5.502	28
Real Estate	0.046	-0.044	0.048	35.977	8
Health Care	0.023	-0.015	0.021	1.437	12
Information	0.001	0.004	0.006	0.874	12
Technology	-0.006	0.013	-0.006	0.513	14

Table 4. Sector Heterogeneity: The table reports results regarding sector heterogeneity. We adopt the usual Global Industry Classification Standard (GICS) and assign each firm to its corresponding sector. For each sector, we compute the average loading in the first PC obtained from PCA on the variance-covariance matrix of the implied market value of assets, as well as the leverage drift, the asset drift, and the 5-year Default Risk Premium (DRP). We also report the number of firms that belong to each sector (N).

risk premium.

We report our results in table 4 and offer graphical evidence of our results as well in figure 9. The loadings in the first PC are generally positive, with the exception of the technology sector. In particular, the loading is around zero for the information sector and relatively small for the energy and health care sectors. Indeed, information, technology, energy, and health care are usually considered defensive or even counter-cyclical sectors due to their tendency to be weakly or even negatively correlated with the business cycle. In contrast, the loading is on average relatively large for the real estate, consumer (both discretionary and services) and material sectors, which typically co-move with the business cycle. In line with this result, we also find that the leverage drift is generally negative, with the exceptions of the information and technology sectors, and is relatively small in magnitude for the energy and health care sectors. The asset drift is on average negative for the technology sector only, substantially smaller for the information and energy sectors, and relatively smaller for health care compared to the cyclical sectors. Importantly, we find that the default risk premium is on average below 1 for defensive sectors and clearly above 1 for cyclical sectors. Overall, we

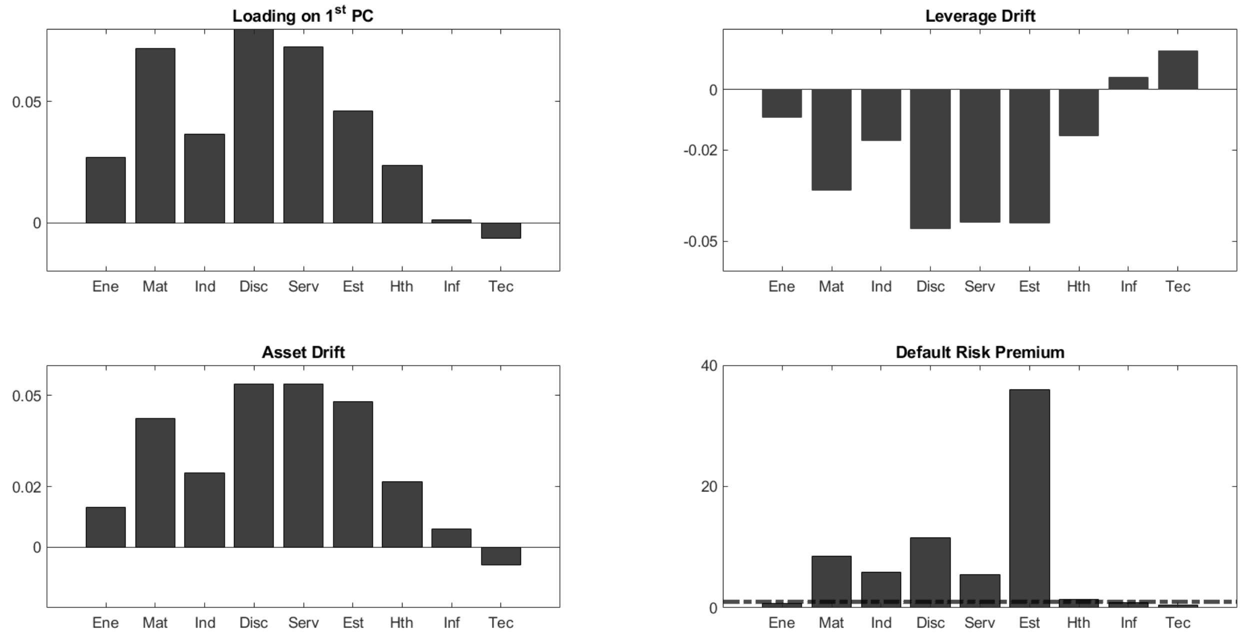

Figure 9. Sector Heterogeneity: The figure displays results regarding sector heterogeneity. We adopt the usual Global Industry Classification Standard (GICS) and assign each firm to the corresponding sector. The figure shows, for each sector, the average loading in the first PC obtained from PCA on the variance-covariance matrix of the implied market value of assets (top-left panel), the leverage drift (top-right panel), the asset drift (bottom-left panel), and the 5-year Default Risk Premium (DRP) (bottom-right panel). The dotted line in the bottom-right panel identifies a DRP equal to 1. The complete corresponding list of sectors is provided in table 4.

show that defensive sectors display a low or even negative correlation with the main common factor that is highly correlated with the business cycle, low asset risk premia, and low default risk premia. This evidence is consistent with the heterogeneity found across firms in terms of relationships with the business cycle and both asset and default risk premia, as highlighted in previous sections.

8. Conclusions

In this paper, we study the default risk premium through the lens of a simple structural model of credit risk. Using information gathered from the credit and equity markets, we infer the daily dynamics of the market value of assets and the default risk premium at the firm level. While we confirm that the premium on default risk is higher than 1 for the vast majority of firms, we show that a significant fraction of firms display a default premium lower than 1. Importantly, we highlight that these firms also display negative asset risk premia and negative correlations with the business cycle. We point out that this heterogeneity matters for the dynamics of the default premium over time and, in particular, for its cyclicality.

Our findings may spark additional investigations proceeding in various directions. First, we only focus on the main latent common factor in order to disentangle the heterogeneity across firms in terms of magnitude and time variation of the premium. However, other factors may exhibit significant co-movements with firm characteristics and economic forces, and so may help explain this heterogeneity in greater depth. Second, we estimate one single barrier for each firm that remains constant throughout the sample period, in line with the underlying theory model that we adopt in the paper. Estimating firm-specific time series of default boundaries may shed light on the relationship between time variation in the barriers and the dynamics of the premium. In addition, by further exploring the link between default and asset risk premia, future research may also deliver new insights about the controversial relationship between default risk and equity returns. Moreover, we believe that our estimation approach may contribute to the growing field of debt *nowcasting*. Nowcasting is an econometric approach that aims at predicting the present and the very near past and future states of key economic variables that are otherwise observed infrequently and with lags. Nowcasting exploits the relationships between different types of information that are released frequently and in a timely manner and an economic variable of interest, to infer (nowcast) the state of the latter.

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Appendix A. From CDS to PD

The CDS is a contract between two parties. The protection buyer purchases protection against the risk of a credit event of a given reference entity, such as a firm, from the protection seller. The protection buyer pays regularly a fixed premium each period until either the credit event occurs or the contract expires, while the protection seller is committed to compensating the buyer for the loss upon the event. The three main credit events that would trigger the settlement of the contract are bankruptcy, default on a payment, and debt restructuring. For simplicity, in this paper we refer only to default for simplicity. The details of a CDS transaction are recorded in the CDS contract, which is usually based on a standardized agreement prepared by the International Swaps and Derivatives Association (ISDA), an association of major market participants. As a priority rule, we select the contracts that adopt the no-restructuring (NR) clause, which has been the standard convention since the CDS Big Bang protocol of April 2009. Otherwise, we include contracts that adopt the modified-restructuring (MR) clause, which was the standard convention before the protocol. As a last resort, we include contracts with full-restructuring or modified-modified restructuring clauses.[8]

The default risk of a reference entity is then priced in the CDS spread, which reflects the amount that the protection buyer is willing to pay for buying insurance against the default of the reference entity. The CDS spread is expressed in basis points as an annualized percentage of the notional value of the transaction and is defined, at the contract's inception date, to equate the expected value of the two contractual legs. The protection buyer pays the CDS premium regularly until either maturity, if the firm does not default, or until the time of the default event. The protection seller, meanwhile, must pay back the protection buyer as soon as the firm defaults, which may occur at any time before maturity.

Then, by assuming the existence of a default-free money market account appreciating at

[8] See Longstaff et al. (2005) for an extensive description of the CDS contractual structure.

a continuous interest rate r, and M periodical payments occurring over one year, the CDS spread γ with time-to-maturity τ, priced at t, solves the following equation:

$$E_t^{\mathcal{Q}}\left[\sum_{m=1}^{M\cdot\tau}\exp\left(-r_t\frac{(m-t)}{M}\right)\frac{\gamma}{M}I_{(t^*>m)}\right]=E_t^{\mathcal{Q}}\left[\int_t^{\tau}\exp(-r(t^*-t))\alpha I_{(t^*=s<\tau)}ds\right],\qquad\text{(A.1)}$$

where t^* stands for the time of default, $I_{(t^*>m)}$ is an indicator function equal to 1 if the firm has not defaulted before the m-th payment and zero otherwise, $I_{(t^*=s<\tau)}$ is an indicator function equal to 1 if the firm defaults at time s before maturity τ and zero otherwise, α is the amount paid by the protection seller to the protection buyer in the case of default (i.e., the loss given default), and $E^{\mathcal{Q}}$ indicates that the expectation is taken under the risk-neutral measure $\mathcal{Q}$.

The observed CDS spreads are the breakeven spreads that equate the value of the premium leg and the value of the protection leg.[9] The value of both the premium and the protection legs, for a given risk-less interest rate and recovery rate, depend only on the default probability. Thus, there is a one-to-one mapping between CDS spread and default probability. In other words, there is one (risk-neutral) default probability implied by the CDS spread, which equates the starting value of the premium leg (protection buyer) and the protection leg (protection seller) in a CDS contract.

Therefore, for a given observed CDS spread γ, termination date, and risk-free rate term structure, we extract the implied probability of default for each firm i, for each day t, and for every time horizon τ that solves the following equation:

$$\sum_{m=1}^{M\cdot\tau}\exp\left(-r_t\frac{(m-t)}{M}\right)\frac{\gamma}{M}(1-PD_t^{\mathcal{Q}}(\tau))=\alpha\int_t^{\tau}\exp(-r(s-t))PD_t^{\mathcal{Q}}(\tau)ds,\qquad\text{(A.2)}$$

[9]See O'Kane and Turnbull (2003) for an extensive overview of CDS valuation.

which is simply obtained from equation (A.1) by applying the expectation operator to the indicator functions. The step from equation (A.1) to equation (A.2) is allowed by the maintained assumption about the size of the loss given default and, as consequence, of the recovery rate at default. This assumption allows, in fact, to build the one-to-one mapping between CDS and PD for a given day, firm, tenor and term-structure of the risk-free interest rate. This assumption is very standard in the credit risk literature, among both academics and practitioners. In making this assumption, we follow the usual conventions for the contractual recovery rate (i.e., recovery rate equal to 40%). In addition, while the model analyzes at which level the value of the assets must fall in order to trigger the default of the company, the post-default events, such as the legal and practical procedures implemented by the bondholders and creditors to recover at least part of their claim over the firm's assets, are out of the scope of the model and certainly beyond the scope of the paper. In other words, the model is useful to determine when the firm defaults- that is, which value the corporate assets must cross from above in order to trigger the default, but it is silent about what happens after the default of the company and finally what is exactly the fraction of the residual assets that the bondholders are able to recover. Therefore, while it is very common and typically used by several well-known applications of the credit risk models in the industry and in the academic research, this assumption is important to adopt this barrier-dependent structural model. Moreover, this assumption is also motivated by a large empirical evidence further than practical convenience.

Hence, given the maintained assumption about the recovery rate and the consequent one-to-one mapping between CDS and PD, we can use information from the CDS market, in conjunction with the information provided by the equity market, to pin down the firm fundamentals and the structural parameters. Specifically, we use data on CDS traded on four different time horizons: 1, 3, 5, and 10 years. Thus, we compute implied probabilities of default with corresponding time horizons τ equal to 1, 3, 5, and 10 years, respectively. In practice, the probability of default for the 1-year time-horizon is computed using the 1-year

CDS spread and the 1-year risk-less interest rate, the probability of default for the 3-year time-horizon is computed using the 3-year CDS spread and the 3-year risk-less interest rate, and so on. The probabilities of default are then related to the model parameters and the state variable in the state-space model using equations (3) and (4) in order to estimate the unknown model parameters and pin down the unobservable state variable. Specifically, we use equation (3) if $\tau < T$ and equation (4) if $\tau = T$, where τ is equal to either the 1-year, 3-year, 5-year, or 10-year, and T is equal to 10 years, which is the longest available maturity in our sample, which we assume to coincide with the maturity of the firm's zero-coupon bond.

Indeed, we acknowledge that the PD is a quantity computed from another variable (the CDS spread) and is not directly observed. However, the state-space model allows to observe the measurement variable with error and the measurement error is key in the identification and estimation of the unknown quantities of the model. In the body of the paper, we explicitly mention that we use the PD as measurement variable, which is observed with noise, and thus we can consider the "observed" PD, that is the PD implied by the CDS spread, to be the "true" PD plus an error. Therefore, the state-space model well accommodates this salient feature of our framework.

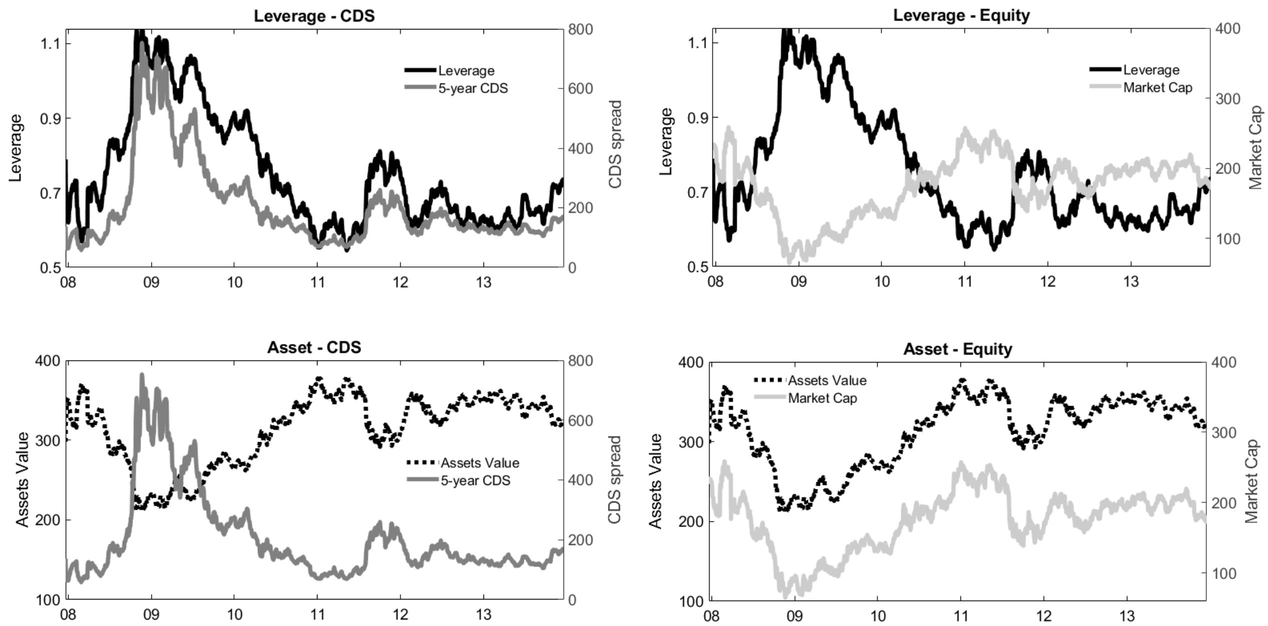

Figure B.10. State Variable: The figure compares market capitalization (light gray line) and 5-year CDS spread (dark gray line) with the market value of leverage (solid black line) and assets (dashed black line) estimated with the non-linear Kalman filter, between December 20, 2007, and December 19, 2013, for a representative firm, which is the US median firm at each day t. At each day t, we select the firm with the median value of implied market value of leverage and collect the corresponding market capitalization, 5-year CDS spread, and implied market value of assets and leverage. Leverage is expressed as $\exp(L)$, where $L = \ln(F/V)$ is the state variable in the state-space model (Section 4.1) and F is the face value of debt. We estimate L and F using the non-linear Kalman filter in conjunction with maximum likelihood, and V is computed using equation (11a). Equity and the value of the assets are expressed in thousands of US dollars and CDS spreads are expressed in basis points.

Appendix B. Model Fit. Additional Evidence

In figure B.10, we plot the dynamics of the market value of leverage and assets that we pin down with our method, and we compare them with the dynamics of the observed market capitalization and 5-year CDS spread. We show that the leverage sharply increases during the Great Recession and tumbles afterwards, increases again during the 2011 sovereign debt crisis, and finally drops. Moreover, the leverage is highly positively correlated with the CDS spread and highly negatively correlated with market capitalization. The opposite of these

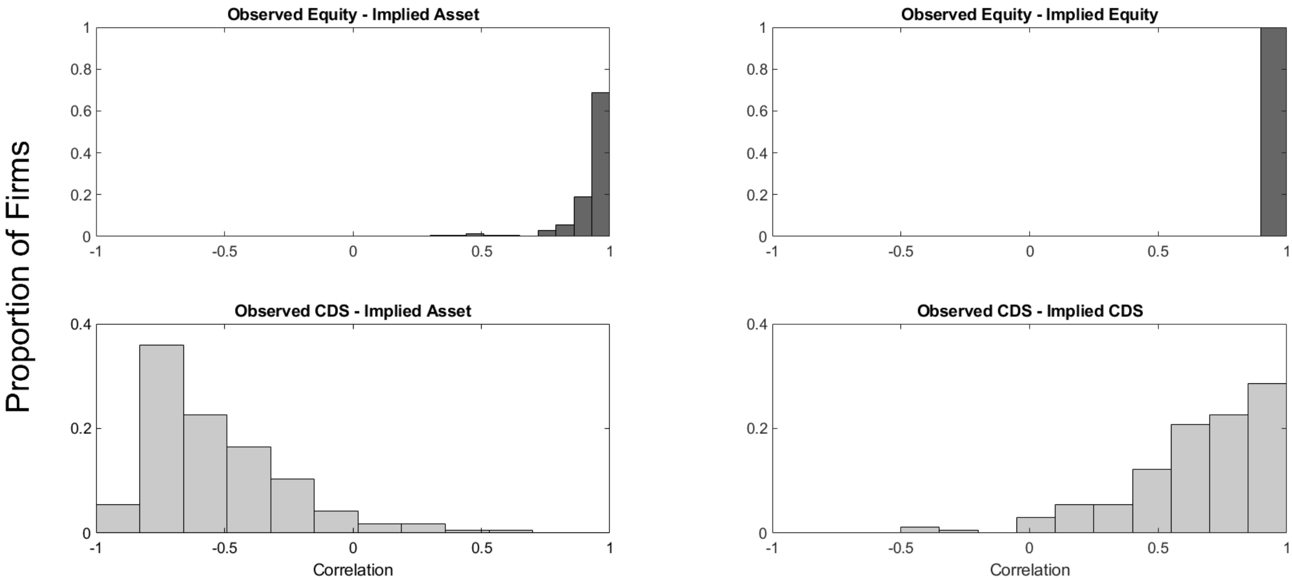

Figure B.11. Model-Implied Data: The figure compares market capitalization and the 5-year CDS spread with both model-implied data and the model-implied market value of assets estimated with the non-linear Kalman filter between December 20, 2007, and December 19, 2013. We obtain the model-implied data from equations (3), (4), (A.2), and (5) using the estimated value of the state variable and model parameters. In the figure, we show the distribution of the firm-specific correlations between model-implied assets and observed market capitalization (top-left panel) and CDS spread (bottom-left panel), between model-implied and observed market capitalization (top-right panel), and between model-implied and observed CDS spread (bottom-right panel).

observations applies to the dynamics of the assets value.

In figure B.11, we plot the distribution of the firm-specific correlations between the implied market value of assets and actual data as well as between model-implied data and actual data. The correlation between market capitalization and implied assets is very close to 1 for the vast majority of the firms, while the correlation between CDS spread and implied assets is highly negative for most of the firms. Moreover, our model estimates fit the equity data very well: the correlation between actual and model-implied equity value is around 1 for all of the sample firms. We generally document positive, though lower, correlations between the model-implied and actual CDS spread.

OLS Regression		
	(1)	(2)
	Equity return	CDS spreads
Constant	0.0028***	0.0299***
	(0.002)	(0.001)
DtD_{t-1}	-0.0003***	-0.0035***
	(0.000)	(0.000)
Region	YES	YES
Year	YES	YES
R^2	0.23	0.35
N	256660	256660

Summary Statistics

Mean	St. Dev.	p1	Median	p99
5.21	1.49	2.60	4.98	10.53

Table B.5. Distance-to-Default: The table reports results from the OLS regression with firm-day observations using daily data on market capitalization and CDS spreads between December 20, 2007, and December 19, 2013, for 164 firms. The dependent variable is the firm-day equity return (column (1)) and 5-year CDS spread (column (2)). The main independent variable is the one-day lagged Distance-to-Default (DtD_{t-1}). Equity return is computed for each firm as the growth rate of its market capitalization, CDS spreads are here expressed in percentage terms (basis points/100), and the DtD is computed from equation (12) using the estimated firm-specific model parameters and state variable. In both columns (1) and (2), we control for region-fixed effects and year-fixed effects. N is the number of observations (firms $\cdot$ days). In the bottom panel, we report summary statistics of the firm-day Distance-to-Default, including the mean and standard deviation as well as the 1-st, 50-th, and 99-th percentiles.

Appendix C. Estimation Results. Time Windows

Model Parameters: (First Period)

	Mean	St.Dev.	p1	Median	p99
Volatility	0.11	0.05	0.06	0.09	0.26
Drift(Lev)	-0.03	0.05	-0.30	-0.02	0.07
Drift(Ass)	0.04	0.06	-0.06	0.03	0.32
Barrier/Debt	0.77	0.07	0.51	0.79	0.88
Barrier/Assets	0.55	0.14	0.14	0.56	0.80

Model Parameters: (Second Period)

	Mean	St.Dev.	p1	Median	p99
Volatility	0.12	0.05	0.06	0.12	0.29
Drift(Lev)	-0.06	0.07	-0.42	-0.05	0.05
Drift(Ass)	0.08	0.08	-0.03	0.06	0.47
Barrier/Debt	0.75	0.08	0.50	0.74	0.87
Barrier/Assets	0.52	0.14	0.10	0.53	0.76

Table C.6. **Estimation Results. Sub-Samples**: The table reports results from the structural model estimation using daily data on market capitalization and CDS spreads for 164 firms. In the top panel, we report results from model estimation using data between December 20, 2007, and December 19, 2010. In the bottom panel, we report results from the model estimation using data between December 20, 2010, and December 19, 2013. For each firm, we estimate the leverage volatility (σ_L) and drift (Drift(Lev) $= \mu_L$), the (log)-barrier-to-debt ratio ($K = \ln(C/F)$), and the face value of debt (F) using a non-linear Kalman filter in conjunction with maximum likelihood, and we compute the asset drift (Drift(Ass) $= \mu_V$) using equation (2). We report results about the default barrier in terms of the face value of debt ($\exp(K) = C/F$), where C is the nominal value of the default barrier, as well as in terms of the value of assets (C/V). We compute the nominal value of the default barrier C and the assets value V using equations (11b) and (11a), respectively. In particular, we use here the firm-specific average assets value over time.

Appendix D. Unscented Kalman filter

The Unscented Kalman filter (UKF) is a non-linear filtering technique that permits the handling of measurement functions, and eventually transition functions, which are not only non-linear but also not differentiable. In fact, the UKF does not require any linearization of the measurement function through partial derivatives to retrieve a linear relationship between the observable and latent variables. Instead, the UKF applies the *unscented transformation* to both the state and measurement variables. The unscented transformation of the prior for the state variable is needed to compute the predicted value of the measurement variable using the non-linear measurement function, to which the *unscented transformation* is then applied to obtain the measurement error after observing the new available data. This error is then used to update the prior for the state variable at the next point in time, using the key Kalman gain. In general, for given priors for the state variable and variance x and p, respectively, and a non-linear measurement function g, the unscented transformation proceeds as follows:

$$x^U = x + \sqrt{c} \cdot [0 \quad \sqrt{p} \quad -\sqrt{p}]$$

$$y = g(x^U)$$

$$y^U = yw$$

$$F^U = yWy^{'}$$

$$Z^U = x^U W y^{'},$$

where the unscented transformation is given by the scale parameter c, the vector w_m, and the matrix W. Details on the unscented transformation can be found in **?**. We now describe the Unscented Kalman filter applied to our state-space model.

The filter is initialized with arbitrary values for the state variable and the conditional state variance: $\{l_{t-1}, p_{t-1}\}$, with $t = 1$, where we use l_{t-1} and p_{t-1} to denote the priors for the state variable and variance. We apply to l_{t-1} the unscented transformation:

$$l_{t-1}^U = l_{t-1} + \sqrt{c} \cdot [0 \quad \sqrt{p_{t-1}} \quad -\sqrt{p_{t-1}}],$$

and we form the prediction about the value of the state variable at t, using l_{t-1}^U, the state equation (8), and taking expectation:

$$E[L_t] = \hat{l}_t = l_{t-1}^U + \mu_L \delta t,$$

where we use $\hat{l}_t$ to denote the expected value of the state variable at t based on information up to $t-1$. We also form a prediction about the conditional state variance:

$$\hat{p}_t = l_{t-1}^U \cdot W \cdot (l_{t-1}^U)' + Q.$$

We next form a prediction about the value of the observable data at t using the predicted value of the state variable at t and the measurement functions g and h; that is, by computing equations (3), (4), (5) and using $\hat{l}_t$:

$$\hat{PD}_t^{\mathcal{Q}}(\tau) = g(\hat{l}_t, \tau), \tag{D.1}$$

$$\hat{S}_t = h(\hat{l}_t, T). \tag{D.2}$$

and stack $\hat{PD}_t^{\mathcal{Q}}(\tau)$ and $\hat{S}_t$ in one vector, Y. Then, we apply the unscented transformation to the predicted value of the measurement variables:

$$Y^U = Yw,$$

and compute the following matrices:

$$F^U = YWY^{'} + R,$$

$$Z^U = \hat{l}_t WY',$$

which we use to compute the Kalman gain:

$$J = Z^U (F^U)^{-1}.$$

We finally combine the Kalman gain and the measurement errors to update the priors for the state variable and variance:

$$l_t = \hat{l}_t + J \cdot \mathbf{e},$$

$$p_t = p_{t-1} - JF^U J',$$

where the vector $\mathbf{e}$ contains the measurement errors, which are obtained by taking the difference between the actual data $PD_t^{\mathcal{Q}}(\tau)$, S_t and the unscented transformation of the predicted value of the data collected in the vector Y^U. As we do in the Extended Kalman filter, we then use l_t and p_t as the priors for the next point in time and iterate the procedure over the entire time series. We report results in table D.7.

State Variable					
Correlation with Implied Assets	Mean	St.Dev.	p1	Median	p99
CDS(1Y)	-0.55	0.24	-0.85	-0.61	0.27
CDS(3Y)	-0.58	0.23	-0.87	-0.63	0.19
CDS(5Y)	-0.53	0.28	-0.88	-0.59	0.48
CDS(10Y)	-0.29	0.39	-0.89	-0.35	0.65
Equity	0.93	0.10	0.41	0.96	1
Model Fit					
Correlation with Implied Data	Mean	St.Dev.	p1	Median	p99
CDS(1Y)	0.60	0.30	-0.25	0.70	0.94
CDS(3Y)	0.67	0.26	-0.16	0.74	0.98
CDS(5Y)	0.66	0.25	-0.19	0.71	0.97
CDS(10Y)	0.54	0.36	-0.30	0.64	0.96
Equity	0.99	0.01	0.94	0.99	1

Table D.7. Unscented Kalman Filter: The table reports results from the Unscented Kalman filter (UKF) using daily data on market capitalization and CDS spreads between December 20, 2007, and December 19, 2013, for 164 firms. In the top panel, we report summary statistics regarding the correlation between the observed data and the model-implied market value of assets estimated with the UKF across firms. We obtain the model-implied market value of assets from equation (11a) using the state variable L estimated with the UKF. In the bottom panel, we report summary statistics regarding the correlation between the observed data and the model-implied data across firms. We obtain the model-implied data from equations (3), (4), (A.2), and (5) using the value of the state variable estimated using the UKF and model parameters. We report the mean and standard deviation, as well as the 1-st, 50-th, and 99-th percentiles.

Appendix E. Additional Results. Yearly Default Barriers

	Mean	St.Dev.	p1	Median	p99
2008	0.75	0.10	0.40	0.76	0.90
2009	0.71	0.08	0.42	0.73	0.89
2010	0.74	0.09	0.41	0.76	0.88
2011	0.77	0.10	0.40	0.79	0.90
2012	0.77	0.12	0.41	0.79	0.90
2013	0.73	0.18	0.41	0.78	0.91

Table E.8. Time-varying barriers: The table reports results on the default boundary estimation using daily data on equity value (Market Capitalization) and CDS spreads, for 164 firms. We estimate a firm-specific default boundary for each year of our sample time-series, by using daily data on equity value (Market Capitalization) and CDS spreads over each year from 2008 to 2013. We report results about the default barrier in terms of the face value of the debt (exp(K) = C/F). For each year, we report the mean, the standard deviation, the 1-st, 50-th, and 99-th percentiles of the firm-specific default boundaries across firms.

Online Appendix:

Alternative State-Space Model

In this Appendix, we present a different version of the state-space model that we describe in the main body of the paper and use to both perform the estimation of the structural parameters and pin down the dynamics of the latent state variable. We limit the presentation of this alternative formulation of the state-space model to the main differences with respect to the original version. We refer the reader to the section 4.1 for the remaining equations of the model and the other details which are common across the two versions.

We now use the CDS spread rather than the CDS-implied probability of default (PD) as observable variable and we assume to observe the CDS spread with noise. Then, the equation (6) becomes:

$$\tilde{\gamma}_{i,t}(\tau) = \gamma_{i,t}(\tau) + \epsilon_{i,t}(\tau), \tag{.1}$$

where $\gamma_{i,t}(\tau)$ is the CDS spread γ with time-to-maturity τ, priced at t, that solves the following equation:

$$\sum_{m=1}^{M\cdot\tau} \exp\left(-r_t \frac{(m-t)}{M}\right) \frac{\gamma}{M}(1 - PD^{\mathcal{Q}}_{i,t}(\tau)) = \alpha_i \int_t^\tau \exp(-r(s-t)) PD^{\mathcal{Q}}_{i,t}(\tau) ds, \tag{.2}$$

where r is the risk-free interest rate, M denotes the number of periodical payments occurring over one year, the default probability is given by

$$PD^{\mathcal{Q}}_{i,t}(\tau) = g\left(L_{i,t}; \mu_{L_i}, \sigma_{L_i}, K_i, \tau\right),$$

and α_i is the amount paid by the protection seller to the protection buyer in the case of default, which coincides with the loss in the case of default, that we set equal to $(1-\exp(K_i))$, under the assumption that the bondholders take over the value of the assets as soon as the firm defaults, when the assets value is equal to the default barrier. Moreover, we still include the equity value as additional observable variable, which we assume to observe with noise as described by equation (7).

We estimate the model to draw inference on the vector of the firm-specific unobservable parameters θ_i by using maximum likelihood in conjunction with a non-linear Kalman filter. For an extensive description of the estimation method, we remind the reader to the section 4.2. With respect to the version of the Kalman filter presented in section 4.2, we now need to modify the equation (9) to take into account that the measurement variable is the CDS spread. We generate a prediction about the value of the observable CDS spread, that we denote by $\hat{\gamma}^{\mathcal{Q}}_{i,t}(\tau)$, using the predicted value of the state variable at t and the equation (.3). Then, $\hat{\gamma}^{\mathcal{Q}}_{i,t}(\tau)$ is the CDS spread γ with time-to-maturity τ, priced at t, that solves the following equation:

$$\sum_{m=1}^{M\cdot\tau} \exp\left(-r_t \frac{(m-t)}{M}\right) \frac{\gamma}{M}(1 - \hat{PD}^{\mathcal{Q}}_{i,t}(\tau)) = \alpha_i \int_t^{\tau} \exp(-r(s-t)) \hat{PD}^{\mathcal{Q}}_{i,t}(\tau) ds, \qquad (.3)$$

where

$$\hat{PD}^{\mathcal{Q}}_{i,t}(\tau) = g(\hat{l}_{i,t}, \tau),$$

.

To update the priors for the state variable and variance, we compute the measurement error after the realization of the actual data, as follows:

$$e_{CDS,i,t} = \tilde{\gamma}^{\mathcal{Q}}_{i,t}(\tau) - \hat{\gamma}^{\mathcal{Q}}_{i,t}(\tau),$$

where $\tilde{\gamma}^{\mathcal{Q}}_{i,t}(\tau)$ is the CDS spread, with time-to-maturity τ, observed at time t for the i-th firm. The Kalman filter and the maximum likelihood algorithm then proceed as usual. We report the model estimation results in table A9.

Model Parameters

Parameter Estimates	Mean	St.Dev.	p1	Median	p99
Volatility	0.12	0.05	0.06	0.12	0.30
Drift(Lev)	-0.03	0.04	-0.16	-0.03	0.08
Drift(Ass)	0.04	0.05	-0.07	0.04	0.21
Barrier/Debt	0.72	0.08	0.50	0.73	0.85
Barrier/Assets	0.52	0.14	0.07	0.53	0.82

State Variable

Correlation with Implied Assets	Mean	St.Dev.	p1	Median	p99
CDS(1Y)	-0.60	0.25	-0.85	-0.61	0.32
CDS(3Y)	-0.59	0.24	-0.86	-0.60	0.25
CDS(5Y)	-0.55	0.29	-0.89	-0.57	0.50
CDS(10Y)	-0.33	0.40	-0.89	-0.32	0.67
Equity	0.92	0.11	0.35	0.96	0.99

Model Fit

Correlation with Implied Data	Mean	St.Dev.	p1	Median	p99
CDS(1Y)	0.60	0.32	-0.26	0.60	0.95
CDS(3Y)	0.66	0.29	-0.13	0.67	0.96
CDS(5Y)	0.62	0.29	-0.36	0.63	0.95
CDS(10Y)	0.57	0.37	-0.50	0.59	0.94
Equity	0.99	0.01	0.91	0.99	1

Table A9. Estimation Results: The table reports results from the structural model estimation, using daily data on market capitalization and CDS spreads between December 20, 2007, and December 19, 2013, for 164 firms. For each firm, we estimate the leverage volatility (σ_L) and drift (Drift(Lev) $= \mu_L$), the (log)-barrier-to-debt ratio ($K = \ln(C/F)$), and the face value of debt (F), using a non-linear Kalman filter in conjunction with maximum likelihood applied to the state-space model described in the Online Appendix. We also compute the asset drift (Drift(Ass) $= \mu_V$) using equation (2). We report results about the default barrier in terms of the face value of debt ($\exp(K) = C/F$) where C is the nominal value of the default barrier, as well as in terms of the value of the assets (C/V). We compute the nominal value of the default barrier C and the value of the assets V using equations (11b) and (11a), respectively. Here in particular, we use the firm-specific average assets value over time. The top panel reports summary statistics of the model parameters estimates. In the middle panel, we report summary statistics of the firm-specific correlations between the observed data and the model-implied market value of the assets estimated with the non-linear Kalman filter. In the bottom panel, we report summary statistics of the firm-specific correlations between the observed data and the corresponding model-implied data. We obtain the model-implied data from equations (3), (4), (A.2) and (5), using the estimated value of the state variable and model parameters. We report the mean, the standard deviation, and the 1-st, 50-th, and 99-th percentiles.